Structural Steelwork Eurocodes Development of A Trans-national Approach

CONTENTS

Structural Steelwork Eurocodes

Development of a Trans-National Approach

Course: Eurocode 4

Lecture 1: Introduction to composite construction of buildings

Summary:

- This lecture contains an introduction to the design and application of composite structures. Also the advantages and fields of use for composite structures are shown.
- Composite construction is popular for buildings and bridges as well because of the following aspects:
  - Economy
  - Architecture
  - Functionality
  - Service and building flexibility
  - Assembly
- Some examples of existing buildings and construction methods are listed.

Pre-requisites:

- None

Notes for Tutors:

- This material comprises one 60 minute lecture
- The tutor should update autonomy national changes of the Eurocode
- All additional modifications of the training packages are under responsibility of the tutor

Objectives:

- To give an overview of possibilities and examples of composite structures illustrated by examples of existing structures.

References:

- [1] Gerald Huber: Non linear calculations of composite sections and semi-continuous joints Doctoral Thesis, Verlag Ernst & Sohn December1999
- [2] ECCS 83 European Convention for Constructional Steelwork: Design Guide for Slim Floors with Built-in Beams
- [3] ECCS 87 European Convention for Constructional Steelwork: Design Manual for composite Slabs
- [4] Tschemmernegg F: Lecture Mixed Building Technology in Buildings, University of Innsbruck
- [5] Helmut Bode: Euro Verbundbau, Konstruktion und Berechnung, 1998
- [6] EN 1994-1-1Draft No.2: Design of composite steel and concrete structures, 2001
- [7] COST C1 Control of the semi–rigid behaviour of civil engineering structural connections Proceedings of the international conference, Liége, 17 to 19 September 1998
- [8] Teaching Modules of A-MBT,1999/2000: Application Centre Mixed Building Technology , Innsbruck -Austria,
- [9] ESDEP: Europäisches Stahlbau - Lehrprogramm CD-Rom, 1996

Contents:

1 General

The most important and frequently encountered combination of construction materials is that of **steel and concrete** applied for buildings as well as bridges [1]. Although very different in nature, these two materials complement one another:

- Concrete is efficient in compression and steel in tension.
- Steel components are relatively thin and prone to buckling, concrete can restrain these against buckling.
- Concrete also gives protection against corrosion provides thermal insulation at high temperatures.
- Steel brings ductility into the structure

The design of structures for buildings and bridges is mainly concerned with the provision and support of horizontal surfaces. In buildings, the floors are usually made of concrete, reinforced by steel to resist tension. As spans increase though, it is cheaper to support the slab, for example by beams, rather than to thicken the slab. In building structures the grid of beams is in turn supported by columns. Both the beams and columns can be conveniently constructed using structural steel sections, normally hot-rolled I- and H- shapes respectively. It used to be customary to design the bare steelwork to carry all the loads, but since the 1950´s it has become increasingly common to connect the concrete slabs to the supporting beams by mechanical devices. These eliminate, or at least reduce, slip at the steel-concrete interface, so that the slab and the steel beam section act together as a composite unit, commonly termed a "composite beam"(Figure 1) [7].

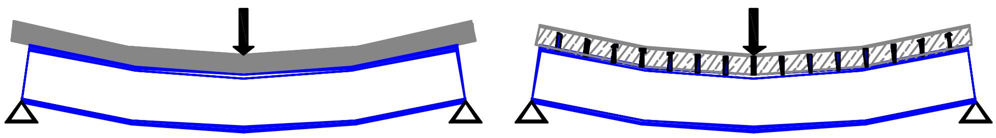


Figure 1 Non – composite and composite beams

In practice, interconnection is achieved by headed studs or other connectors which are welded or shot-fired to the structural steel and enclosed of concrete [8].

Composite members as conventional composite beams, composite columns and steel-deck composite slabs have been in use for a considerable number of years. Simplifying assumptions for the interaction between the concrete slab and the steel beam helped to establish composite construction as an easy to handle extension of the bare steel construction. As the application of this technology proved its efficiency, large-scale research projects were started world-wide with the aim of further improvement.

One research avenue concentrated on the interaction between the steel beam and the concrete slab. Obviously the advantages of a beam acting compositely are the higher stiffness and higher load resistance in comparison with its non-composite counterpart. In a first simplifying step the interaction has been assumed to be infinitely stiff preventing any slip between the two construction elements. As usual the most economic way is not the extreme one but a middle course (not to mention that a fully-rigid interaction strictly cannot be realised in practice, it would require at least a very large number of shear connectors and hence high costs). In contrast, completely ignoring the strengthening effect of the concrete slab by a clear separation to the steel beam results in high material costs, again resulting in a lack of economy. Today numerous studies based on tests and numerical simulations provide a solid background to understand the so-called *incomplete interaction* between the steel and concrete sections within a composite beam.

2 Aspects of use of composite structures

A universal way of design involves not only the optimisation of upper load resistance (strength), stiffness and ductility, but also inclusion of architectural, economical, manufactural and utilisation aspects of beams, slabs and columns.

2.1 Architectural

Designing composite structures offers a lot of architectural variations to combine different types of composite elements.

In addition to reductions in the dimensions of the beams

- longer spans
- thinner slabs
- more slender columns

offer flexibility and more generous opportunities for design.

2.2 Economical

Enormous potential of cost saving results from smaller dimensions (higher stiffness results in less deflections, longer spans and less overall height) and quicker erection.
The advantageous ratio of span width to height (l/h=35) can be beneficial:

- Reduction of height reduces the total height of the building
  Saving area of cladding
- Longer spans with the same height (compared to other methods of construction)
  -->Column free rooms can be used more flexibly
- Additional storeys with the same total height of building

Composite structures are easy to erect and have quicker times of erection
--> saving of costs, earlier completion of the building,
-->lower financing costs
-->ready for use earlier thus increasing rental income

2.3 Functionality

Conventional steel structures use applied fire protection systems to insulate the steel from the heat of the fire. Modern steel and composite structures can provide fire resistance by using principles of reinforced concrete structures in which the concrete protects the steel because of its high mass and relatively low thermal conductivity.

Just as composite slabs may resist fire, also composite beams can be used with unprotected flanges, but then the space between the flanges has to be filled with concrete and additional reinforcement. This not only maintains relatively low temperatures in the web and upper flange, but also provides flexural strength, compensating for the reduction contribution from the hot lower flange.

2.4 Service and building flexibility

Composite structures are adaptable. They may readily be modified during the life of the building. This is especially true when the slab is used with framed structures. It is then always possible to create a new staircase between two floors by simply adding the necessary trimmer beams.

Recent developments and changes in communications, information and computing technology have shown the importance of being able to modify quickly the building´s service arrangements. Furthermore, in commercially let buildings or in multi-shared properties it has to be possible to modify the services without violating the privacy of the other occupants. In order to solve this problem, engineers have to choose between several solutions. There are generally three alternatives for accommodating the services:

- in the ceiling
- within a false floor
- in a coffer box running along the walls

The gap between the soffit and the bottom flange of a composite beam constitutes an ideal zone in which services can be located.

2.5 Assembly

Composite floors are now the preferred approach for a wide range of structures, offering the designer and clients the following advantages:

- **Working platform.**

Before concreting, the decking provides an excellent safe working platform which speeds the construction process for other trades.

- **Permanent shuttering:**

The steel deck spans from beam to beam, forming permanent formwork to the concrete, the need of temporary props is often not necessary. The decking constitutes a good vapour barrier.
The soffit remains clean after concreting and the use of colour-coated steel sheets can give an attractive aesthetical aspect to the ceiling although painting can cause problems with through-deck stud welding.

- **Steel reinforcement:**

The steel reinforcement provided by the cross-section of the deck itself is usually sufficient to resist sagging moments. Fabric reinforcement may be provided in the slab to resist shrinkage or temperature movements or to provide continuity over intermediate supports. (hogging moments). Composite action is obtained by the profiled shape or by mechanical means provided by indentation or embossment of the steel profile.

- **Speed and simplicity of construction:**

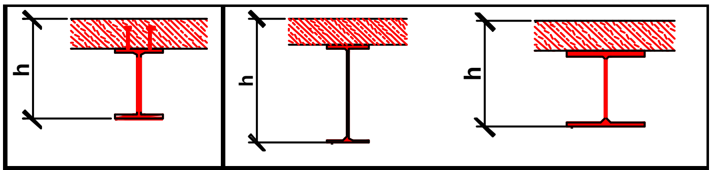
The unique properties of the steel deck combining high rigidity and low weight, ease considerably the transportation and the storage of the material on site. Often one lorry is capable of carrying up to 1500 m^2 of flooring. A team of four men can set up to 400 m^2 of decking per day. Panels are light, pre-fabricated elements that are easily transported and set in place by two or three men.

- **Quality controlled products:**

Steel components of composite structures are manufactured under factory controlled conditions. This allows the establishment of strict quality procedures and less random work on the construction site. This results in a greater accuracy of construction.

2.6 Comparison with other methods

It is necessary to apply composite elements in the design to profit from the available advantages and to use synergy effects. Thus composite structures have a higher stiffness and load capacity with the same dimensions compared to bare steel.



	Composite beam	Steel beam without any shear connection	
Steel cross section	IPE 400	IPE 550	HE 360 B
Construction height [mm]	560	710	520
Load capacity	100%	100%	100%
Steel weight	100%	159%	214%
Construction height	100%	127%	93%
Stiffness	100%	72%	46%

Table 1 Composite beam- steel beam [8]

In Table 1 a composite beam is compared with two types of steel beams without any shear connection to the concrete slab. The load capacity is nearly the same but a difference in stiffness and construction height is shown.

Generally the cross section dimensions of composite structures are much less than reinforced concrete or a bare steel framework .
Table 2 for example compares the sizes of quite large composite columns and beams with reinforced concrete counterparts under the same loading conditions.

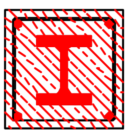
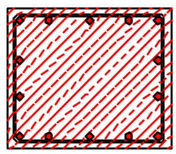
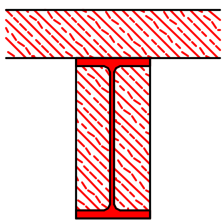
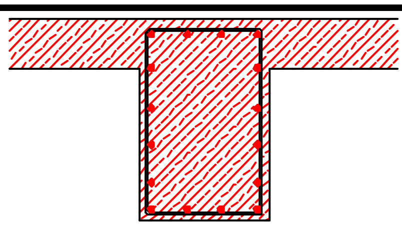
	Composite	Reinforced concrete
Column		
Dimensions [cm]	70 / 70	80 / 120
Beam		
Dimensions [cm]	160 / 40	160 / 120

Table 2 Comparison of composite structures–reinforced concrete [8]

3 Construction methods

Traditionally two counteracting methods of construction could be observed both connected with special advantages but also disadvantages worth mentioning.

- **Conventional concrete construction method** shows up very well considering styling, freedom of form and shapes, easy to handle onsite, thermal resistance, sound insulation and resistance against chemical attacks. In contrast to this it behaves poorly in view of the ratio between resistance and dead load, time-consuming shuttering and extension of the construction time due to hardening of concrete. Furthermore, as concrete itself is unable to take tensile forces reinforcement has to be provided which again is very time-consuming.

The primary advantage of

- **Construction in steel** is the high ratio between bearing capacity and weight. As the fabrication can be done in advance independently of the weather the erection is very simple with small tolerances. Fire resistance of bare steel constructions may be problematic. It can only be solved by using more of material or by cost-intensive preventive measures. Ultimately also the need for higher educated personnel has to be mentioned as a disadvantage of the steel construction.

So comparing these two methods a combination of both presents itself as the most economic way. More than only picking out the advantages of each method even new advantages can be gained. So for example, in composite construction higher bearing capacities can be achieved than in steel and concrete. But also stiffness and plastic redistribution can be improved by combining steel with concrete. On the one hand this enables advantage to be taken of the plastic reserves of the system and on the other hand to reduce safety factors due to the good-natured inherent ductility of the failure modes.

Speaking about *composite construction* in the following it should be mentioned that in many cases actually *mixed building technology* is the most efficient solution. Strictly **composite** only means the interaction of two materials **within one construction element** (e.g. a concrete-filled tubular steel column) whereas the philosophy of **mixed building technology** includes the **combination of construction elements or members** built up with different construction methods (e.g. concrete column in combination with a composite beam and a prefabricated slab).

See also clause 4: Definitions and terminology

Building up a composite structure in a very economic way can be divided into the following operations:

- First of all a conventional skeleton structure in steel, braced or unbraced, will be erected. If hollow steel sections are used for the columns the reinforcement cages already can be positioned in the shop.

- Also all brackets, finplates and vertical shear studs (non-headed bolts or shot-fired nails) for the load transfer between the steel and the concrete encasement have to be prepared in the shop to speed up the erection on site requiring a detailed planning stage. After arranging the columns the bare steel beams are simply hinged in between.

- Prefabricated concrete elements or profiled steel sheeting are spanning from beam to beam, serving both as shuttering and as a working platform.

Finally, by concreting the slabs and the columns in one process the stiffness and resistance of the columns and beams increases and the joints are automatically transformed from hinges to semi-continuous restraints.

3.1 Construction elements

Figure 2 shows the principles of the composite construction method. Slabs spanning between a grid of beams which are supported by the columns. So the floor itself consists of the floor beams and the slab. Figure 3 illustrates the different construction elements on site. The following subsections deal with the individual construction elements commonly used in composite or mixed building technology.

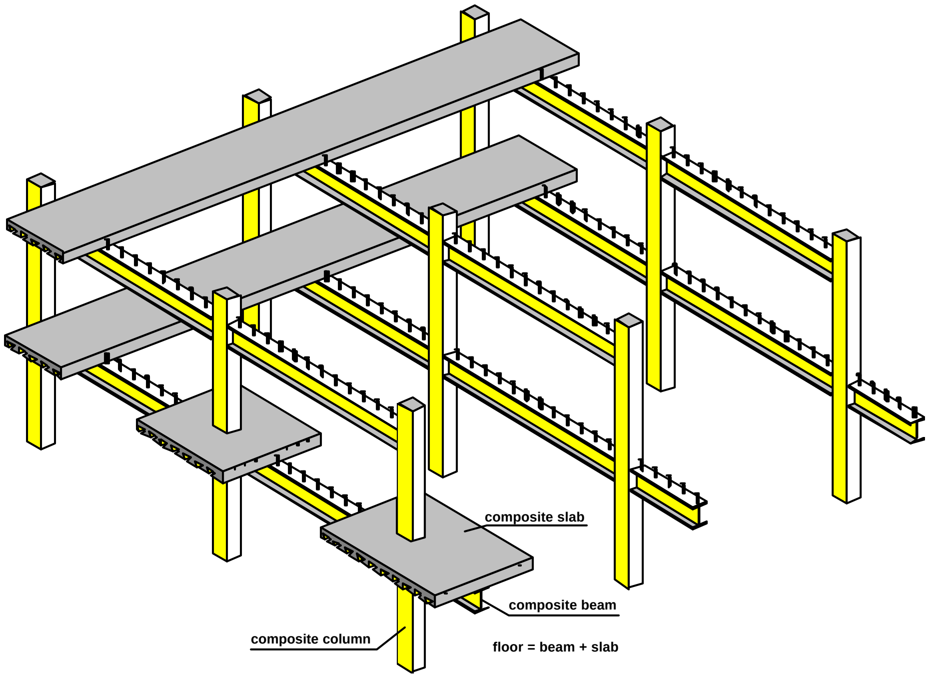


Figure 2 Construction elements [1]

Figure 3 Columns and slabs on site (Citibank Duisburg, Germany) [8]

3.2 Slabs

3.2.1 Reinforced concrete slabs

Depending on the complexity of the floor shape, the time schedule and the capabilities of the prefabrication shops, reinforced concrete slabs can be manufactured

- on site using shuttering
- using partially prefabricated elements
- using fully prefabricated elements

For all these variations, illustrated in Figure 4, there is the possibility to use normal or light weight concrete. In the case of fully prefabricated slabs attention has to be paid to the fact that only a small part of in situ concrete within the clearances can be activated by the beams to act compositely.

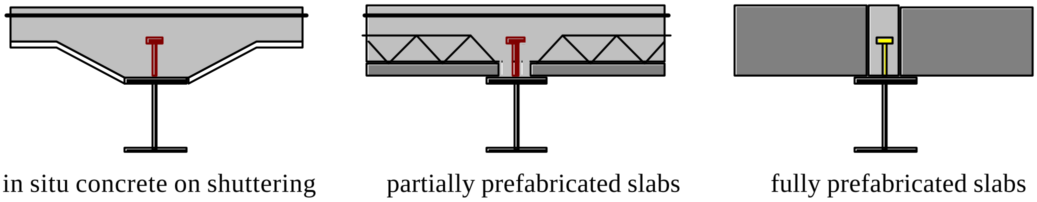


Figure 4 Types of concrete slabs [1]

3.2.2 Pre-stressed concrete slabs

In recent years pre-stressed prefabricated hollow core slabs have often been applied for very wide spans between the steel beams. These elements originally were intended to be used between insulated rigid supports like stiff concrete walls. So extending the application one has to be conscious of the following problems:

- flexible supports like beams (steel, concrete or composite) lead to transverse bending
- the unintended composite action results in transverse shear stresses
- local heating of the beam flanges acting as supports may cause a failure of anchorage and reduced shear resistance of the concrete ribs
- attention has to be paid to the differential thermal strains very close to the ends
- unintended restraints would require reinforcement in the upper layer
- the impact of beam deflection on fracture of the slabs is strongly reduced when the beams are pre-cambered – to become straight again by slabs self weight- and when the common deflection criteria for steel structures are met. It is also good practice to use an intermediate rubber or felt layer between the slabs and steel beam bottom plate. Concrete fillings of the ends of the hollow cores do have a favourable effect on the resistance to shear failure. Therefore, bending is still the governing design criterion for –usually- slender prestressed slabs

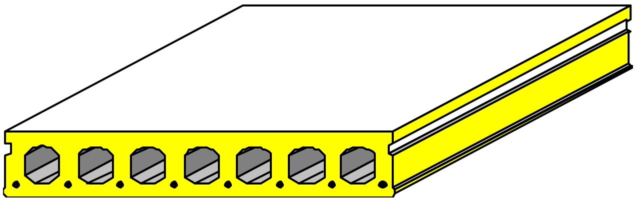


Figure 5 Pre-stressed prefabricated hollow core slab [1]

3.2.3 Profiled steel sheeting

Both conventional trapezoidal metal decking and special steel sheeting for composite slabs are in use. If there are no measures to ensure a composite action the steel profiles either are provided for the full vertical action (deep steel decks; the concrete in between only serves for the load distribution) or they are only used as so-called lost shuttering neglecting the contribution they may make in the final state. Both extremes again will lead to an uneconomic use of both materials. In a composite slab there are several possibilities to provide an interlock between steel and concrete:

- **Chemical** interlock is very brittle and unreliable therefore it must not be considered in the calculations
- **Frictional** interlock is not able to transfer large shear forces (see figure 6)
- **Mechanical** interlock by interlocking embossments of the steel decking. (see figure 7)
- **End anchorage** like headed bolts, angle studs or end-deformations of the steel sheeting brings a very concentrated load introduction at the ends and therefore a sudden increase from the bare steel to the composite resistance (see figure 8).

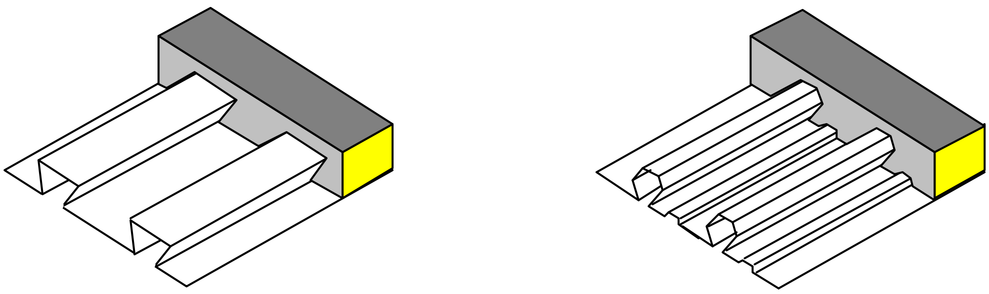


Figure 6 Frictional interlock in composite slabs [1]

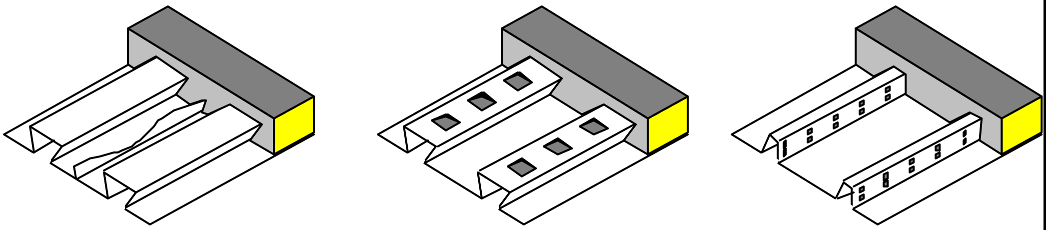


Figure 7 Mechanical interlock in composite slabs [1]

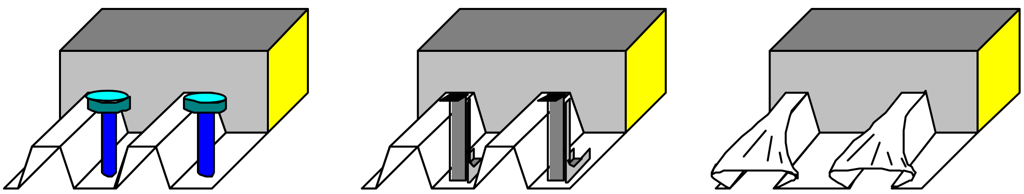


Figure 8 End anchorage for composite slabs [1]

3.3 Beams

The second element within the floor are the beams supporting the slabs and carrying the loads to the columns . Depending on the grid of beams the slabs therefore are spanning in one direction. Following the philosophy of mixed structures those beams can be realised in steel, concrete, steel-concrete composite or even other materials or their combination. In the following only steel-concrete composite floor beams will be treated in detail.

In a composite beam within the sagging moment region the concrete slab is activated in compression by shear connectors. Headed studs dominate in practical application. The advantage is the combination of a relatively large stiffness with a very large deformation capacity. Therefore, in contrast to block dowels, headed studs can be arranged with constant spacing in between which considerably facilitates the application. The disadvantage lies in the problems of weldability, especially when using galvanised plates or coated steel flanges but also regarding water in between the sheeting and the flange. *[6] 6.7.6*

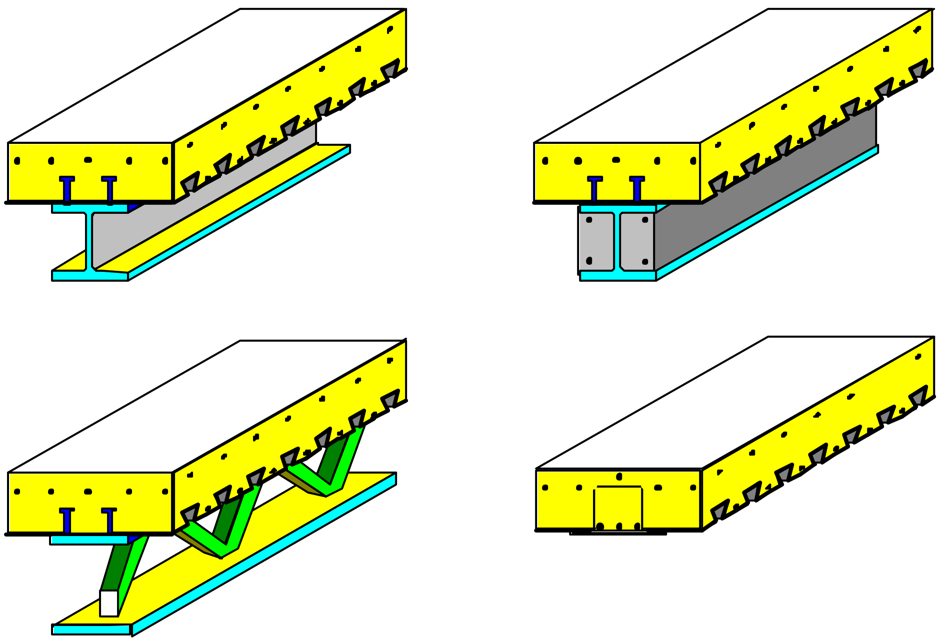


Figure 9 Conventional and innovative composite beams [1]

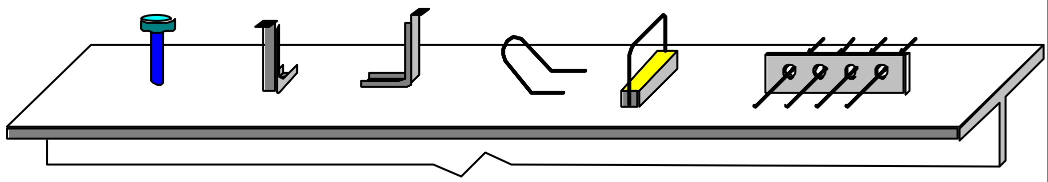


Figure 10 Types of shear connectors [1]

Figure 11 Welding of the shear connectors (headed studs) [9]

The high bearing capacity and stiffness of composite beams allows the construction of very wide column free rooms with comparatively little construction height. Until now composite beams have only been built single span or continuous, rigid composite connections to the columns have been avoided because of missing knowledge. For the steel parts, rolled IPE-, HEA-, HEB-, channel - but also built-up sections are used for conventional spans. For higher column spacing castellated beams and trusses are brought into action. In special cases the steel beam sections may be partially encased e.g. in view of fire protection.

The slim-floor technology, where the beams are fully integrated into the slabs, has brought a sensational boom of the composite construction method starting in the Scandinavian countries. It gives the possibility of a flat ceiling without downstand beams. Slim floor systems have the same advantages as conventional concrete flat slabs whilst avoiding the well-known problems of punching shear at the column heads. The combination of slim-floors with hollow column sections seems to promise a great future.

See the mentioned examples in clause 5

3.4 Columns

Beside the possibility to realize pure steel or concrete columns the bearing behaviour of composite columns mainly is dominated by the structural steel part in it.
They are commonly used where large normal forces are combined with the wish for small sections. As the composite columns may be prefabricated or at least prepared in the shop the construction time can be drastically reduced compared to in-situ concrete. A decisive advantage over bare steel columns is the very high fire resistance of composite columns without any

preventive measures.

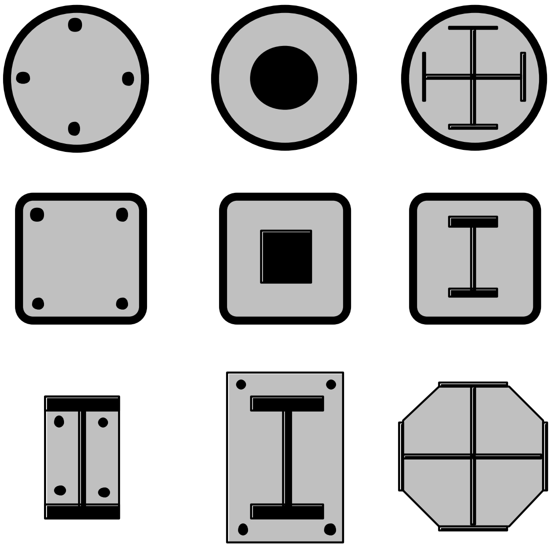


Figure 12 Examples of composite columns [1]

Figure 13 Tubular column with shot fired nails as shear connectors and reinforcement on site (Citibank Duisburg, Germany) [8]

For the steel section rolled I-profiles as well as rectangular and tubular hollow sections are suitable. I-sections can be fully or partially encased, where only the chambers are filled with concrete. Hollow sections have the advantage that no further shuttering is necessary for concreting, they are favoured by architects and they behave very well with regard to fire resistance. For very high loads even steel cores within hollow steel sections are used. A survey of practical composite column types is shown in Figure 12, for an application of a composite column see in Figure 13.

To ensure sufficient composite action between the steel and concrete part, shear connectors have to be placed in the areas of concentrated load introduction, therefore at the level or a little bit below the floors. Considering rolled sections again headed bolts or angle studs can be used. For hollow column sections non-headed bolts put in through holes and welded at the section surface stood the tests, additionally serving as bar spacers not hampering concreting. However the welding of these shear connectors is relatively time-consuming. As a really economic alternative together with Hilti the nailing technique (Figure 14) has been developed for the use as shear connectors in hollow column sections. The placement is easy and very fast and the resistance proved to be amazingly high with good ductility.

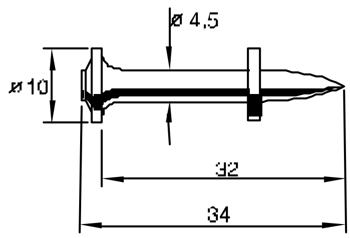


Figure 14 HILTI Nail X-DSH32 P10

3.5 Joints

Traditionally the joints have simply been regarded as a part of the column and have not been considered in the global calculation. However as a joint strictly consists of parts of the column, the beams and the slabs, connecting elements and sometimes also includes stiffening elements the real behaviour can only be taken into consideration by defining the joint as a separate element within the structure additional to the beams and column elements. On one hand this enables more efficient constructions but on the other hand the influence of the joints on the global behaviour is so important that the old-fashioned philosophy of perfect hinges or fully continuous restraints does not describe the real behaviour of a semi continuous joint. (See Figure 15) According to this new approach, joints may be assessed with regard to the following three main characteristics.

- **stiffness:**A joint with vanishing rotational stiffness and which therefore carries no bending moment is called a hinge. A rigid joint is one whose rigidity under flexure is more or less infinite and which thus ensures a perfect continuity of rotations. In between these two extreme boundaries we speak about semi-rigid joints.

- **moment resistance:**In contrast to a hinge, a joint whose ultimate strength is greater than the ultimate resistance (ultimate strength) of the parts whose linkage it ensures is called a full strength joint. Again a partial strength joint represents a middle course between these extremes. (For simplicity from now on “resistance” will mostly be used for the ultimate resistance value; the terms “resistance” and “strength” are used in the Eurocodes with an identical meaning.)

- **rotation capacity (ductility):** Brittle behaviour is characterised by fracture under slight rotation, usually without plastic deformations. Ductile behaviour is characterised by a clear non-linearity of the moment-rotation curve with a large plateau before fracture. It usually indicates the appearance of plastic deformations. The ductility coefficient is the ratio between the ultimate rotation and the elastic rotation limit. Semi-ductility falls in between brittle and ductile behaviour.

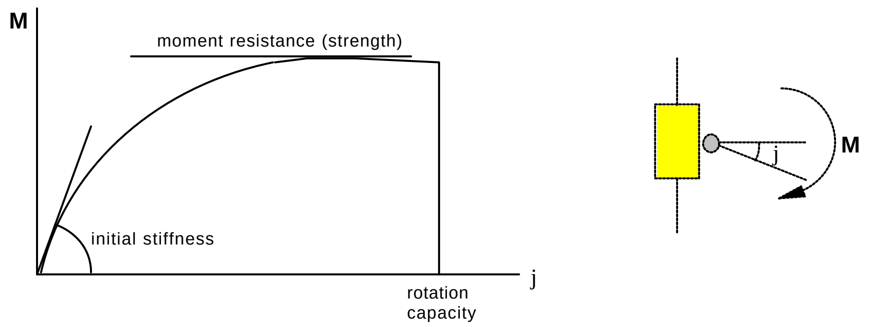


Figure 15 Joint response [1]

Starting analysing full-scale joints it could be seen very quickly that the number of influencing parameters is too large. So world-wide the so-called component method is accepted as the best method to describe the joint behaviour analytically. In contrast to the common finite element method (FEM), which often fails to consider local load introduction problems, the joint here is divided into logical parts exposed to internal forces. So while the FEM works on the level of strains and stresses, the component method concentrates on internal forces and deformations of so-called component springs.

In recent years all over the world extensive testing programs have been performed for studying the non-linear behaviour of individual components and their assembly to gain the non-linear moment-rotation reaction of the whole joint formed by these components. Further as the connection between beams and hollow column sections is problematic with regard to the transfer of vertical shear forces due to the inaccessibility of the column's interior, several efforts have been made to develop a connection providing sufficient bearing capacity which either can be prepared already in the shop or which can easily be placed in site. The problem increases due to the eccentricity of the imposed loads due to tolerances in combination with the relatively thin steel sheets. An example of a connection type is illustrated in figure 16.

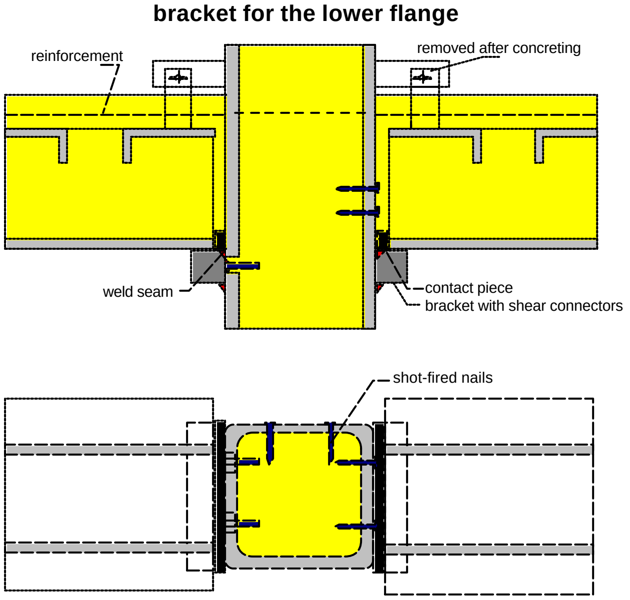


Figure 16 Example for the vertical shear transfer between beams and composite columns [1]

- A simple **bracket** can be welded to the column surface supporting either the beam's upper or lower flange. Bearing on the lower flange requires fire protection to the bracket and in those cases where no suspended ceiling is used the architect might refuse to use such an ugly bracket. Using a bracket for the upper flange, one has to put up a more difficult erection, especially when aiming at a rigid connection in the final state.

Summarising it should be emphasised that the most economic way of erection is to start with single span steel beams (propped or unpropped) hinged to the columns. By placing contact pieces and reinforcement semi-continuous restraints are very simply formed after hardening of concrete at the final state.

4 Definitions and terminology

[6] 1.4.2(1)

As some principle terms in composite construction are often used in the wrong context and therefore lead to misunderstandings they will be now defined:

4.1 Composite member

A structural member with components of concrete and of structural or cold-formed steel, interconnected by shear connection so as to limit the longitudinal slip between concrete and steel and the separation of one component from the other

4.2 Shear connection

An interconnection between the concrete and steel components of a composite member that has sufficient strength and stiffness to enable the two components to be designed as parts of a single structural member

4.3 Composite beam

A composite member subjected mainly to bending

4.4 Composite column

A composite member subjected mainly to compression or to compression and bending

4.5 Composite slab

A bi-dimensional horizontal composite member subjected mainly to bending in which profile steel sheets:

- are used as permanent shuttering capable of supporting wet concrete, reinforcement and site loads, and
- subsequently combine structurally with the hardened concrete and act as part or all of the tensile reinforcement in the finished slab

4.6 Composite frame

A framed structure in which some or all of the elements are composite members and most of the remainder are structural steel members.

4.7 Composite joint

A joint between composite members, in which reinforcement is intended to contribute to the resistance and the stiffness of the joint.

4.8 Shear-slip characteristic of a single connector

Leaving the old-fashioned point of view that an element (connector, joint, ...) is either infinite rigid or completely pinned, fully ductile or absolutely brittle the idealised relationship between shear force and displacement of a connector leads to three dominating characteristics:

- initial stiffness
- resistance (bearing capacity, strength)
- deformation capacity

The slip is defined as the relative displacement between the two connected materials in the interface layer in the direction of the beam's axis. An uplift between steel and concrete has to be prevented by anchorages (e.g. headed studs) or other elements able to carry tensile forces like stirrups.

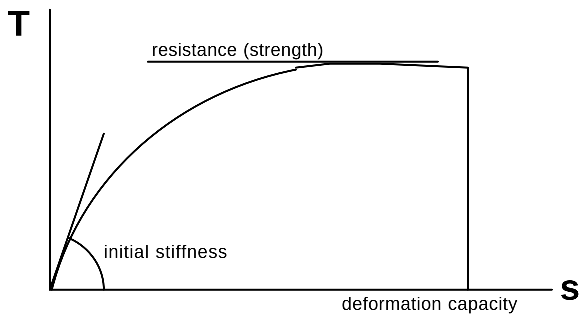


Figure 17 Shear-slip curve of a connector [1]

Building a beam or slab by positioning shear connectors one behind the other, the global behaviour of the beam is decisively influenced by these three characteristics. So the degree of shear connection of a beam apart from the number of shear connectors directly is linked to the resistance (short "strength" or "resistance") of a single connector.(See Figure 17) The shear interaction depends on the initial stiffness of the used connectors and their number. The deformation capacity of the whole beam is bound up with that of the individual connector itself.

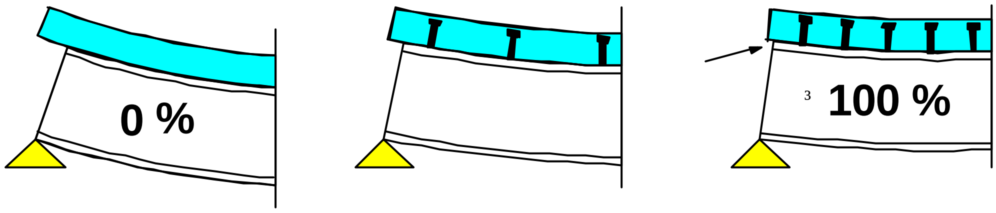
4.9 Degree of shear connection (resistance)

The degree of shear connection gives the ratio between the **bearing capacity of the shear connection** and that of the composite section itself, which is dominated by the weaker part (either steel or concrete). Assuming ideal plastic behaviour depending on the ratio between steel and concrete resistance the degree of shear connection η can be expressed by the formula given in Figure 18

$$h = \frac{N_{i,Rd}}{N_{Rd,concrete}} \qquad h = \frac{N_{i,Rd}}{N_{Rd,steel}}$$

$$\text{with} \quad N_{i,Rd} = \sum_{dowel\ number} T_{Rd,i}$$

Figure 18 Degree of shear connection



no shear connection
big end slip, big steel beam

partial shear connection

full shear connection
many dowels

Figure 19 Degree of shear connection [1]

No shear connection (η=0) means that both section parts act completely separately. In the case of full shear connection (η≥100%) there is sufficient bearing capacity provided by the dowels to achieve failure of the section itself (yielding of all layers). In between these two extreme boundaries we speak about partial shear connection (0≤η≤100%) which commonly results in an optimum of material and costs (Figure 19). For partial shear connection the bearing capacity of the beam is limited by the failure of the shear connection.
It is important to mention that so far partial shear connection according to EC4 is only licensed observing the following conditions: Ductile shear connectors, static loading, sagging moments, limited span. *[6] 6.7*

It should be noted that full shear connection does not mean that there is no slip in the interface. So no slip strictly can only be achieved by a very high degree of shear connection depending on the stiffness of the connection elements themselves. Therefore raising the degree of shear connection above 100% by adding more shear connectors does not increase the beam's resistance at the **ultimate limit state** any more but brings a reduction of slip and deflection at the serviceability limit state. However attention has to be paid not to exceed the capacity of the shear in the concrete flanges which would lead to brittle failure.

4.10 Shear interaction (stiffness)

Whether a shear interaction is complete (rigid, stiff) or incomplete (semi-rigid, weak, soft) depends on the shear connectors themselves and their number in relation to the stiffness of the composite parts (steel beam, concrete slab). Therefore a clear definition between these two cases cannot be given.

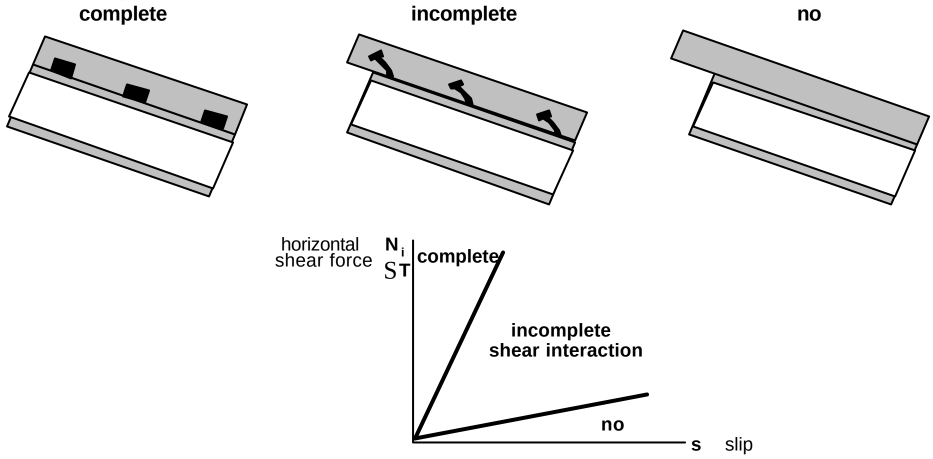


Figure 20 Shear interaction [1]

Ideal rigid interaction means that there is no **relative displacement** (slip) between the composite parts within the shear interface. As the shear connectors act as parallel springs, an increase of shear connection goes hand in hand with an increase of shear interaction (reduction of slip). Therefore strictly an infinitely rigid interaction would only be possible for infinite stiff shear connectors or an infinitely large number of them (and therefore not realizable in practice). So the term complete interaction has to be understood as sufficiently small displacements the effects of which may be neglected. So for the incomplete interaction, as a term related to the **serviceability limit state**, the relative displacements in the dowel layer have to be taken into consideration by a jump in the strain distribution. The Bernoulli-hypothesis of plane cross sections remaining plane in the deformed state is only valid for the two individual section parts steel and concrete separately, but not for the overall composite cross section.

For incomplete interaction the slip is accompanied by an increase of mid-span-deflection in comparison to a beam with an infinitely stiff shear interface as demonstrated in figure 21. In EC4 [6] this effect is taken into consideration in an approximate manner by a linear interpolation depending on the degree of shear connection:

$$\frac{\delta}{\delta_c} = 1 + \nu \cdot (1 - \eta) \cdot \left(\frac{\delta_a}{\delta_c} - 1 \right)$$

where δ_a is the deflection of the bare steel beam
δ_c is the deflection of the composite beam assuming infinite rigid shear connection
η is the degree of shear connection
ν depends on the type of shear action (0,3 or 0,5)

As already mentioned that means that in the code 100% shear connection roughly simplifying is set equal to infinite stiff shear interaction.

[6] 7.2.2.2(4)

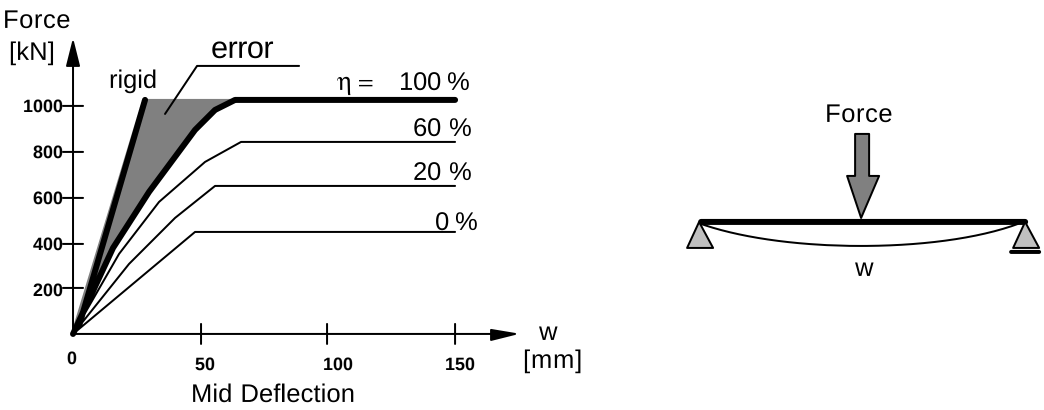


Figure 21 Influence of partial interaction on the beam deflection [1]

5 Examples

5.1 Millennium Tower (Vienna – Austria) [1], [8]

As a first reference to an existing building , where composite structures have been successfully been applied, the MILLENNIUM TOWER in Vienna can be mentioned.

This 55 storey high-rise tower (Figure 23) with a ground area of about 1000 m^2 and a height of more than 202 metres including the antenna (highest building in austria) could be realized within a construction time of only 8 months- from May to December 1998 (Figure 24).

This means a construction progress of 2 to 2,5 storeys per week!

The horizontal stabilisation is given by an internal concrete core containing the elevators and the stairways. Around that two overlapping circles of slim floors are spanning to very slender composite columns at the facade.(See figure 22)

Thanks to the really effective restraint action of the semi continuous joints between the slim floor beams and the tubular composite columns by a specially developed moment-connection the overall slab thickness could be reduced to only 19 cm. This results in reduced material consumption, foundation and facade costs. Special calculations have been done for the serviceability limit state in view of vibrations and differential shrinkage between the external composite frames and the internal concrete core.

Finally also the nailing technique developed in Innsbruck has been applied at the Millennium Tower for the first time: the vertical shear transfer between the tubular column steel section and the internal concrete encasement is provided by shot-fired nails, which are fixed easily from outside without any welding and by penetrating the tube are reaching into the column interior.

Apart from a lot of initial research to be able to realize such an innovative project the acceptance by the planners, the architects, the construction firm onsite and the owner of the building has been enormous and causes a lot of expectancy for the future.

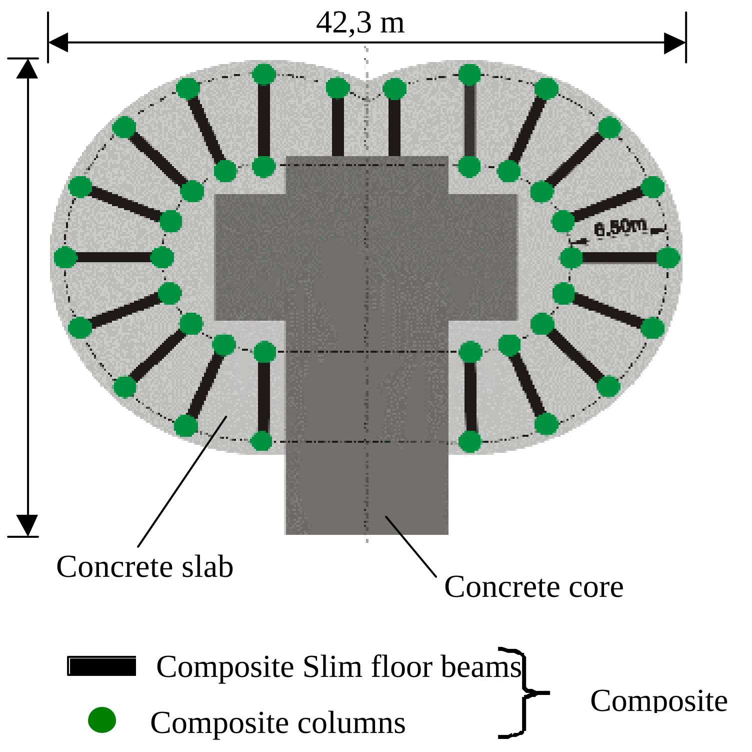


Figure 22 Millennium Tower Vienna (Austria), plan view

Figure 23 Millennium Tower Vienna (Austria) [8]

Figure 24 Construction progress (total erection time: 8 months) [8]

5.2 Citibank Duisburg (Duisburg – Germany)

Figure 25 Citibank Duisburg (Germany) [8]

The office building of the Citibank in Duisburg (Germany) has a total height of 72m, 15 storeys and a total ground floor of 14500 m^2.

It is a typical example of mixed building technology. The internal reinforced concrete core is intended to carry horizontal forces and was erected with a maximum speed of 3 m per day. The composite columns and slabs around followed the core in nearly the same speed, so a very fast progress of construction was possible.

5.3 Parking deck “DEZ” (Innsbruck- Austria) [8]

Figure 26 General view of the parking deck [8]

A further example for a composite structure is the new parking deck in Innsbruck (Austria), which shows how the technology leads to new solutions in the design phase as well as in the execution and construction. The structural requirements and boundary conditions are pointed out briefly and their solution is explained by a suitable system choice.

The parking house is a 4 storey building with ground dimensions of 60 x 30 m.

The particularity is the 26 cm thick slim floor slab which is semicontinuously connected with the composite columns.

Maximum span length of composite slim floor beams : 10,58 m

Also a particularity of the building is the 4,8 m cantilever and the very slim columns (composite columns: Ø=355 mm).

This building is an example of simplifying the process of erection. By using columns over 2 storeys and partially prefabricated slabs the time of erection could be minimized.

Figure 27 Erection of composite columns over 2 storeys

Figure 28 Assembly of slim-floor beams and prefabricated concrete

slabs

Figure 29 shows the cross section of the slim-floor beam and slab.

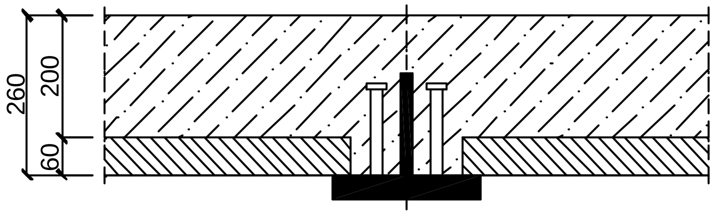


Figure 29 Cross section of the slim-floor beam and slab

- 200 mm concrete slab
- 60 mm prefabricated concrete elements
- steel beam : web: 165/20 mm, flange 245/40 mm
- headed studs Ø 22 m

6 Future developments

A lot of research all over Europe has improved existing composite systems and has led to the development of new technologies e.g. Slim-floor slabs with semi continuous connections to the columns, new steel sheets or systems to minimize the time of erection and assembly.

Other developments concerning the real behaviour of composite structures and elements are published in COST-C1 project.[7]

7 Concluding Summary

Composite construction is popular for buildings and bridges as well because of the following aspects:

- Economy
- Architecture
- Functionality
- Service and building flexibility
- Assembly

Therefore composite constructions should be strengthened to take an important place beside conventional steel construction by using the common Eurocodes with national applications, the modules of SSEDTA 1 and the following modules of SSEDTA 2 for additional support.

Structural Steelwork Eurocodes
Development of a Trans-National Approach

Course: Eurocode 4

Lecture 2 : Introduction to EC4

Summary:

- A number of terms used in EC4 have a very precise meaning
- The principal components for composite construction are concrete, reinforcing steel, structural steel, profiled steel sheet, and shear connectors.
- Material properties for each component are defined in other Eurocodes.
- Guidance is given on what methods of analysis, both global and cross-sectional, are appropriate.
- EC4 is based on limit state design principles
- The Ultimate Limit State is concerned with collapse
- The Serviceability Limit State is concerned with operational conditions. These relate specifically to deflections and crack control, and EC4 provides guidance for controlling both.
- EC4 is structured on the basis of element type, and detailed procedures for the design of beams, columns and slabs are given in separate sections.

Pre-requisites:

- None

Notes for Tutors:

This material comprises one 30 minute lecture.

Objectives:

- To describe the structure of EC4.
- To explain some specific technical terms and define principal notation
- To identify the principal components and corresponding material characteristics for composite construction.
- To introduce the principles of limit state design in relation to composite steel and concrete construction.
- To outline the principles for analysis and design for both ultimate and serviceability conditions for composite beams, columns and slabs.

References:

- EC4: EN 1994-1-1: Eurocode 4: Design of Composite Steel and Concrete Structures Part 1.1: General rules and rules for buildings.

Contents:

1. Structure of Eurocode 4 Part 1.1

The arrangement of sections within EC4-1-1 is based on a typical design sequence, starting with basic data on material properties and safety factors, then considering issues related to methods of analysis, before detailing the requirements for element design (at both ultimate and serviceability limit states).

EC4 is organised into a number of Sections as follows:

Section 1 General
Outlines the scope of EC4, defines specific terms, and provides a notation list.

Section 2 Basis of Design
Outlines design principles and introduces partial safety factors

Section 3 Materials
Specifies characteristic strengths for concrete, steel (reinforcing and structural), and shear connectors

Section 4 Durability
Specifies particular requirements for corrosion protection of composite elements, in relation to the interface between steel and concrete, and galvanising standards for profiled steel sheets for composite slabs.

Section 5 Structural Analysis
This outlines appropriate methods of global analysis and their potential application, and defines the effective width and section classification.

Section 6 Ultimate limit states
This provides detailed rules regarding detailed sizing of individual structural elements (beams and columns), including shear connectors. The design of composite slabs is covered in Section 9.

Section 7 Serviceability limit states
Sets out limits on deflections and requirements to control cracking

Section 8 Composite joints in frames for buildings
Provides detailed procedures for designing joints.

Section 9 Composite slabs with profiled steel sheeting for buildings
Provides specific guidance for the use of composite decking, and sets out detailed procedures for verification at both ultimate and serviceability limit states for both shuttering and the composite slab.

Section 10 Execution
Provides guidance on the site construction process. This specifies minimum standards of workmanship as implicitly assumed in the rest of EC4.

Section 11 Standard tests
Describes procedures for testing shear connectors and composite floor slabs where standard design data is not available.

2. Terminology

The Eurocodes define a number of terms which, although often used generally in a rather loose way, have more precise meanings in the context of EC4. These terms are clearly defined and include the following: *cl. 1.4.2*

- 'Composite member' refers to a structural member with components of concrete and structural or cold-formed steel, interconnected.by shear connection to limit relative slip.

- 'Shear connection' refers to the interconnection between steel and concrete components enabling them to be designed as a single member.
- 'Composite beam' is a composite member subject mainly to bending.
- 'Composite column' is a composite member subject mainly to compression or combined compression and bending.
- 'Composite slab' is a slab in which profiled steel sheets act as permanent shuttering and subsequently act to provide tensile reinforcement to the concrete.
- 'Execution' refers to the activity of creating a building, including both site work and fabrication.
- 'Type of building' refers to its intended function (eg a dwelling house, industrial building)
- 'Form of structure' describes the generic nature of structural elements (eg. beam, arch) or overall system (eg. Suspension bridge)
- 'Type of construction' indicates the principal structural material (eg. steel construction)
- 'Method of construction' describes how the construction is to be carried out (eg prefabricated)
- 'Composite frame' is a framed structure in which some or all of the elements are composite.
- 'Composite joint' is a joint between composite members in which reinforcement is intended to contribute to its resistance and stiffness.
- Type of framing:

Simple	joints do not resist moments
Continuous	joints assumed to be rigid
Semi-continuous	connection characteristics need explicit consideration in analysis

- 'Propped structure or member' is one in which the weight of concrete applied to the steel elements is carried independently, or the steel is supported in the span, until the concrete is able to resist stress.
- 'Unpropped structure or member' is one in which the weight of concrete is applied to the steel elements which are unsupported in the span..

3. Notation/Symbols

A complete list of symbols is included in EC4. The most common of these are listed below: *cl. 1.6*

Symbols of a general nature:

L, l	Length; span; system length
N	Number of shear connectors; axial force
R	Resistance; reaction
S	Internal forces & moments; stiffness
δ	Deflection; steel contribution ratio
λ	Slenderness ratio
χ	Reduction factor for buckling
γ	Partial safety factor

Symbols relating to cross-section properties:

A	Area
b	Width
d	Depth; diameter
h	Height
i	Radius of gyration
I	Second moment of area
W	Section modulus
φ	Diameter of a reinforcing bar

Member axes

The following convention is adopted for member axes:

x-x	along the length of the member
y-y	axis of the cross-section parallel to the flanges (major axis)
z-z	axis of the cross-section perpendicular to the flanges (minor axis)

Symbols relating to material properties:

E	Modulus of elasticity
f	Strength
n	Modular ratio

EC4 also makes extensive use of subscripts. These can be used to clarify the precise meaning of a symbol. Some common subscripts are as follows:

c	Compression, composite cross-section, concrete
d	Design
el	Elastic
k	Characteristic
LT	Lateral-torsional
pl	Plastic

Normal symbols may also be used as subscripts, for example:

Rd	Design resistance
Sd	Design values of internal force or moment

Subscripts can be arranged in sequence as necessary, separated by a decimal point – for example:

$N_{pl.Rd}$	Design plastic axial resistance.

4. Material properties

4.1 Concrete

Properties for both normal weight and lightweight concrete shall be determined according to EC2, but EC4 does not cover concrete grades less than C20/25 or greater than C60/75. *cl. 3.1*

4.2 Reinforcing steel

Properties for reinforcing steel shall be determined according to EC2, but EC4 does not cover reinforcement grades with a characteristic strength greater than 550N/mm^2.. *cl. 3.2*

4.3 Structural steel

Properties for structural steel shall be determined according to EC3, but EC4 does not cover steel grades with a characteristic strength greater than 460N/mm^2.. *cl. 3.3*

4.4 Profiled steel sheeting for composite slabs

Properties for steel sheeting shall be determined according to EC3, but EC4 also restricts the type of steel to those specified in certain ENs. *cl. 3.4*

The recommended minimum (bare) thickness of steel is 0,7mm

4.5 Shear connectors

Reference is made to various ENs for the specification of materials for connectors. *cl. 3.5*

5. Structural analysis

General guidance is given on what methods of analysis are suitable for different circumstances. *cl. 5.1.2*

5.1 Ultimate Limit State

For the Ultimate Limit State, both elastic and plastic global analysis may be used, although certain conditions apply to the use of plastic analysis.

When using elastic analysis the stages of construction may need to be considered. The stiffness of the concrete may be based on the uncracked condition for braced structure. In other cases, some account may need to be taken of concrete cracking by using a reduced stiffness over a designated length of beam. The effect of creep is accounted for by using appropriate values for the modular ratio, but shrinkage and temperature effects may be ignored. *cl. 5.1.4*

Some redistribution of elastic bending moments is allowed.

Rigid-plastic global analysis is allowed for non-sway frames, and for unbraced frames of two storeys or less, with some restrictions on cross-sections. *Cl. 5.1.5*

Cl. 5.3.4

A similar distinction is made between sway and non-sway frames, and between braced and unbraced frames as for steel frames, and reference is made to EC3 for definitions.

5.2 Properties and classification of cross-sections

The effective width of the concrete flange of the composite beam is defined, although more rigorous methods of analysis are admitted. *cl. 5.2*

Cross-sections are classified in a similar manner to EC3 for non-composite steel sections. *cl. 5.3*

5.3 Serviceability Limit State

Elastic analysis must be used for the serviceability limit state. The effective width is as defined for the ultimate limit state, and appropriate allowances may be made for concrete cracking, creep and shrinkage. *cl. 5.4*

6. Ultimate Limit State

cl. 6

The ultimate limit state is concerned with the resistance of the structure to collapse. This is generally checked by considering the strength of individual elements subject to forces determined from a suitable analysis. In addition the overall stability of the structure must be checked.

The ultimate limit state is examined under factored load conditions. In general, the effects on individual structural elements will be determined by analysis, and each element then treated as an isolated component for design. Details of individual design checks depend on the type of member (eg beam, column) and are described in other parts of this course.

The ultimate limit state design for composite connections and composite slabs are dealt with in Sections 8 and 9 respectively.

6.1 Beams

For beams, guidance is given on the applicability of plastic, non-linear and elastic analysis for determining the bending resistance of the cross-section, with full or partial interaction. *cl. 6.3*

Procedures for calculating the vertical shear resistance, including the effects of shear buckling and combined bending and shear.

Beams with concrete infill between the flanges enclosing the web are defined as partially encased, and separate considerations apply to the design for bending and shear for these. *cl. 6.4*

In general, the top flange of the steel beam in composite construction is laterally restrained against buckling by the concrete slab. However, in the hogging bending zones of continuous beams, the compression flange is not restrained in this way, and procedures for checking lateral-torsional buckling for such cases are given. If a continuous composite beam satisfies certain conditions defined in EC4, such checks are unnecessary. *cl. 6.5*

Detailed procedures are given for the design of the longitudinal shear connection, including the requirements for the slab and transverse reinforcement. A range of different connector types is considered. *cl. 6.7*

6.2 Columns

Various types of composite columns, including encased sections and concrete-filled tubes, are covered. Simplified procedures are given for columns of doubly symmetrical cross-section and uniform throughout their length. Guidance is given on the need for shear connection and how this can be achieved. *cl. 6.8*

7. Serviceability Limit State

Serviceability requirements are specified in relation to limiting deflections and concrete cracking. Other less common serviceability conditions relating to control of vibrations and limiting stresses are not included in EC4. *cl. 7.1*

7.1 Deflections

At the serviceability limit state, the calculated deflection of a member or of a structure is seldom meaningful in itself since the design assumptions are rarely realised. This is because, for example: *cl. 7.2*

- the actual load may be quite unlike the assumed design load;
- beams are seldom "simply supported" or "fixed" and in reality a beam is usually in some intermediate condition;

The calculated deflection can, however, provide an index of the stiffness of a member or structure, i.e. to assess whether adequate provision is made in relation to the limit state of deflection or local damage. Guidance is given on calculating deflections for composite beams, including allowances for partial interaction and concrete cracking. No guidance is given regarding simplified approaches based on limiting span/depth ratios.

No reference is given to limiting values for deflections in EC4. It is therefore recommended that calculated deflections should be compared with specified maximum values in Eurocode 3, which tabulates limiting vertical deflections for beams in six categories as follows: *EC3 Table 4.1*

- roofs generally.
- roofs frequently carrying personnel other than for maintenance.
- floors generally.
- floors and roofs supporting plaster or other brittle finish or non-flexible partitions.
- floors supporting columns (unless the deflection has been included in the global analysis for the ultimate limit state).
- situations in which the deflection can impair the appearance of the building.

The deflections due to loading applied to the steel member alone, for example those during the construction stage for unpropped conditions, should be based on the procedures of EC3 using the bare steel section properties.

Deflections due to subsequent loading should be calculated using elastic analysis of the composite cross-section with a suitable transformed section. Where necessary, methods of allowing for incomplete interaction and cracking of concrete are given

7.2 Concrete Cracking

Concrete in composite elements is subject to cracking for a number of reasons including direct loading and shrinkage. Excessive cracking of the concrete can affect durability and appearance, or otherwise impair the proper functioning of the building. In many cases these may not be critical issues, and simplified approaches based on minimum reinforcement ratios and maximum bar spacing or diameters can be adopted. Where special conditions apply, for example in the case of members subject to sever exposure conditions, EC4 provides guidance on calculating crack widths due to applied loads. Limiting crack widths are specified in relation to exposure conditions. *cl. 7.3*

8. Composite Joints

cl. 8

The guidance given applies principally to moment-resisting beam-column connections. It relates to moment resistance, rotational stiffness, and rotation capacity. The inter-dependence of global analysis and connection design is described, but where the effects of joint behaviour on the distribution of internal forces and moments are small, they may be neglected. Guidance is given on joint classification as rigid, nominally pinned, or semi-rigid for stiffness, and as full strength, nominally pinned or partial strength in relation to moment resistance.

Detailed guidance is given in relation to design and detailing of the joint, including slab reinforcement.

9. Composite Slabs

cl. 9

Detailed guidance is given in relation to the design of composite slabs, for both ultimate and serviceability limit states. This includes construction stages when the steel sheeting is acting as permanent shuttering and, in an unpropped condition, must resist the applied actions due to wet concrete and construction loads. In this case reference is made to EC3 Part 1.3.

Calculation procedures are given for determining the resistance of composite slabs in relation to flexure, longitudinal shear and vertical shear. Principles for determining stiffness for calculating deflections are stated, and conditions in which detailed calculations can be omitted are specified in relation to span:depth ratios.

10. Concluding Summary

- A number of terms used in EC4 have a very precise meaning
- The principal components for composite construction are concrete, reinforcing steel, structural steel, profiled steel sheet, and shear connectors.
- Material properties for each component are defined in other Eurocodes.
- Guidance is given on what methods of analysis, both global and cross-sectional, are appropriate.
- EC4 is based on limit state design principles
- The Ultimate Limit State is concerned with collapse
- The Serviceability Limit State is concerned with operational conditions. These relate specifically to deflections and crack control, and EC4 provides guidance for controlling both.
- EC4 is structured on the basis of element type, and detailed procedures for the design of beams, columns and slabs are given in separate sections.

Structural Steelwork Eurocodes
Development of
A Trans-National Approach 2

Course: Eurocode 4

Lecture 3: Structural modelling and design

Summary:

- The lecture presents the different design steps for a composite structure, including the structural modelling, the structural analysis and the verification of the frame under serviceability and ultimate limit states (SLS and ULS).
- In particular the way on how the frame and member idealisation is achieved for structural modelling is contemplated.
- The frame classification which determines the structural analysis and design process together with the limitations of Eurocode 4 in this field are also covered by the lecture.
- The SLS and ULS design requirements are specified.
- Finally a flow chart gives a general overview of the full design process and indicates how the different design steps are covered by lectures 4 to 9.

Pre-requisites:

- Generalities about composite construction.
- Basic knowledge on frame analysis and the design of structural members (resistance and plasticity aspects).
- The preliminary reading of SSEDTA 2 "Eurocode 4" lecture n°1 is recommended.

Notes for Tutors:

This material comprises a 90-minute lectures.

Objectives:

The student should:

- Know about the successive design steps of a composite building frames, including the structural modelling and the analysis and structural design process.
- Have indications on how to idealise and classify a composite structure.
- Know about the limitations of Eurocode 4 as far as analysis and design are concerned.
- Be aware of the structural requirements of Eurocode 4 for members (slabs, beams, columns and joints) under serviceability and ultimate limit states.

References:

- [1] Eurocode 2 : Design of concrete structures
  EN 1992-1:200x, CEN (European Normalisation Centre), Brussels.
- [2] ESDEP lectures on composite structures (Volume 10)

Contents:

1. Introduction

The structural design of a building frame is a long process involving a number of successive steps.

At the beginning of this process, the geometry of the structure (general dimensions of the building, height of the storeys, number of bays, …) is usually known; it generally results from a preliminary architectural design based on the requirements of the future building's owner.

The location of the building (country, landscape, altitude, …) and its foreseen utilisation mode (offices, machineries, dance halls, …) allows the designer to define the basic load cases as well as the predominant load combinations.

On this basis, the structural design may start by a so-called "structural modelling" and is then followed by an "actual design" aimed at ensuring that the structure is economically able to satisfy the design requirements expressed in normative documents. In the latter step, the adequate response of the building under service loads (serviceability limit states) and factored loads (ultimate limit states) has be to verified.

In the present lecture, generalities about the structural modelling and the design process are presented and a general overview of the contents of the Lectures 4 to 9 is given in the flowchart format.

2. Structural modelling

The frame design and analysis process of frames is conducted on a model based on many assumptions including those for the structural model, the geometric behaviour of the structure and of its members and the behaviour of the sections and of the joints.

EC3 5.2.2

EC3 5.2.3

Guidelines for the simplified modelling of building structures for analysis and design are given in the informative Annex H [1] of the second amendment to Eurocode 3, Part 1-1. Simplified models for buildings subjected to predominantly static loading are proposed which may be adopted as alternatives to more sophisticated models. It does not cover methods intended either for seismic design nor for " stressed skin " design which are dealt with in other specific Eurocodes [2,3].

EC3 Annex H

While the essential aspects of frame modelling are cited in the present lecture, reference to the relevant parts of Eurocode 3 Part 1-1 and to Annex H should be made by the designer.

All of these recommendations of Annex H apply equally to steel and composite frames.

2.1 Structural concept

EC3 Annex H

The layout of the structure should be based on the requirements for the intended use of the building, including resistance to the actions that are likely to occur.

One is required to identify the following categories of structural elements:

- ***main structural elements***: including main frames, their joints and their foundations that form the routes by which vertical and horizontal forces acting on the building are transferred to the ground;
- ***secondary structural elements***: such as secondary beams or purlins, that transfer loads to the main structural elements;
- ***other elements***: elements that only transfer loads to the main or secondary elements. Examples are sheeting, roofing and partitions.

In cases where these three categories of elements are subject to different safety requirements, they should be modelled separately, if necessary.

2.1.1 Spatial behaviour

EC3 Annex H

As an alternative to analysing the main structure as a one three-dimensional framework, it may be analysed as two series of independent plane frames running in two horizontal directions approximately at right angles to each other, see Figure 1, provided each such plane frame has sufficient out-of-plane restraint to ensure its lateral stability.

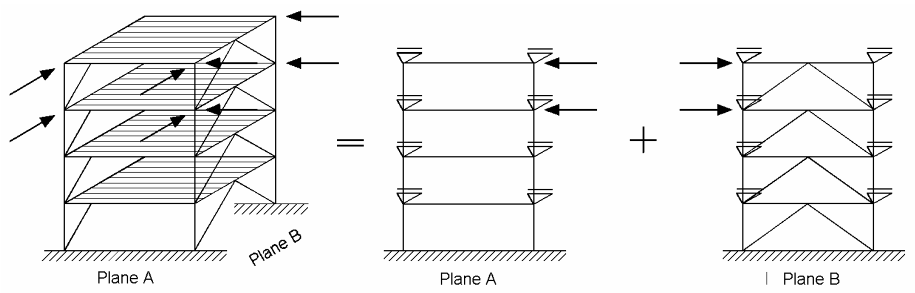


Figure 1 - Reduction of a three-dimensional framework to plane frames

2.1.2 Resistance to horizontal forces

This aspect is treated in Section 2.3 where the Braced/Unbraced and Sway/Non-sway classifications are explained.

When the configuration of the structure is such that the building is sensitive to possible eccentricity of horizontal loading, relative to the centre of resistance to twisting of the structure, the effects of applying only part of the horizontal loading should also be taken into account.

EC3 Annex H
5.2.4.3(5)

2.1.3 Ground-structure interaction

Whether ground-structure interaction should be accounted for or not depends on the significance of the effects on the internal forces and moments in the structural elements of the foundation settlements resulting from the loading on the ground. EC3 Annex H

The following procedure is proposed in Annex H for examining ground-structure interaction :

- As a first step, the structure may be analysed assuming that the ground is rigid. From this analysis, the loading on the ground should be determined and the resulting settlements should be calculated. EC3 5.2.2.3
- The resulting settlements are applied to the structure in the form of imposed deformations and the effects on the internal forces and moments should be evaluated.
- When the effects are significant, the ground-structure interaction should be accounted for. This may be done by using equivalent springs to model the soil behaviour.

No criteria are given in Annex H, or elsewhere in the code, to decide on the significance of the ground-structure interaction. It is suggested here that when they do not reduce the resistance of the structure by more than 5%, they can be considered to be insignificant and can be neglected in design. This criterion is the same as that given for the classification of rigid joints. EC3 6.4.2.2(2)

2.1.4 Modelling of frames

The following guidelines are taken from Annex H:

- The members and joints should be modelled for global analysis in a way that appropriately reflects their expected behaviour under the relevant loading. EC3 Annex H 2.3.1(4)
- The basic geometry of a frame should be represented by the centrelines of the members.
- It is normally sufficient to represent the members by linear structural elements located at their centrelines, disregarding the overlapping of the actual widths of the members.
- Alternatively, account may be taken of the actual width of all or some of the members at the joints between members. Methods which may be used to achieve this are proposed in Annex H. They include one involving special flexible joints.

2.1.5 Framing and joints

In Eurocode 3 and Eurocode 4, the term ***framing*** is used to distinguish between the various ways that joint behaviour can be considered for global analysis. 1.4.2
It is recognised that in general, due to joint deformations, deflected shapes of members will be discontinuous at the joints. Depending upon the effects of this discontinuity, a distinction can be made between the following cases:

- The discontinuity may be neglected, i.e. the joints are assumed to be rigid, and the frame may be analysed as continuous. This is called ***continuous framing***.
- The discontinuity may be taken into account by assuming a pinned (hinged) joint model, taking advantage of possible rotations without considering joint moment resistances. This is called ***simple framing***,
- The discontinuity at the joints may also be accounted for by using ***semi-continuous framing***. This is where a joint model (i.e. a semi-rigid joint model) is used in which its moment-rotation behaviour is taken into account more precisely.

The use of a continuous, or of a simple type of framing, must be justified by an appropriate choice of joint type (“ rigid ” and “ simple ” joint classifications respectively). Whilst it is likely that for the analysis of many typical frames, a choice of only one of these framing possibilities will be made throughout the frame for the beam to column joints, the use of different framing types for various parts of a given frame can be envisaged. EC3 5.2.2(2)

Joint modelling is introduced in Lecture 9 on “Joints” .

2.2 Main structural elements

As stated in Lecture 1, a typical composite building is composed of a number of different main structural elements which allow the transfer all the loads applied to the structure to the foundation. These are: EC3 5.1.3

- the composite slabs;
- the composite beams;
- the steel or composite columns.

These elements are illustrated in Figure 2.

Two other important types of structural elements also require special consideration:

- the joints which allow the transfer of internal forces between any connected members;
- the bracing system, when existing, which transfers the horizontal forces acting on the structure to the ground. However modelling and design procedures for bracing systems are not addressed in the SSEDTA course. EC3 5.5.3

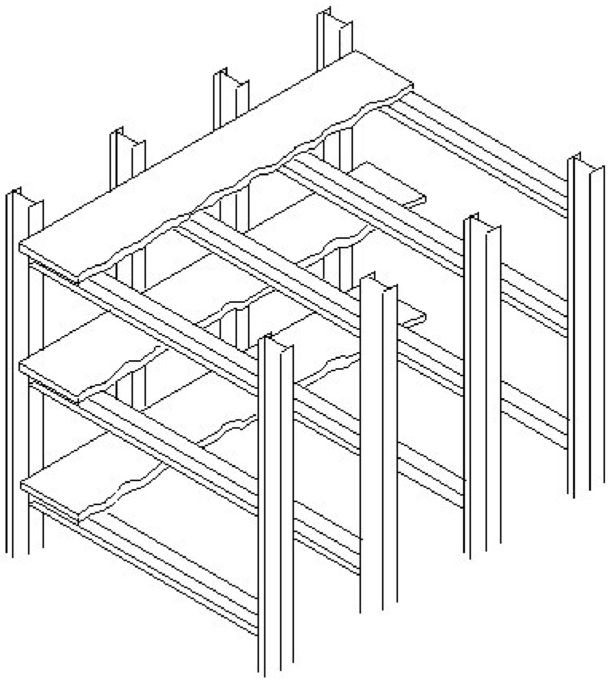


Figure 2 - Main structural elements

In Figure 1, the reduction of a three-dimensional framework has presented as the usual way to model the structure for practical design purposes. This principle, which is easily applied to steel structures, requires further attention as far as composite buildings are concerned. The specific reason for this is the presence at each floor of a two-dimensional composite slab.

In order to overcome this difficulty, and to avoid the need for three-dimensional modelling and design of the frame, the following procedure is usually followed:

- the slab is assumed to span in a principal direction and is designed accordingly.
- the three-dimensional framework is then reduced, as previously explained, to plane frames which are studied independently each from the other.
- in order to enable such a "dissociation" into plane frames, the concept of effective width for the composite slabs is introduced.
  As a result, a composite beam is constituted by a steel profile and an effective slab; both components are connected together by shear connectors, as explained in Lecture 1.

In the following paragraphs details about the concept of "slab effective width" are given. The notion of an equivalent modular ratio which is helpful for the elastic derivation of the elastic properties of composite elements (beams or columns) is introduced.

2.2.1 Effective width of slab 5.2.2

In a composite beam, the transfer of the shear stress between the steel beam and concrete slab fully mobilises the slab only if the width of slab between adjacent steel beams, $2b_i$, is not too great (Figure 3). In reality the induced stress distribution in the slab is not uniform: it is higher close to the steel beam and diminishes progressively away from the beam. This phenomenon is known as *shear lag*.

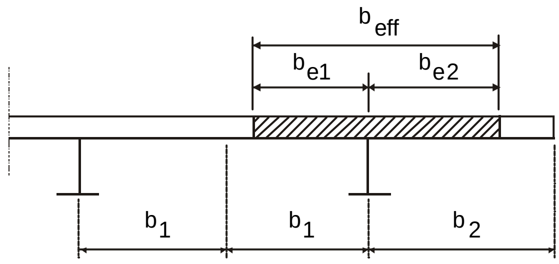


Figure 3 - Effective width of slab for beam

In order to treat a composite floor as an assembly of independent tee-sections, the concept of an effective width b_{eff} of the slab is introduced. Thus a width of slab is associated with each beam such that the normal flexural constraint calculated by Navier's assumption and applied to the composite section thus defined, would provide the same maximum constraint as that originating in the actual non-uniform distribution. The value of b_{eff} depends, in quite a complex manner, on the relation of the spacing $2b_i$ to the span L of the beam, on the type of load, on the type of supports to the beam, on the type of behaviour (elastic or plastic) and on other factors besides. That is why in the domain of building, most of the design codes are satisfied with simple safe formulae. Eurocode 4 proposes the following expression:

$$b_{eff} = b_{e1} + b_{e2}$$

with $b_{ei} = min\ (L_o/8; b_i)$

where L_o is equal to the distance measured between consecutive points of contraflexure in the bending moments diagram.

In the case of a beam on two supports the length L_o is equal to the span L of the beam. For continuous beams, L_o representing the length of the beam subject to positive moments determined from Figure 4. One can discern there an effective width of slab under a positive bending moment, based on a length L_o used to represent the length of the beam subject to positive moments and an effective width under a negative moment (in the region of intermediate supports), based on a length L_o, representing the length of the negative moments beam. It should be noted, in this latter case, that the effective width reduces to the longitudinal reinforcement bars only (those included in this width) if one allows that concrete is not at all resistant to tension stress.

It will be noted that the L_o lengths from two adjacent zones partially overlap; that is explained by the fact that, in practice, one has to consider not the bending moments diagram generated by a single load combination but from envelope diagrams which present the same type of interference.

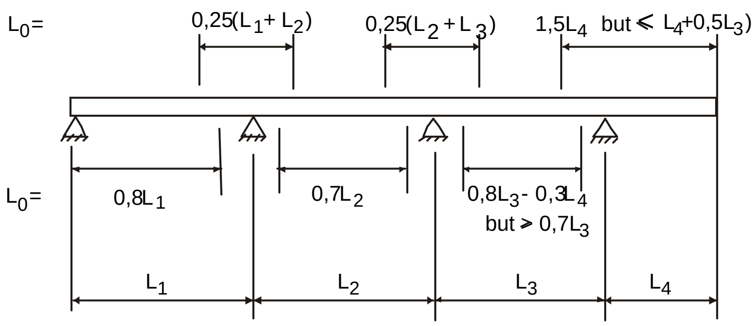


Figure 4 - Lengths L_0 for the determination of effective width

This effective width serves both to check the resistance of transverse sections and to determine the elastic properties of these sections.

2.2.2 Equivalent modular ratio

2.2.2.1 Generalities 5.2.3

The mechanical and geometrical properties of a composite section are required for the calculation of the internal stresses and the deformations.

In an elastic design approach, the concrete in compression and the steel are assumed to behave in a linearly elastic fashion. Where Eurocode 4 permits the use of the uncracked flexural stiffness, $(EI)_1$, the concrete in tension may be considered uncracked. Where the flexural stiffness of the cracked section, $(EI)_2$, must be used, the strength of concrete in tension is ignored.

Even after cracking has occurred, the section derives stiffness from the concrete. This "tension stiffening" is due to the uncracked concrete between cracks. This effect is not taken into account in the calculation of section stiffness in this lecture. It is, however, taken into account indirectly in the calculation of deflections and crack widths.

In calculating the section properties of the composite section in the elastic range, use is made of the concept of the transformed section. Using this concept, the steel-concrete composite section is replaced by an equivalent homogeneous section in steel. For a section subjected to positive bending, the concrete flange of area A_c is replaced with a fictitious steel flange of area A_c/n, where *n* is the modular ratio (see Section 2.2.2.3.). The fictitious steel flange is of similar depth to the concrete flange, see Figure 5. Geometrical properties are readily calculated for the transformed section, and strains may be obtained using the elastic modulus for steel. Use is again made of the modular ratio in calculating elastic stresses in the concrete flange of the original composite section as shown in Figure 6.

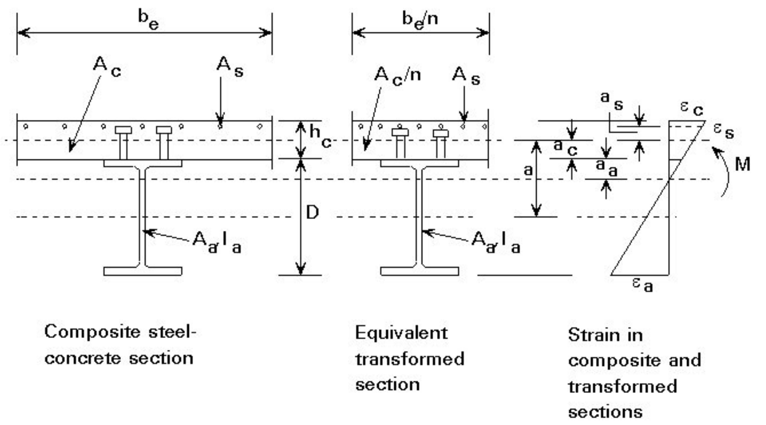


Figure 5 - The transformed section

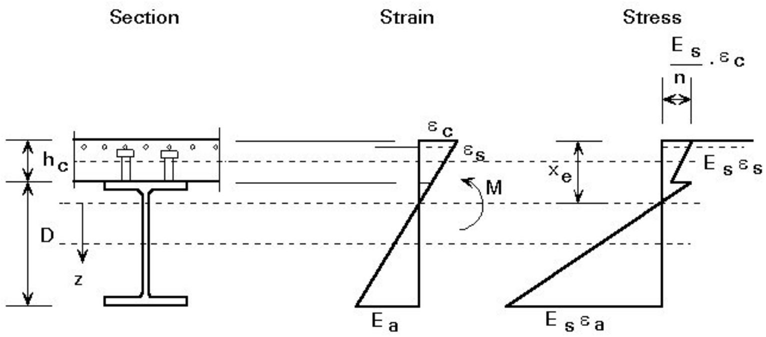


Figure 6 - Elastic strains and stresses in the composite section

2.2.2.2 Elastic Moduli

Young's modulus for steel

The value of the elastic modulus for structural steel, E_a, is given as $210x10^3\ N/mm^2$ in Eurocode 4. A value of $200x10^3 N/mm^2$ is given in Eurocode2 [1] for the elastic modulus of reinforcing steel, E_s. For simplicity, the Eurocode 4 value, $210x10^3\ N/mm^2$, may be adopted for structural and for reinforcing steel alike.

Elastic modulus for concrete - short-term

Concrete is a non-linear, non-elastic material. It does not display a unique or constant value of elastic modulus, as shown in Figure 7, and sustains permanent deformation on removal of load. When subjected to a constant stress, concrete strains increase with time - a phenomenon known as creep - see Figure 8. It is also subject to changes of volume caused by shrinkage (or swelling), and by temperature changes.

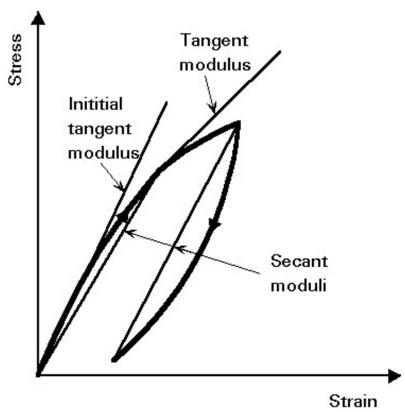


Figure 7 - Stress-strain curve for concrete, showing various moduli

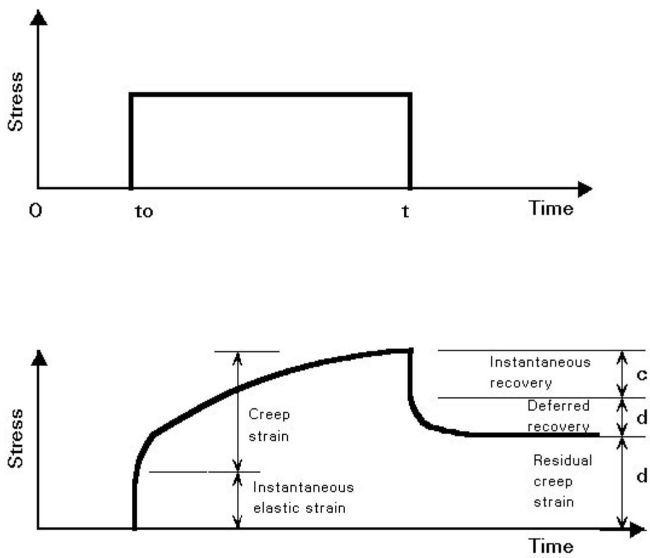


Figure 8 - Creep in concrete

Notwithstanding this non-linearity, it is necessary to be able to quantify the relationship between stress and strain in order to obtain a realistic estimate of deformations. Various elastic moduli are shown in Figure 7.

These are:

- an initial tangent modulus;
- a tangent modulus corresponding to a given stress level;
- a secant modulus;
- and a "chord" mo dulus.

The values of a number of these moduli are seen to depend on the reference stress level. They are, in addition, affected by the rate of loading. The value used in design codes is generally a secant modulus corresponding to a specified rate of loading.

An estimate of the mean value of the secant modulus E_{cm} for short-term loading, for normal-weight concretes, can be obtained from Table 1 for the range of concrete strengths normally used in composite construction.

Strength Class Concrete (Normal weight concrete)		20/25	25/30	30/37	35/45	40/50	45/55
Characteristic compressive strength - cylinder - cube	 f_{ck} $f_{ck, cube}$	 20 25	 25 30	 30 37	 35 45	 40 50	 45 55
Associated mean tensile strength	f_{ctm}	2,2	2,6	2,9	3,2	3,5	3,8
Secant modulus of elasticity	E_{cm}	29	30,5	32	33,5	35	36

Table 1 - Mean values of secant modulus E_{cm} for short term loading for normal weight concretes

Elastic modulus for concrete – long term

Time-dependent deformation of concrete may be calculated. This item is not addressed here but the reader will find useful information in the relevant sections of Eurocode 2 [1].

2.2.2.3 Modular ratio

In the calculation of the geometrical properties of the section, and of stresses, reference is made to the modular ratio, n. This is the ratio E_a/E_c, where E_a is the elastic modulus of structural steel, and E_c is that of the concrete. The effect of the modular ratio on stresses is illustrated in Figure 6. 5.2.3(1)

For the calculation of long-term effects in buildings, sufficiently accurate results will be obtained by using an effective modulus for concrete, $E_{c'}$, in the calculation of the modular ratio. The effective modulus is the short-term modulus for concrete modified for the effects of creep.

Eurocode 4 gives three sets of values for short-term and long-term modular ratios. These values are listed in order of increasing simplicity in Table 2. It will not usually be necessary to resort to method (a) in that table, which involves explicit calculation of the creep coefficient f. The choice of method should take account of the purpose of the analysis and the accuracy required. It is noted in Eurocode 4 that the value of the modular ratio has much less influence on the accuracy of calculated action effects than on calculated stresses or deformations. Method (c), which adopts the same high value of the modular ratio for both short-term and long-term effects, could thus be used conveniently for the global analysis of structures; this would remove the need for separate analyses for these two conditions.

Option	Short-term effects	Long-term effects	Comments
(a)	Secant modulus E_{cm} (Table 1)	Various, depending on concrete grade	This method takes account of concrete grade and age.
(b)	6	18	Takes no account of concrete grade, but of concrete age
*(c)	15	15	Takes no account of concrete grade or age

* Restricted to beams, the critical sections of which are Class 1 or 2

Table 2 - Values of the modular ratio

2.3 Frame classification

2.3.1 Braced and unbraced frames 5.1.9.3

When bracing is provided it is normally used to prevent, or at least to restrict, sway in multi-storey frames. Common bracing systems are trusses or shear walls (Figure 9).

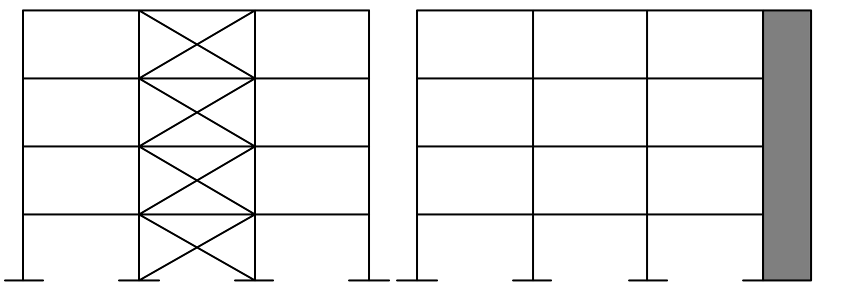


Figure 9 - Common bracing systems

For a frame to be classified as a **braced frame**, it must possess a bracing system which is adequately stiff.

When it is justified to classify the frame as **braced**, it is possible to analyse the frame and the bracing system separately as follows :
The frame without the bracing system can be treated as fully supported laterally and as having to resist the action of the vertical loads only.
The bracing system resists all the horizontal loads applied to the frames it braces, any vertical loads applied to the bracing system and the effects of the initial sway imperfections from the frames it braces and from the bracing system itself.

It should be noted that in a frame with a truss type or frame type bracing system, some members participate in the bracing system in addition to being part of the frame structure (without bracing).

For frames without a bracing system, and also for frames with a bracing system but which is not sufficiently stiff to allow classification of the frame as braced, the structure is classified as **unbraced**. In all case of unbraced frames, a single structural system, consisting of the frame and of the bracing when present, shall be analysed for both the vertical and horizontal loads acting together as well as for the effects of imperfections.

2.3.2 Braced and unbraced classification criteria

The existence of a bracing system in a structure does not guarantee that the frame structure is to be classified as braced. Only when the bracing system reduces the horizontal displacements by at least 80% can the frame be classified as braced.

If no bracing system is provided: the frame is ***unbraced***.

When a bracing system is provided the following applies :

- when $Y_{br} > 0{,}2\ Y_{unbr}$: the frame is classified as ***unbraced***,

- when $Y_{br} \leq 0{,}2\ Y_{unbr}$: the frame is classified as ***braced***,

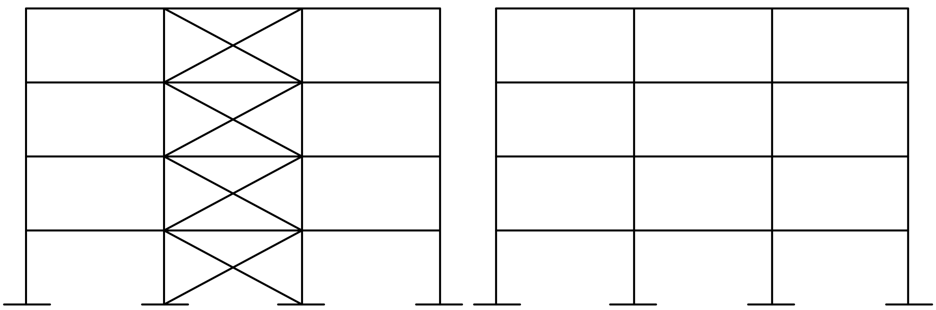
where:

Y_{br} is the lateral flexibility of the structure with the bracing system.
Y_{unbr} is the lateral flexibility of the structure without the bracing system.

2.3.3 Sway and non-sway frames 5.1.9.2

The term **non-sway frame** is applicable when the frame response to in-plane horizontal forces is sufficiently stiff for it to be acceptable to neglect any additional forces or moments arising from horizontal displacements of its nodes. The global second-order effects (i.e. the P-Δ sway effects) may be neglected for a non-sway frame.

When the global second-order effects are not negligible, the frame is said to be a **sway frame**.



Braced frame (may be sway if bracing is very flexible)   Unbraced frame (may be non-sway if not sensitive to horizontal loads)

Figure 10 - Braced and unbraced frame

Normally a frame with bracing is likely to be classified as ***non-sway***, while an unbraced frame is likely to be classified as ***sway***. However, it is important to note that it is theoretically possible for an unbraced frame to be classified as non-sway (this is often the case of one storey portal frame buildings) while a frame with bracing may be classified as sway (possible for multi-storey buildings) (see Figure 10).

When a frame is classified as ***non-sway***, a first-order analysis may always be used.

When a frame is classified as ***sway***, a second-order analysis shall be used. A procedure involving iterations on a first-order elastic analysis is usually adequate for this purpose. Furthermore, on condition that the structure meets certain conditions, a first-order analysis (without any iteration process) may be used either by making a nominal correction to member end forces to allow for the global second-order effects or by analysing for vertical loads and for sway load effects (to be magnified for design) separately.

It should be noted that bracing systems which are themselves frames (or sub frames) must also be classified as sway or non-sway .

2.3.4 Sway and non-sway classification criteria

The classification of a frame structure (or bracing system) as sway or non-sway is based on the value of the ratio of the design value of the total vertical load V_{Sd} applied to the structure to its elastic critical value V_{cr} producing sway instability (failure in the sway mode).

Obviously, the doser that the applied load is to the critical load, the greater is the risk of instability and the greater are the global second-order effects on the structure (the P- Δ effects).

The classification rule is as follows :

- V_{Sd} / V_{cr} *£ 0,1* the structure is classified as ***non-sway***.

- $V_{Sd} / V_{cr} > 0{,}1$ the structure is classified as ***sway***.

This rule can also be expressed in the following way:

- $l_{cr} = \frac{V_{cr}}{V_{Sd}} \geq 10$ the structure is classified as ***non-sway***.

- $l_{cr} = \frac{V_{cr}}{V_{Sd}} < 10$ the structure is classified as ***sway***.

3. The design process

As for all other building structures the design of composite frames is a three-step procedure which implies successively:

- the pre-design of the main structural components, i.e. slabs, beams, columns and joints;
- the structural frame analysis aimed at defining the distribution of internal forces (bending moments, shear and axial forces, ...) and displacements for different loading stages and combinations;
- the design checks both at serviceability and ultimate limit states

The pre-design of a structure usually requires the designer to have had experience from past projects which progressively improve his ability to determine initial layouts and approximate member sizes. No guidelines on how to pre-design a composite structure are given in the present lecture notes.

The manner in which the structural frame analysis has to be carried out is discussed in Lecture 5. As the design of the slab may be achieved independently of the rest of the framed structure (see Lecture 4), the structural frame analysis considered in the present course therefore only involves beam, column and joint components.

The design checks which have to be performed under service and ultimate loads to ensure the adequacy of the structure to the design requirements are described in Lecture 4 (slabs), Lecture 6 (simply supported beams), Lecture 7 (continuous beams), Lecture 8 (columns) and Lecture 9 (joints) where both principles and application rules are given.

In the present lecture, only the general design requirements for composite structures at serviceability and ultimate limit states are briefly presented.

4. Generalities about design requirements

As a preliminary and general remark, it is worthwhile recognising that, for composite construction, it is necessary to distinguish between the building stage and the final stage of construction when evaluating the structural performance. It is also important to incorporate the appropriate live loads at each stage of the erection process. 5.1.4.2

4.1 Verifications at serviceability limit state (SLS) 7

The structural requirements of the whole under service loads relate to the control and the limitation of the following values: 7.1(2)

- the transverse displacements of the composite beams;
- the cracking of the concrete;
- the beam vibrations, especially for large span beams.

For building structures, the design requirements under service loads are often of a conventional nature and the designer will try as much as possible to avoid performing a detailed structural analysis or a precise cross-section verification.

For instance, the influence of the concrete shrinkage on the transverse beam displacements is only taken into consideration for simply supported beams with a "span-to-total section depth" ratio higher than *20* and providing that the free shrinkage deformation likely to occur exceeds 4×10^{-4}. Similarly it is permitted to simplify the elastic analyses of the structural frame through the adoption of a single equivalent modular ratio n'' for the concrete which associates the creep deformations under long-term actions and the instantaneous elastic deformations.

In addition Eurocode 4 does not adopting a kind of "admissible stress" design criterion under service loads; this means that a partial yielding of the cross-section components is allowed:

- either at mid-span (where this partial yielding has a limited influence on the beam transverse displacements);
- or at the supports in continuous beams (the influence on the transverse beam displacements being then considered in a conventional way).

Experience shows that a concentration of plastic deformations at specific locations probably never occurs because of the nature of the loading applied to the beams in building frames and the high proportion of permanent loads.

4.2 Verifications at ultimate limit state (ULS) 6

As all the other types of building structures, the composite building frames may reach their collapse due to a lack of resistance or a lack of stability. Appropriate resistance and stability verifications have therefore to be carried out in addition to the frame analysis to ensure the structural and economic adequacy of the structure with the design requirements at ultimate limit states.

The nature of these verifications may vary according to the structural system and a so-called "frame classification" is preliminary required (see Section 2.3 in the present Lecture).

In Eurocode 4 design application rules are only provided for non-sway-frames in which the risk of frame instability affecting the structure as a whole is fully prevented.

As a result only local instability checks on constitutive members will have to be carried out in addition to usual resistance checks for cross-sections.

Those applicable to simple and continuous beams are listed in the following paragraphs.

Specific design checks for beams

The ultimate limit state design checks relate to:

- The resistance of critical sections, defined as the points of maximum bending moment (section I-I on Figure 11, or points where a concentrated load is applied in addition to a distributed load), maximum shear (section II-II at external supports) or where the combined effect of bending moment and shear is likely to be greatest (sections III-III on Figure 11). Points where there is a sudden change of section and/or mechanical properties, other than a change due to cracking of the concrete, should also be considered.
- The strength of the longitudinal shear connection (line IV-IV);
- The longitudinal shear strength of the transversally reinforced concrete slab (line V-V and VI-VI);
- The resistance to lateral-torsional buckling under negative bending moments, with lateral displacement of the bottom flange of the steel section (buckled position VII);

- The shear strength and buckling resistance of the web (in zones with high shear forces, close to the critical sections II-II and III-III) and possibly the crippling strength of the web at points where concentrated loads or reactions are applied (for example at a support when the web of the steel beam is not transversally stiffened).

In Lectures 6 and 7 where the design guidelines on how to practically perform these resistance and stability checks are carefully described, the two basic concepts introduced in Sections 2.2.1. and 2.2.2. and known as "the slab effective" width" and the "equivalent modular ratio" are extensively used.

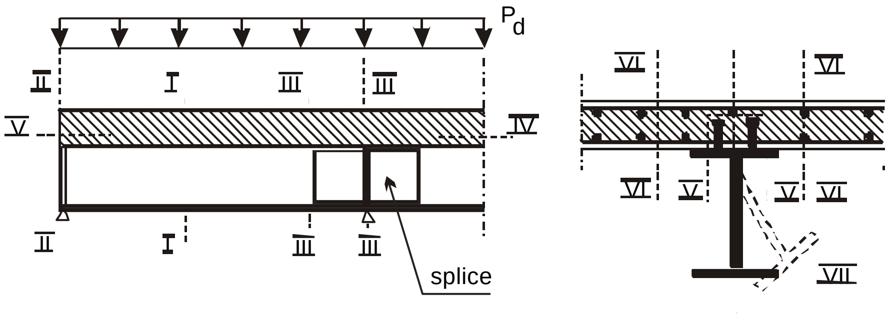


Figure 11 - Ultimate limit state design checks

5. Modelling and design flow chart

The whole structural modelling, analysis and design of non-sway composite frame described in the present Lecture is summarised in the following flow chart (Table 3). At each step in the flow chart, the number of the Lecture reflecting the specific topic is indicated in the left hand column or within the individual cell.

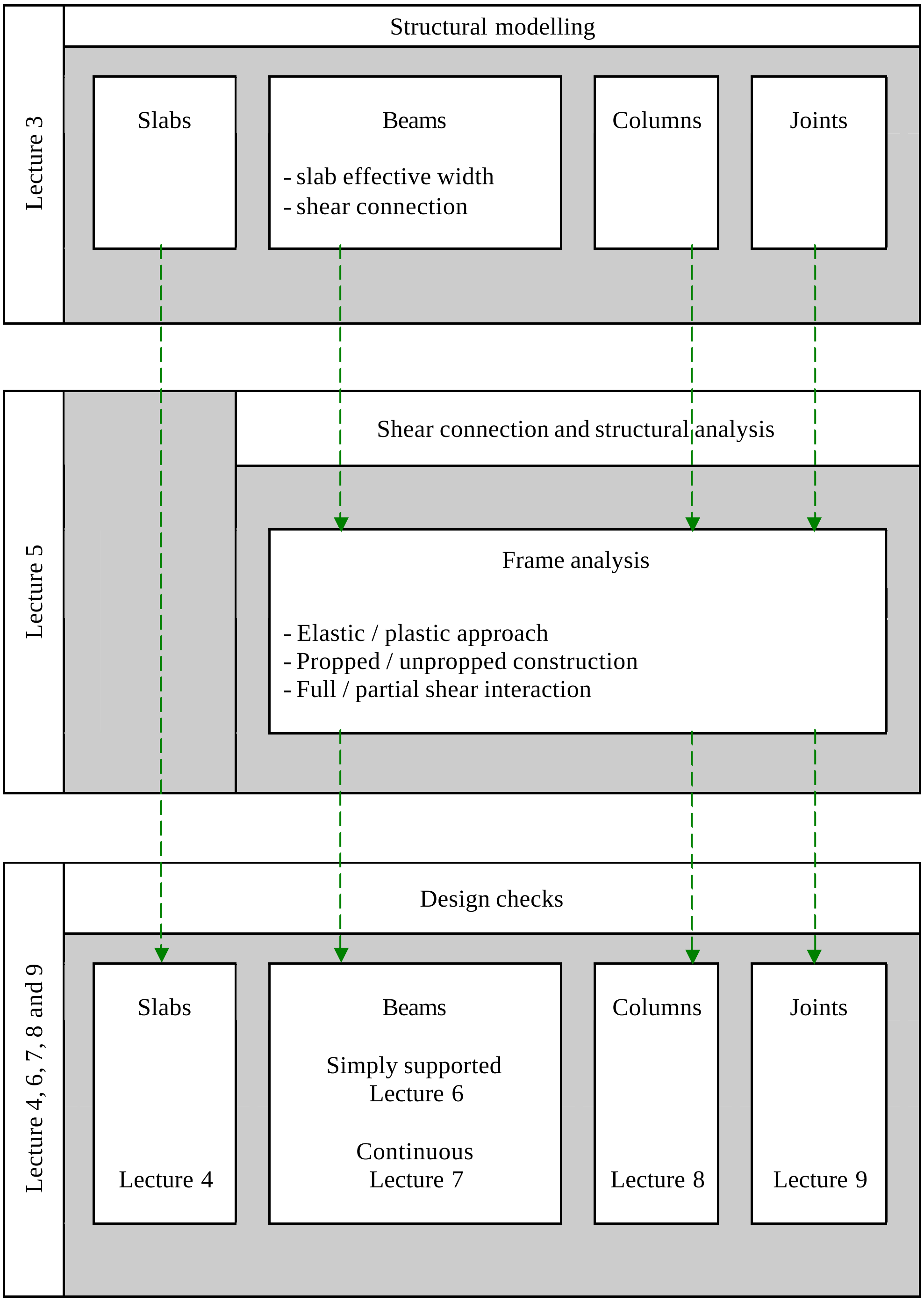


Table 3 - Flow chart of actions.
Modelling, analysis and design process

6. Conclusions

This lecture has introduced the reader to the concepts involved in the representation of a composite structure into a form which is suitable for analysis. This is an integral part of the design process. The simplifications which are required to make the process practical have been explained.

Distinctions have been drawn between braced and unbraced frames, and between sway and non-sway frames, in a way in which their implications for design are apparent.

General remarks regarding the checks required at both serviceability and ultimate limit states have been summarised.

Finally the whole design process has been set out in a tabular form. References to the individual lectures in this series, in which each component is considered in detail, is clearly indicated. This diagram thus establishes a framework for the following chapters.

Structural Steelwork Eurocodes
Development of
A Trans-national Approach 2

Course: Eurocode 4

Lecture 4: Composite Slabs with Profiled Steel Sheeting

Summary:

- Composite floors are frequently used in multi-storey building construction.
- A composite slab comprises steel decking, reinforcement and cast in situ concrete. When the concrete has hardened, it behaves as a composite steel-concrete structural element.
- Modern profiled steel may be designed to act as both permanent formwork during concreting and tension reinforcement after the concrete has hardened.
- Design of composite slabs requires consideration of the performance of the steel sheeting as shuttering during construction and as reinforcement to the hardened concrete slab.
- Loading, analysis for internal forces and moments, and section verification are explained.
- The shear connection between the steel sheeting and concrete is of particular importance. This is usually determined by tests.
- Design methods - the semi-empirical m-k method and the partial interaction method are explained.

Pre-requisites:

- Familiarity with EC2 and EC3

Notes for Tutors:

This material comprises one 45 minute lecture.

Objectives:

The student should:

- Appreciate the advantages of composite floors
- Recognise that the design of composite slabs requires consideration of the construction and in service conditions
- Be aware of the analysis methods available for determining internal forces and moments
- Know how to perform design checks at the serviceability and ultimate limit states
- Understand the basis of the semi-empirical and partial interaction design approaches.

References:

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Contents:

1. Introduction

The widespread popularity of steel in multi-storey building construction is in part due to the use of composite floors. A composite slab comprises steel decking, reinforcement and cast in situ concrete (Figure 1). When the concrete has hardened, it behaves as a composite steel-concrete structural element. Modern profiled steel may be designed to act as both permanent formwork during concreting and tension reinforcement after the concrete has hardened. After construction, the composite slab consists of a profiled steel sheet and an upper concrete topping which are interconnected in such a manner that horizontal shear forces can be transferred at the steel-concrete interface.

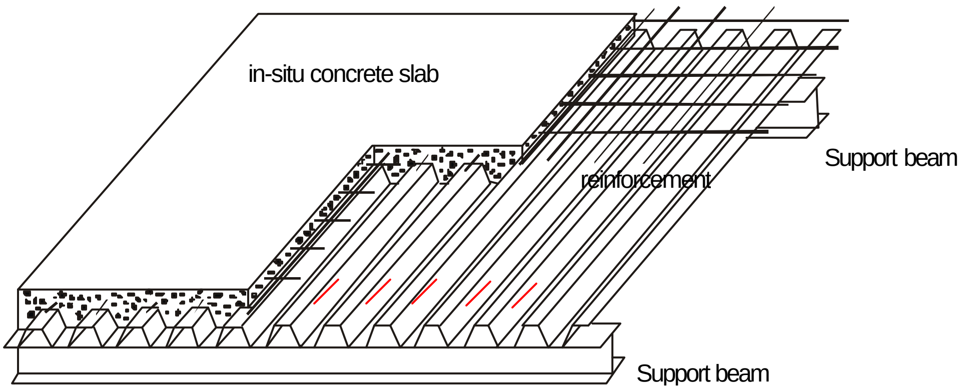


Figure 1 Composite slab with profiled sheeting

Composite floor construction is essentially an overlay of one-way spanning structural elements. The slabs span between the secondary floor beams, which span transversely between the primary beams. The latter in turn span onto the columns. This set of load paths leads to rectangular grids, with large spans in at least one direction (up to 12, 15 or even 20 m).

Composite slabs are supported by steel beams, which normally also act compositely with the concrete slab. The spacing of the beams, and therefore the slab span, depends on the method of construction. If the beam spacing is below about 3,5m, then no temporary propping is necessary during concreting of the slab. In this case, the construction stage controls the design of the metal decking. Due to the short slab span, the stresses in the composite slab in the final state after the concrete has hardened, are very low. For such floors, trapezoidal steel sheets with limited horizontal shear resistance and ductility are most often used. They have the lowest steel weight per square metre of floor area. For other floor layouts where the lateral beam spacing is much larger, props are necessary to support the metal decking during concreting. Due to the longer slab span, the final composite slab is highly stressed. As a result this final state may govern the design. In this case the steel sheeting will require good horizontal shear bond resistance. Re-entrant profiles are often used leading to greater steel weight per square metre of floor area.

Composite floor construction used for commercial and other multi-storey buildings, offers a number of important advantages to the designer and client:

- speed and simplicity of construction
- safe working platform protecting workers below
- lighter construction than a traditional concrete building
- less on site construction
- strict tolerances achieved by using steel members manufactured under controlled factory

conditions to established quality procedures.

The use of profiled steel sheeting undoubtedly speeds up construction. It is also often used with lightweight concrete to reduce the dead load due to floor construction. In the UK and North America, this use of lightweight concrete is common practice for commercial buildings.

1.1 Profiled decking types

Numerous types of profiled decking are used in composite slabs (Figure 2). The different types of sheeting present different shapes, depth and distance between ribs, width, lateral covering, plane stiffeners and mechanical connections between steel sheeting and concrete. The profiled sheeting characteristics are generally the following:

- Thickness between 0,75mm and 1,5mm and in most cases between 0,75mm and 1mm;
- Depth between 40mm and 80mm; (deeper decks are used in slim floor systems - lecture 10).
- Standard protection against corrosion by a thin layer of galvanizing on both faces.

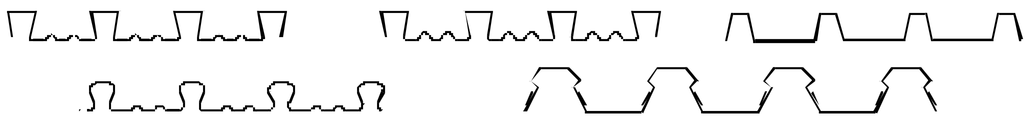


Figure 2 Types of profiled decking

Profiled decking is cold formed: a galvanised steel coil goes through several rolls producing successive and progressive forming. The cold forming causes strain hardening of the steel resulting in an increase of the average resistance characteristics of the section. Generally, a S235 grade coil has a yield limit of approximately 300 N/mm^2 after cold forming.

1.2 Steel to concrete connection

The profiled sheeting should be able to transfer longitudinal shear to concrete through the interface to ensure composite action of the composite slab. The adhesion between the steel profile and concrete is generally not sufficient to create composite action in the slab and thus an efficient connection is achieved with one or several of the following (Figure 3):

- Appropriate profiled decking shape (re-entrant trough profile), which can effect shear transfer by frictional interlock;
- Mechanical anchorage provided by local deformations (indentations or embossments) in the profile;
- Holes or incomplete perforation in the profile;
- Anchorage element fixed by welding and distributed along the sheet;
- End anchorage provided by welded studs or another type of local connection between the concrete and the steel sheet;
- End anchorage by deformation of the ribs at the end of the sheeting.

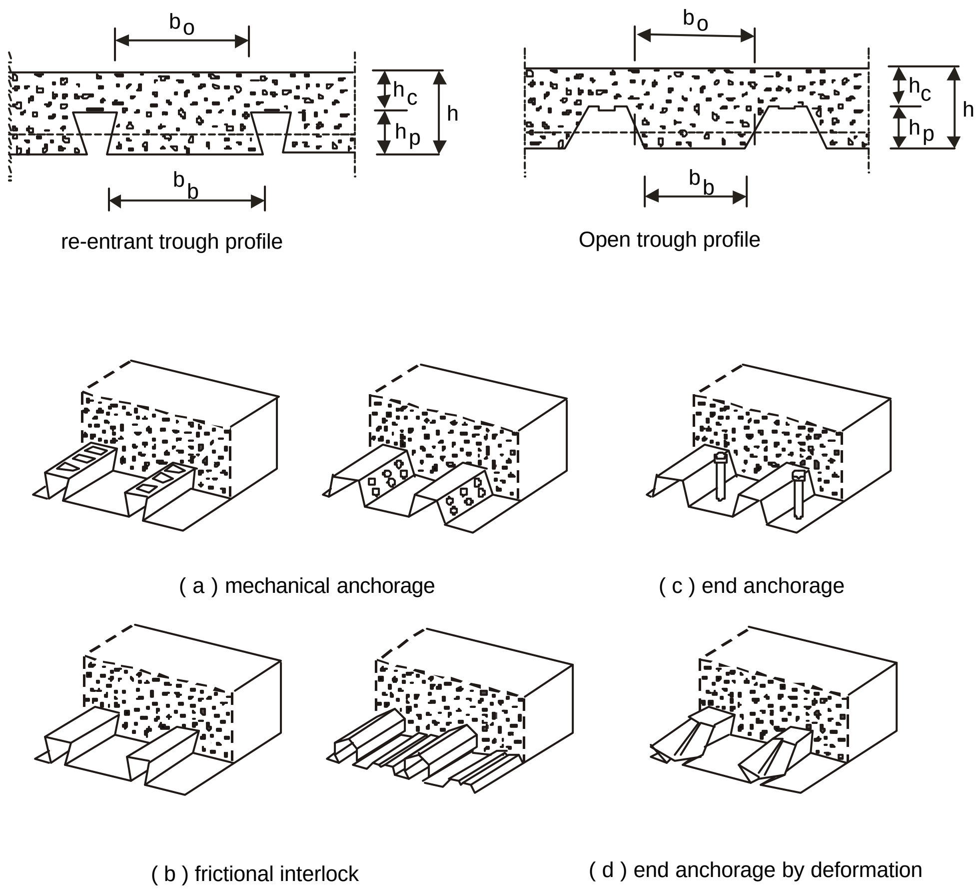


Figure 3 Typical forms of interlock in composite slabs

1.3 Reinforcement of the slab

It is generally useful to provide reinforcement in the concrete slab for the following reasons:

- Load distribution of line or point loads;
- Local reinforcement of slab openings;
- Fire resistance;
- Upper reinforcement in hogging moment area;
- To control cracking due to shrinkage.

Mesh reinforcement may be placed at the top of the profiled decking ribs. Length of, and cover to, reinforcement should satisfy the usual requirements for reinforced concrete.

1.4 Design Requirements

Design of composite slabs is treated in chapter 9 of *Eurocode 4*. The code concerns the design *9.1.1*
of the slabs in multi-storey buildings, which are generally mainly loaded with static loads, or the slabs of industrial buildings subjected to mobile loading. Composite slabs can also be used in structures subject to repetitive or suddenly applied loads causing dynamic effects. However, particular care is needed with construction details in order to avoid any damage of the composite action.

Referring to figure 3, the overall depth of the composite slab, h, should not be less than 80 mm. The thickness h_c of concrete above the ribs of the decking should be greater than 40mm to ensure ductile behaviour of the slab and sufficient cover of reinforcing bars. If the slab acts compositely with a beam, or is used as a diaphragm, the minimum total depth h is 90mm and the minimum concrete thickness h_c above decking is increased to 50mm. *9.2.1*

The nominal size of aggregate depends on the smallest dimension on the structural element within which concrete is poured, and should not exceed the least of: *9.2.2*

- 0,40 h_c where h_c is the depth of concrete above the ribs
- $b_o/3$, where b_o is the mean width of the rib (minimum width for re-entrant profiles);
- 31,5 mm (sieve C 31,5).

These criteria ensure that the aggregate can penetrate easily into the ribs.

Composite slabs require a minimum bearing of 75mm for steel or concrete and 100mm for other materials. *9.2.3*

2. Composite slab behaviour

Composite behaviour is that which occurs after a floor slab comprising of a profiled steel sheet, plus any additional reinforcement, and hardened concrete have combined to form a single structural element. The profiled steel sheet should be capable of transmitting horizontal shear at the interface between the steel and the concrete. Under external loading, the composite slab takes a bending deflection and shear stresses appear at the steel-concrete interface.

If the connection between the concrete and steel sheet is perfect, that is if longitudinal deformations are equal in the steel sheet and in the adjacent concrete, the connection provides *complete interaction*. If a relative longitudinal displacement exists between the steel sheet and the adjacent concrete, the slab has incomplete interaction. The difference between the steel and adjacent concrete longitudinal displacement can be characterised by the relative displacement called *slip*.

Composite slab behaviour is defined with the help of a standardised test as illustrated in figure 4: a composite slab bears on two external supports and is loaded symmetrically with two loads P applied at ¼ and ¾ of the span. Calling the deflection at mid-span of the slab d, the load-deflection curve, P-d, is an effective representation of the slab behaviour under load. This behaviour depends mainly on the steel-concrete connection type (shape, embossment, connectors, …).

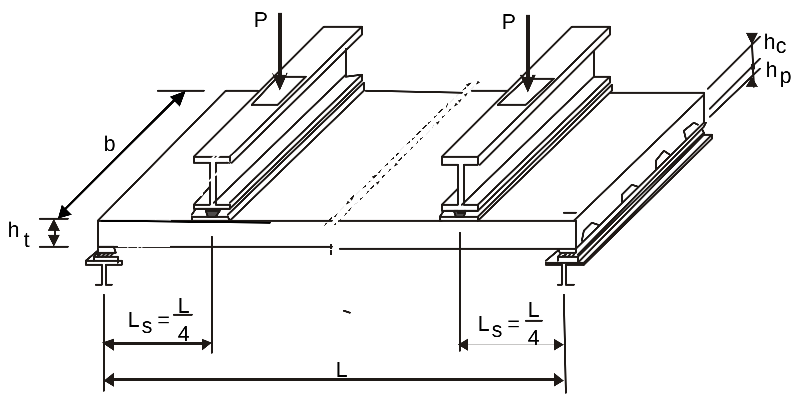


Figure 4 Standardised test

Two types of movement can be identified at the steel-concrete interface :

- local *micro-slip* that cannot be seen by the naked eye. This micro-slip is very small and allows the development of the connection forces at the interface;
- interface global *macro-slip* that can be seen and measured and depends on the type of connection between the concrete and steel.

Three types of behaviour of the composite slab can be identified (Figure 5) :

- *Complete interaction* between steel and concrete: no global slip at the steel-concrete interface exists. The horizontal transfer of the shear force is complete and the ultimate load P_u is at its maximum, the composite action is complete. The failure can be brittle, if it occurs suddenly or *ductile* if it happens progressively.
- Zero *interaction* between concrete and steel: global slip at the steel-concrete interface is not limited and there is almost no transfer of shear force. The ultimate load is at its minimum and almost no composite action is observed. The failure is *progressive*.
- *Partial interaction* between concrete and steel: global slip at the steel-concrete interface is not zero but limited. The shear force transfer is partial and the ultimate lies between the ultimate loads of the previous cases. The failure can be *brittle* or *ductile*.

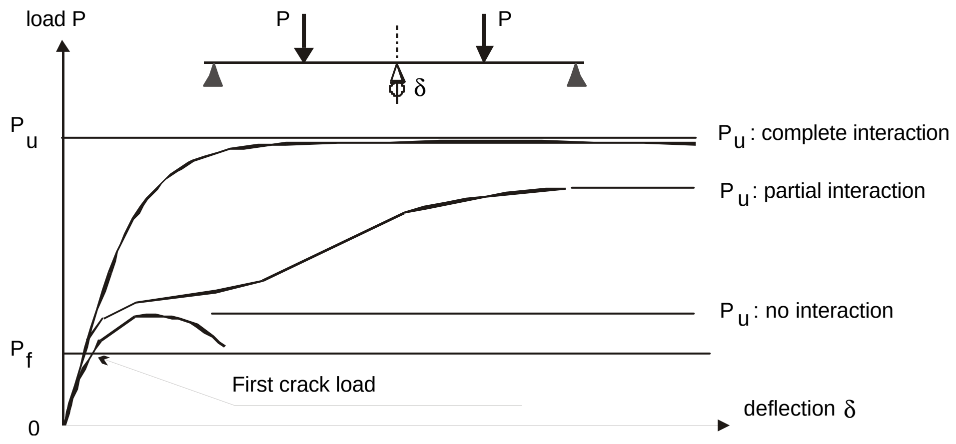


Figure 5 : composite slab behaviour

The composite slab stiffness, represented by the first part of the *P-d* curve, is different for each type of behaviour. This stiffness is at its highest for complete interaction and its lowest for zero interaction.

Three types of link exist between steel and concrete:

- *Physical-chemical link* which is always low but exists for all profiles;
- *Friction link* which develops as soon as micro slips appear;
- *Mechanical anchorage link* which acts after the first slip and depends on the steel-concrete interface shape (embossments, indentations etc)

From *0* to P_f, the physical-chemical phenomena account for most of the initial link between the steel and concrete. After first cracking, frictional and mechanical anchorage links begin to develop as the first micro-slips occur. Stiffness becomes very different according to the effectiveness of the connection type.
Composite slab failure can happen according to one of the following collapse modes (Figure 6).

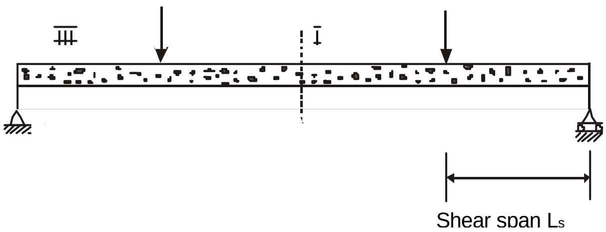


Figure 6 Composite slab failure mode types

- *Failure type I* : The failure is due to an *excessive sagging moment* (section I), that is the bending resistance of the slab $M_{pl.Rd}$; this is generally the critical mode for moderate to high spans with a high degree of interaction between the steel and concrete.
- *Failure type II* : The failure is due to *excessive longitudinal shear*; the ultimate load resistance is reached at the steel concrete interface. This happens in section II along the shear span L_s.
- *Failure type III* : The failure is due to an excessive vertical shear near the support (section III) where vertical shear is important. This is only likely to be critical for deep slabs over short spans and subject to heavy loads

The composite slab failure may be (Figure 7) *Brittle* in which case the failure occurs suddenly generally without observable important deformations or *ductile*, that is the failure happens progressively with significant deformation at collapse and preceded by signs of distress. *11.3.5(1)*

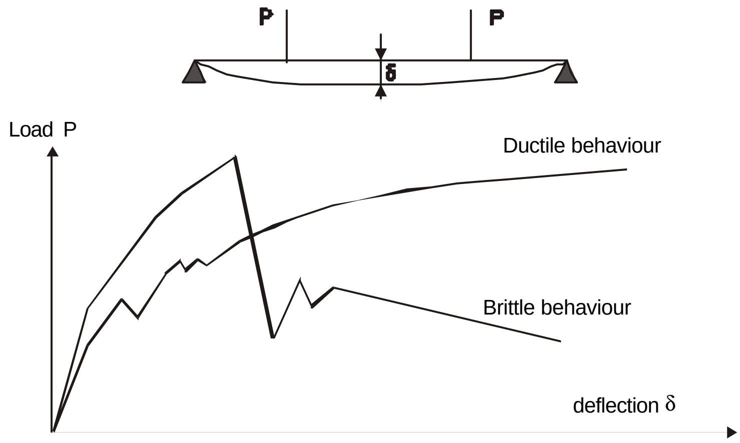


Figure 7 Load deflection response of brittle and ductile slabs

The brittle or ductile mode of failure depends on the characteristics of the steel-concrete interface. Slabs with open trough profiles experience a more brittle behaviour, whereas slabs with re-entrant trough profiles tend to exhibit more ductile behaviour. However, profiled decking producers ameliorate brittle behaviour with various mechanical means, such as embossments or indentations and the use of dovetail forms. Shear connectors between beam and slab also influence the failure mode.

3. Design conditions, actions and deflection

Two design conditions should be considered in composite slab design. The first relates to the situation during construction when the steel sheet acts as shuttering and the second occurs in service when the concrete and steel combine to form a single composite unit. *9.3.1*

3.1 Profiled steel as shuttering

The profiled steel must resist the weight of wet concrete and the construction loads. Although the steel deck may be propped temporarily during construction it is preferable if no propping is used. Verification of the profiled steel sheeting for the ultimate and serviceability limit states should be in accordance with part 1.3 of *Eurocode 3*. Due consideration should be given to the effect of embossments or indentations on the design resistances.

At the *ultimate limit state*, a designer should take the following loads into account :
- Weight of concrete and steel deck;
- Construction loads;
- Storage load, if any;
- 'ponding' effect (increased depth of concrete due to deflection of the sheeting).

Construction loads represent the weight of the operatives and concreting plant and take into account any impact or vibration that may occur during construction. According to *Eurocode 4*, in any area of 3m by 3m, in addition to the weight of the concrete, the characteristic construction load and weight of surplus concrete ('ponding' effect) should together be taken as 1,5kN/m^2. Over the remaining area, a characteristic loading of 0,75kN/m^2 should be added to the weight of concrete. These loads should be placed to cause the maximum bending moment and/or shear (Figure 8). *9.3.2.(1)*
These minimum values are not necessarily sufficient for excessive impact or heaping concrete, or pipeline or pumping loads. If appropriate, provision should be made in design for the additional loading. Without the concrete, the sheet should be shown by test or calculation to be able to resist a characteristic load of 1kN on a square area of side 300mm or to a linear characteristic load of 2kN/m acting perpendicularly to the rib on a width of 0,2m. This load represents the load due to an operative.

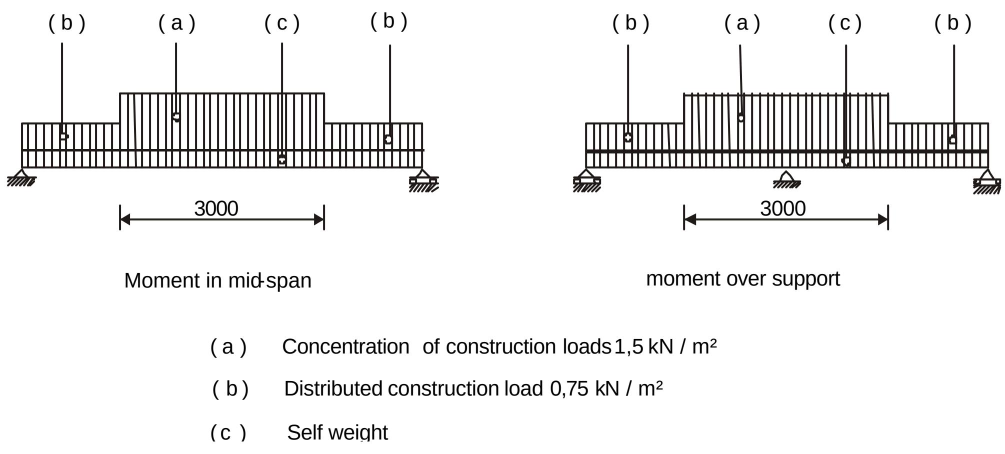


Figure 8 Load arrangements for sheeting acting as shuttering

The deflection of the sheeting under its own weight plus the weight of wet concrete, but excluding construction loads, should not exceed *L/180* where *L* is the effective span between supports. *9.6 (2)*

If the central deflection *d* of the sheeting under its own weight plus that of the wet concrete, calculated for serviceability, is less than 1/10 of the slab depth, the ponding effect may be ignored in the design of the steel sheeting. If this limit is exceeded, this effect should be allowed for; for example by assuming in design, that the nominal thickness of the concrete is increased *9.3.2(2)*

over the whole span by *0,7d*.
Propping can dramatically reduce deflections, props being considered as supports in this context. Use of propping is discouraged as it hinders the construction process and adds time and costs to the project.

3.2 Composite slab

The composite check corresponds to the situation of the slab after the concrete has hardened and any temporary propping has been removed. The loads to be considered are the following :

- Self-weight of the slab (profiled sheeting and concrete);
- Other permanent self-weight loads (not load carrying elements);
- Reactions due to the removal of the possible propping;
- Live loads;
- Creeping, shrinkage and settlement;
- Climatic actions (temperature, wind...).

For typical buildings, temperature variations are generally not considered.

Serviceability limit state checks include the following :

- Deflections;
- Slip between the concrete slab and the decking at the end of the slab called end slip;
- Concrete cracking.

3.2.1 Deflections

The limiting values recommended by *Eurocode 3* are $L/250$ under permanent and variable long duration loads and $L/300$ under variable long duration loads. If the composite slab supports brittle elements (cement floor finishes, non-flexible partitions, etc...), the last limit is then $L/350$.
The deflection of the sheeting due to its own weight and the wet concrete need not be included in this verification for the composite slab. This deflection already exists when the construction work (partitions, floor finishes, door and window frames...) is completed and has no negative influence on these elements. Moreover the bottom of the slab is often hidden by a ceiling.
In practice two span conditions arise for composite slabs. They are either an internal span (for a continuous slab) or an external slab (edge span of a continuous slab or simply-supported slab). *9.8.2(3)*

For an internal span, the deflection should be determined using the following approximations : *9.8.2(4)*

- The second moment of inertia should be taken as the average of the values for the cracked and uncracked section. ;
- For concrete of normal density, an average value of the modular ratio ($n=E_a/E_c$) for both long and short-term effects may be used.

3.2.2 End slip

For external spans, end slip can have a significant effect on deflection. For non-ductile behaviour, initial end slip and failure may be coincident while for semi-ductile behaviour, end slip will increase the deflection. *9.8.2(5)*
Where test behaviour indicates initial slip at the desired service load level for the non-anchored slab, end anchorage (studs, cold formed angles...) should be used in external slabs. Such end slip is considered as significant when it is higher than 0,5mm. Generally no account need be taken of the end slip if this 0,5mm end slip is reached for a load exceeding 1,2 times the desired service load. Where end slip exceeding 0,5mm occurs at a load below 1,2 times the design service load, *9.8.2(6)*

then end anchors should be provided or deflections should be calculated including the effect of the end slip.

3.2.3. Concrete cracking

The crack width in hogging moment regions of continuous slabs should be checked in accordance with *Eurocode 2*. In normal circumstances, as no exposure to aggressive physical or chemical environments and no requirements regarding waterproofing of the slab exist, cracking can be tolerated with a maximum crack width of 0,3mm. If the crack width is higher than this limit, reinforcement should be added according to usual reinforced concrete rules. *9.8.1(1)*

Where continuous slabs are designed as a series of simply supported beams, the cross-sectional area of anti-crack reinforcement should not be less than 0,2 % of the cross-sectional area of the concrete on top of the steel sheet for unpropped construction and 0,4 % for propped construction. *9.8.1(2)*

4. Analysis for internal forces and moments

4.1 Profiled steel sheeting as shuttering

According to *Eurocode 4*, elastic analysis should be used due to the slenderness of the sheeting cross-section. Where sheeting is considered as continuous, flexural stiffness may be determined without consideration of the variation of stiffness due to parts of the cross-section in compression not being fully effective. The second moment of area is then constant and is calculated considering the cross-section as fully effective. This simplification is only allowed for global analysis and hence not for cross-section resistance and deflection checks.

4.2 Composite slabs

The following methods of analysis may be used : *9.4.2 (1)*

- *Linear analysis without moment redistribution* at internal supports if cracking effects are considered ;
- *Linear analysis with moment redistribution* at internal supports (limited to 30 %) without considering concrete cracking effects ;
- *Rigid-plastic analysis* provided that it can be shown that sections where plastic rotations are required have sufficient rotation capacity ;
- *Elastic-plastic analysis* taking into account non-linear material properties.

The application of linear methods of analysis is suitable for the serviceability limit states as well as for the ultimate limit states. Plastic methods should only be used at the ultimate limit state. A continuous slab may be designed as a series of simply supported spans. In such a case, nominal reinforcement should be provided over intermediate supports. *9.4.2 (2)* *9.4.2 (4)*

Where uniformly distributed loads, as is generally the case or line loads perpendicular to the span of the slab, are to be supported by the slab, the effective width is the total width of the slab. Where concentrated point or line loads parallel to the span of the slab are to be supported by the slab, they may be considered to be distributed over an effective width smaller than the width of the slab. *Eurocode 4* gives some explanations on the calculation of these effective widths. To ensure the distribution of line or point loads over the width considered to be effective, transverse reinforcement should be placed on or above the sheeting. This transverse reinforcement should be designed in accordance with Eurocode 2 for the transverse bending moments. If the characteristic imposed loads do not exceed 7,5kN for concentrated loads and 5,0kN/m² for distributed loads, a nominal transverse reinforcement may be used without calculation. This nominal transverse reinforcement should have a cross-sectional area of not less *9.4.3* than 0,2% of the area of structural concrete above the ribs and should extend over a width of not less than the effective width. Reinforcement provided for other purposes may fulfil all or part of this requirement.

5. Verification of sections

5.1 Verification of profiled steel sheeting as shuttering at ultimate limit state (ULS)

The construction load case is one of the most critical. The sheeting, which is a thin steel element, should resist to construction and wet concrete loads (see figure 8).

Verification of the profiled steel sheeting is not treated in detail in *Eurocode 4*. Reference is made to part 1.3 of *Eurocode 3* for that verification. *9.5.1*

For each planar element completely or partially in compression, an effective width should be calculated to account for the effects of local buckling. After calculating the effective widths of all planar elements in compression, the determination of the cross-section properties (effective second moment of area I_{eff} and effective section modulus W_{eff}) can be obtained.
Bending moment resistance of the section is then given by:

$$M_{Rd} = f_{yp} \frac{W_{eff}}{g_{ap}} \qquad \textbf{(1)}$$

5.2 Verification of profiled steel sheeting as shuttering at serviceability limit state (SLS)

The deflection is determined with the effective second moment of area of the sheeting calculated as explained above (5.1). The deflection of the decking under uniformly distributed loads (p) acting in the most unfavourable way on the slab (Figure 9) is given by the following:

$$d = k \frac{5}{384} p L^4 \frac{1}{EI_{eff}} \qquad \textbf{(2)}$$

where L is the effective span between supports.

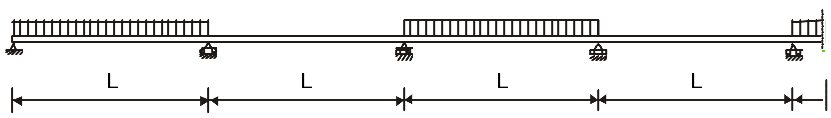


Figure 9 Most unfavourable loading

k coefficient is

k = 1,00 for a simply supported decking;
k = 0,41 for a decking with two equal spans (3 supports);
k = 0,52 for a decking with three equal spans;
k = 0,49 for a decking with four equal spans.

5.3 Verification of composite slab at ultimate limit state (ULS)

5.3.1 Verification of the sagging bending resistance

Type I failure is due to sagging bending resistance. That failure mode is reached if the steel sheeting yields in tension or if concrete attains its resistance in compression. In sagging bending regions, supplementary reinforcement in tension may be taken into account in calculating the composite slab resistance. *9.7.2*

Material behaviour is generally idealised with rigid plastic "stress-block" diagrams. At the ultimate limit state, the steel stress is the design yield strength f_{yp}/γ_{ap}, the concrete stress is its design strength $0{,}85 f_{ck}/\gamma_c$ and the reinforcement steel stress is also its design strength f_{sk}/γ_s.

Anti-cracking reinforcement or tension reinforcement for hogging bending can be present in the depth of the concrete slab. This reinforcement is usually in compression under sagging bending and is generally neglected when evaluating resistance to sagging bending.

Two cases have to be considered according to the position of the plastic neutral axis.

Case 1 – Plastic neutral axis above the sheeting

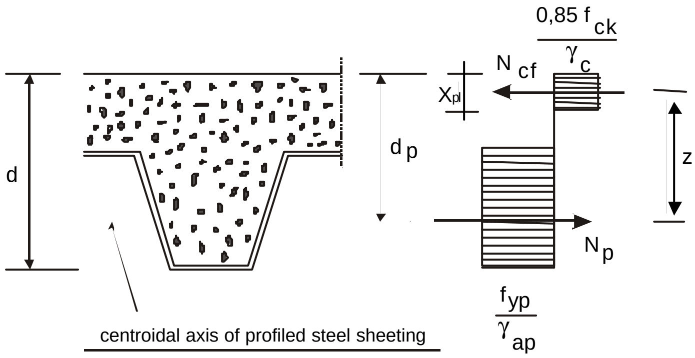


Figure 10 Stress distribution for sagging bending if the neutral axis is above the steel sheet

The resistance of concrete in tension is taken as zero. The resulting tension force N_p in the steel sheeting is calculated with the characteristics of the effective steel section A_{pe}. This force is equated to the resulting compression force in the concrete N_{cf} corresponding to the force acting on the width b of the cross-section and the depth x_{pl} with a stress equal to the design resistance:

$$N_p = A_{pe}\frac{f_{yp}}{\gamma_{ap}} \qquad \textbf{(3)}$$

and

$$N_{cf} = b\, x_{pl}\, \frac{0{,}85\, f_{ck}}{\gamma_c} \qquad \textbf{(4)}$$

Equilibrium gives x_{pl} as:

$$x_{pl} = \frac{\dfrac{A_{pe}\, f_{yp}}{\gamma_{ap}}}{\dfrac{0{,}85\, b\, f_{ck}}{\gamma_c}} \qquad \textbf{(5)}$$

If d_p is the distance from the top of the slab to the centroid of the effective area of the steel sheet (Figure 10), the lever arm z is then:

$$z = d_p - 0.5x \qquad \textbf{(6)}$$

and the design resistance moment is equal to:

$$M_{ps.Rd} = N_p\, z \qquad \textbf{(7)}$$

or

$$M_{ps.Rd} = A_{pe} \frac{f_{yp}}{\gamma_{ap}}(d_p - \frac{x}{2}) \qquad \textbf{(8)}$$

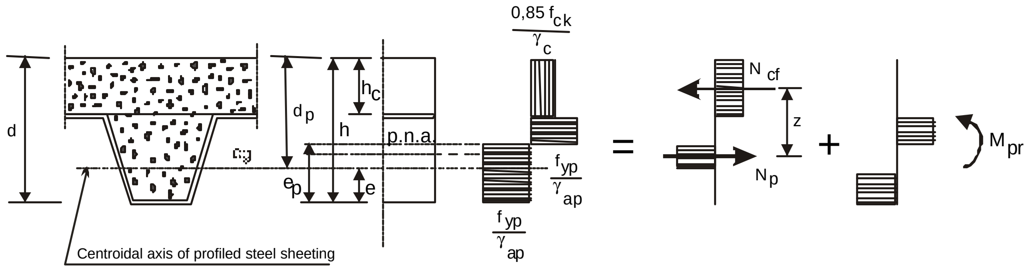
The effective area A_p of the steel decking is the net section obtained without considering the galvanising thickness (generally 2 x 0,020 = 0,04 mm) and the width of embossments and indentations.

Case 2 – Plastic neutral axis in steel sheeting

If the plastic neutral axis intercepts the steel sheeting, a part of the steel sheeting section is in compression to keep the equilibrium in translation of the section. For simplification, the concrete in the ribs as well as the concrete in tension is neglected. *9.7.2(6)*

As shown in Figure 11, the stress diagram can be divided in to two diagrams each representing one part of the design resistant moment M_{psRd}:

- The first diagram depicts the equilibrium of the force N_{cf}, corresponding to the resistance of the concrete slab (depth h_c) balanced by a partial tension force N_p in the steel sheeting. The lever arm z depends on the geometrical characteristics of the steel profile. The corresponding moment to that diagram is $N_{cf}.z$. The calculation of the lever arm z by an approximate method is explained below.
- The second diagram corresponds to a pair of equilibrating forces in the steel profile. The corresponding moment is M_{pr} , called the *reduced plastic moment* of the steel sheeting, and must be added to $N_{cf}z$.



p.n.a. : plastic neutral axis c.g. : centre of gravity

Figure 11 Stress distribution for sagging bending if the neutral axis is inside the steel sheet

The bending resistance is then :

$$M_{ps.Rd} = N_{cf}\, z + M_{pr} \qquad \textbf{(9)}$$

Compression force in the concrete is:

$$N_{cf} = \frac{0{,}85\, f_{ck}}{g_c} b h_c \qquad \textbf{(10)}$$

Some authors have proposed an approximate formula where M_{pr}, reduced (resistant) plastic moment of the steel sheeting can be deduced from M_{pa}, design plastic resistant moment of the effective cross-section of the sheeting. That formulae calibrated with tests (Figure 12) on 8 steel profile types is the following :

$$M_{pr} = 1{,}25\, M_{pa}\left(1 - \frac{\frac{N_{cf}}{A_p f_{yp}}}{g_{ap}}\right) \leq M_{pa} \qquad \textbf{(11)}$$ 9.7.2(6)

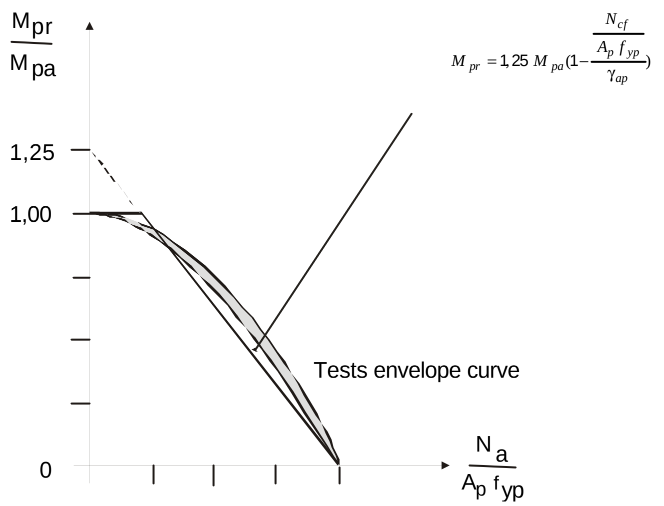


Figure 12 Experimental relation between M_{pa} and M_{pr}

The lever arm is calculated with the following formula :

$$z = h_t - 0{,}5h_c - e_p + (e_p - e)\frac{\frac{N_{cf}}{A_p f_{yp}}}{g_{ap}} \qquad \textbf{(12)}$$

with :

e_p : distance of the plastic neutral axis of the effective area of the sheeting to its underside;

e : distance from the centroid of the effective area of the steel sheet to its underside.

5.3.2 Verification of the hogging bending resistance

Type I failure is due to hogging bending resistance and the plastic neutral axis is generally in the depth of sheeting. Usually, the steel sheeting is ignored as it is in compression and may buckle and its contribution is low in comparison to the force of compression of the concrete contained in the ribs.

In the slab of depth h_c, concrete is in tension and its resistance is neglected. Only reinforcing bars in the slab carry the tension due to hogging bending. Design negative resistance is given 9.7.2(7)
by yielding (f_{ys} / g_s) of the reinforcement (Figure 13).

Design resistance of the reinforcement bars is:

$$N_s = A_s f_{ys} / g_s \qquad \textbf{(13)}$$

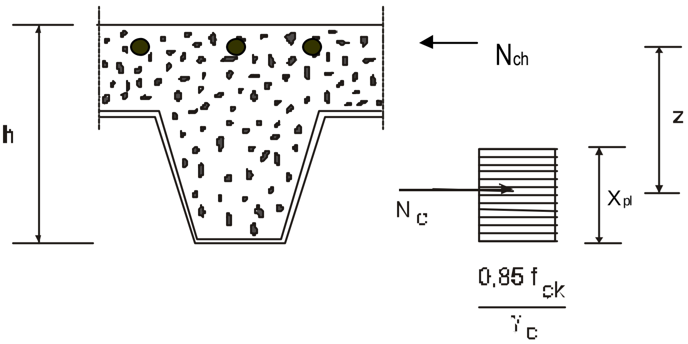


Figure 13 Stress distribution for hogging bending

Internal force in the concrete is approximately:

$$N_c = 0{,}85 b_c \; x_{pl} \frac{f_{ck}}{\gamma_c} \qquad \textbf{(14)}$$

where b_c is the width of the concrete in compression, taken as the average width of the concrete ribs over 1m for simplicity.

Equilibrium gives the depth of concrete in compression x_{pl} as:

$$x = \frac{A_s \dfrac{f_{ys}}{g_s}}{0{,}85 b_c \dfrac{f_{ck}}{g_c}} \qquad \textbf{(15)}$$

If z is the lever arm of the resulting internal forces N_{ch} and N_c, the bending resistance is:

$$M_{ph.Rd} = \frac{A_s f_{ys}}{g_s} z \qquad \textbf{(16)}$$

Reinforcement should be sufficiently ductile to allow rotations in the yielded sections. High yield reinforcing steel will usually satisfy this criterion, providing the depth of the concrete slab is not too great.

5.3.3 Longitudinal shear

Type II failure corresponds to the resistance to longitudinal shear. The verification method is to *9.7.3*
evaluate the average longitudinal shear resistance t_u existing on shear span L_s *and compare this with the applied force*. This resistance t_u depends on the type of the sheeting and must be established for all proprietary sheeting as the value is a function of the particular arrangements of embossment orientation, surface conditions etc.

The design resistance of the slab against longitudinal shear is determined by a standardised
semi-empirical method called the *m-k method* originally proposed by Porter and Ekberg (1976). *11.3*
This method does not refer to the average resistance t_u but uses the vertical shear force V_t to check the longitudinal shear failure along the shear span L_S. The direct relationship between the vertical shear and the longitudinal shear is only known for elastic behaviour, if the behaviour is elastic-plastic, the relationship is not simple and the *m-k* method, which is a semi-empirical approach, is used.

The *partial interaction* method is an alternative to the *m-k method*. This method presents a *9.7.3(6)* more satisfactory model but is only available for ductile composite slabs and is difficult to apply for very low levels of interaction.

m-k method

The semi-experimental method *m-k* uses a linear parametric formulae in which all the determining parameters are present: 11.3.5

$$V_{LR} = F(f_{ck}, L_s, d_p, b, A_p, V_t) \quad \textbf{(17)}$$ 9.7.3(3)

where:

V_{LR} : longitudinal shear resistance;
V_t: vertical shear force;
d_p : average depth of the composite slab.

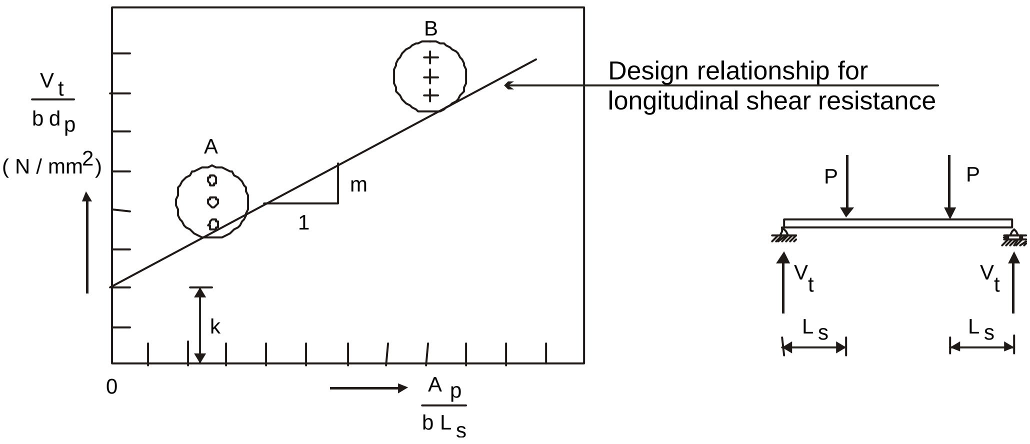


Figure 14 Derivation of m and k from test data

Figure 14 shows the *m-k* line determined with six full-scale slab tests separated into two groups for each steel profile type. The ordinate is a stress dimension term and depends on the vertical shear force V_t including the self-weight of the slab. The abscissa is a non-dimensional number and represents the ratio between the area of the sheeting and the longitudinal shear area. Multiplying this ratio by f_y/t_u and with regards to the vertical axis, a direct relationship is established with the longitudinal shear load capacity of the sheeting. 10.3.1.5

According to *Eurocode 4*, the maximum design vertical shear $V_{t.Sd}$ for a width of slab b, is limited due to the longitudinal shear resistance to $V_{L.Rd}$ given as :

$$V_{L.Rd} = b.d_p\left(m\frac{A_p}{bL_s} + k\right)\frac{1}{g_{VS}} \quad \textbf{(18)}$$ 9.7.3(3)

where k and m (in N/mm^2) are the ordinate at the origin and the slope of the *m-k* line and g_{VS} is a partial safety factor equal to 1,25.

The factors m and k are obtained from standardised full-scale tests. The m and k values depend on the sheeting type and the dimensions of the section of the slab and are generally given by profiled steel manufacturers.

Eurocode 4 does not consider any concrete influence and the characteristic value for each group is deemed to be the one obtained by reducing the minimum value by 10%. The straight line through these characteristic values of both groups forms the design relationship. Concrete is neglected because it is generally observed for buildings that its resistance has no influence if f_{ck} is between 25 and 35MPa.

For design, L_s depends on the type of loading. For a uniform load applied to the entire span L of a simply supported beam, L_s equals L/4. This value is obtained by equating the area under the shear force diagram for the uniformly distributed load to that due to a symmetrical two point load system applied at distance L_s from the supports. For other loading arrangement, L_s is obtained by similar assessment. Where the composite slab is designed as continuous, it is permitted to use an equivalent simple span between points of contraflexure for the determination of shear resistance. For end spans, however, the full exterior span length should be used in design.

The longitudinal shear line (Figure 15) is only valid between some limits because, depending on the span, the failure mode can be one of the three described earlier.

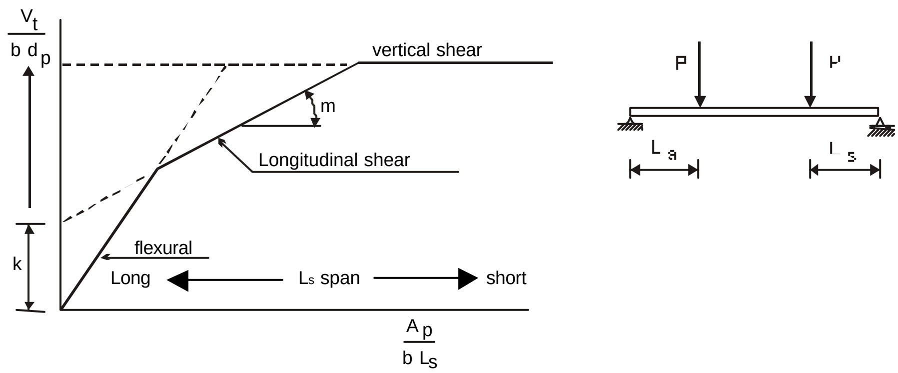


Figure 15 Relationship between failure mode and span

If longitudinal shear resistance of the slab is not sufficient, it can be increased by the use of some form of end anchorage, such as studs or local deformations of the sheeting. Clause 9.7.4 of Eurocode 4 permits enhanced performance due to these end anchorages.

Partial connection method

The *partial connection method,* or t_u *method,* can also be used for the verification of the resistance to longitudinal shear. This method should be used only for composite slabs with ductile behaviour. 9.7.3(2)

The method is based on the value of the design ultimate shear stress $t_{u.Rd}$ acting at the steel-concrete interface. This stress value leads to a design partial interaction diagram. In this diagram the bending resistance M_{Rd} of a cross-section at a distance L_x from the nearer support is plotted against $L_{x.}$

t_u is either given by the profile steel manufacturer or by results from standardised tests on composite slabs. The design partial interaction diagram is given in Figure 16 . *11.3.6*

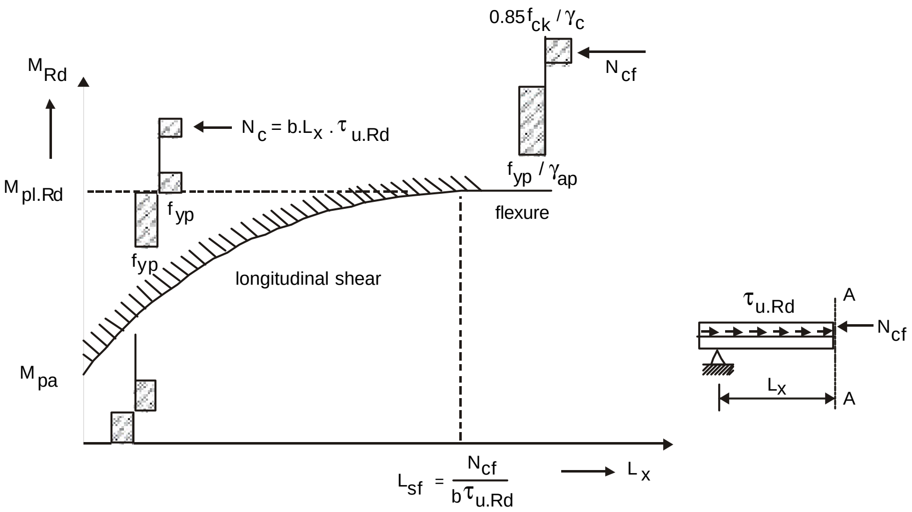


Figure 16 Design partial interaction diagram

Where there is no connection ($L_x = 0$), it is assumed that the steel sheeting supports the loading. The stress diagram is bi-rectangular and the resistance moment equals M_{pa} (the design plastic resistance moment of the effective cross-section of the sheeting). For full connection, stress diagrams correspond to the design resistant moment $M_{pl.Rd}$. Between these two diagrams, the stress distribution corresponds to a partial connection.

The minimum length L_{sf} to obtain full interaction is given by :

$$L_{sf} = \frac{N_{cf}}{b.t_{u.Rd}} \qquad \textbf{(19)}$$

where N_{cf} is the minimum of either the design resistant force of the concrete slab of depth h_c or of the steel sheeting :

$$N_{cf} = \min(\frac{0{,}85\, f_{ck} b h_c}{g_c} ; \frac{A_p\, f_{yp}}{g_{ap}}) \qquad \textbf{(20)}$$

For L_x ³ L_{sf} , the shear connection is full, so the bending resistance (type I failure mode) is critical. For $L_x < L_{sf}$, the shear connection is partial, so the longitudinal shear resistance (type II failure mode) is critical.

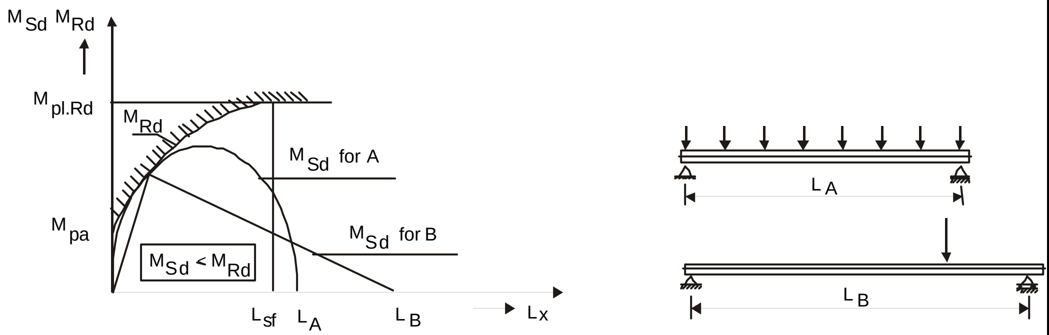


Figure 17 Verification procedure

The verification procedure is illustrated in Figure 17 for two slabs with different types of loading and span. Resistant moment diagrams and design bending moment diagrams are plotted against L_x on the same axis system. For any cross-section of the span, the design bending moment M_{Sd}

cannot be higher than the design resistance M_{Rd}.

5.3.4 Vertical shear verification

Type III failure corresponds to the resistance to vertical shear. This type of failure can be critical where steel sheeting has effective embossments (thus preventing Type II failure) and is characterised by shearing of the concrete and oblique cracking as is observed for reinforced concrete beams. The cracking is developed in the sheared area in a 45° direction to the mean plane of the slab.

Eurocode 4 currently refers the designer to EN 1992-1-1:2001. Previous drafts gave more detail and suggested the vertical shear resistance $V_{v.Rd}$ of a composite slab over a width equal to the distance between centres of ribs should be determined from: *9.7.5*

$$V_{v.Rd} = b_o d_p k_1 k_2 t_{Rd} \quad \textbf{(21)}$$

with :

b_o : mean width of the concrete ribs (minimum width for re-entrant profile) (Figure 18);
t_{Rd} : basic shear strength to be taken as $0{,}25\ f_{ctk}/g_c$;
f_{ctk} : approximately equal to $0{,}7$ times mean tension resistance of the concrete f_{ctm}
A_p : is the effective area of the steel sheet in tension within the considered width b_o;

$k_1 = (1{,}6 - d_p) \geq 1$ with d_p expressed in m

$k_2 = 1{,}2 + 40\, r$

$r = A_p / b_o d_p < 0{,}02$

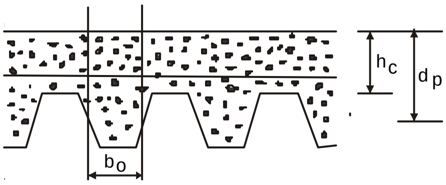


Figure 18 Cross section used for vertical shear resistance

5.3.6 Punching shear resistance

Under a heavy concentrated load, the slab can fail by punching i.e. vertical shear on the perimeter of the concentrated load. Figure 19 represents that kind of failure. *9.7.6*

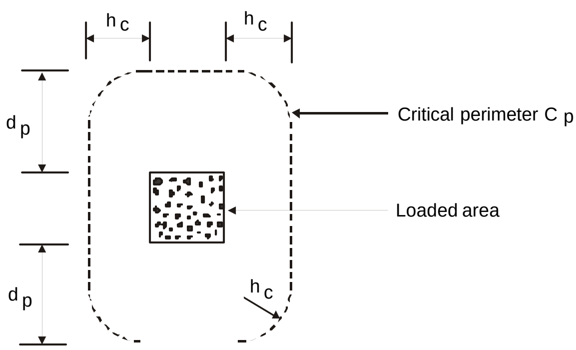


Figure 19 Punching failure

Eurocode 4 again now only refers to *Eurocode 2* whereas the previous draft presented the following formula to evaluate the punching shear resistance $V_{p.Rd}$.

$$V_{p.Rd} = C_p h_c k_1 k_2 t_{Rd} \tag{22}$$

where C_p is the critical perimeter determined by considering the perimeter of the load application surface and a 45° load dispersion.

5.4 Elastic properties of cross-sections for serviceability limit state verification (SLS)

Elastic analysis is normally used to calculate the deflection of slabs at the serviceability limit state. In this case the average value of the cracked and uncracked cross-sectional stiffness should be used. End slip may be eventually considered.

In a cross-section where the concrete in tension is considered as cracked, as the cross-section shown in Figure 20 under sagging loading, the second moment of area I_{cc} can be obtained from:

$$I_{cc} = \frac{bx_c^3}{12n} + \frac{bx_c(\frac{x_c}{2})^2}{n} + A_p(d_p - x_c)^2 + I_p \tag{23}$$

with :
I_p : second moment of area of the profiled sheeting ;
n : modular ratio
x_c : position of the elastic neutral axis to the upper side of the slab obtained by the next formula :

$$x_c = \frac{nA_p}{b}\left(\sqrt{1 + \frac{2bd_p}{nA_p}} - 1\right) \tag{24}$$

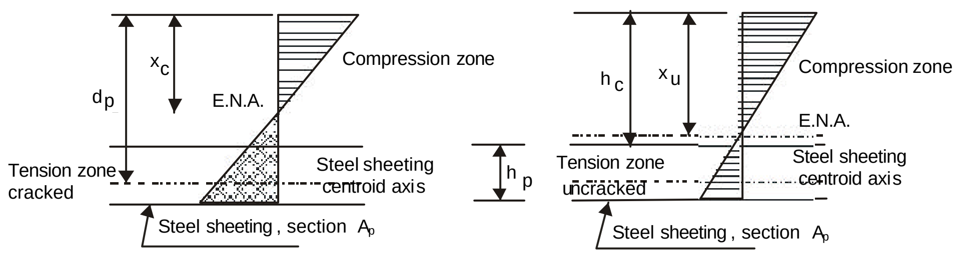


Figure 20 Second moment of inertia calculation for cracked and uncracked cross-sections (sagging moment)

In a section under sagging moment, considering the concrete in tension as not cracked, the second moment of area I_{cu} is given by :

$$I_{cu} = \frac{bh_c^3}{12n} + \frac{bh_c(x_u - \frac{h_c}{2})^2}{n} + \frac{b_m.h_p^3}{12n} + \frac{b_m.h_p}{n}(h_t - x_u - \frac{h_p}{2})^2 + A_p(d_p - x_u)^2 + I_p \tag{25}$$

where
$x_u = \sum A_i z_i / \sum A_i$ is the position of the elastic neutral axis to the upper side of the slab.

In these formulae giving the second moment of area, the modular ratio n can be considered as the average value of the short and long term modular ratio:

$$n = \frac{E_a}{E'_{cm}} = \frac{E_a}{\frac{1}{2}(E_{cm} + \frac{E_{cm}}{3})} \qquad \textbf{(26)}$$

6. Concluding summary

Composite slabs, consisting of a cold-formed steel sheet acting in combination with in-situ concrete, are widely used in steel framed buildings. The steel sheet acts as shuttering during construction and later as steel reinforcement for the concrete slab. Reasons for the popularity of this form of floor construction have been given.

Design of composite slabs requires consideration of two conditions – first, the steel sheeting as a relatively thin bare steel section supporting wet concrete and construction operatives and later as a combined structural element. Appropriate loadings and methods of analysis for each case have been explained and design checks at the serviceability and ultimate limit states introduced.

The performance of a composite slab is dependent on the effectiveness of the shear connection between the concrete and steel sheeting. A semi-empirical design method is usually used, the basis and explanation of which has been provided.

Structural Steelwork Eurocodes
Development of
A Trans-National Approach 2

Course: Eurocode 4

Lecture 5: Shear Connectors and Structural Analysis

Summary:

- The lecture gives information on how to determinate the internal forces in a composite structure at ultimate and serviceability limit states.
- Particular concern is given to moment redistribution aspects resulting from plasticity, cracking, creep and shrinkage phenomena in the composite cross-section.
- The evaluation of the deflection under service loads is also addressed , together with concrete cracking and vibrations aspects.

Pre-requisites:

- Basic knowledge on frame analysis aspects (elasticity, plasticity, first and second order effects, moment redistribution, etc)
- The preliminary reading of SSEDTA 1 "Eurocode 3" lecture n°4 on frame analysis is recommended.

Notes for Tutors:

This material comprises two 60-minute lectures.

Objectives:

The student should:

- Have some knowledge about the need for and the performances of traditional shear studs (resistance, ductility).
- Be able to select and apply the available analysis procedures for composite frames (elastic, plastic) at ultimate limit states.
- Have sufficient knowledge to check the resistance, the displacements, the concrete cracking in the slabs and the vibrations of composite structures at serviceability limit states.

References:

- ESDEP lectures on composite structures (Volume 10)

Contents:

1. Shear connectors

1.1 General

1.1.1 The forces applied to connectors

Consider a composite beam. If there is no mechanical connection between the concrete slab and the steel beam (the effect of natural bond is ignored) then the two materials will slide relative to one another and the bending stresses in the section will be as shown in Figure 1a. Clearly, if longitudinal shear resistance is provided by some form of connection so that the stresses at the interface of the two materials are coincident, then the beam acts as a fully composite section. If it is assumed that the fully connected composite beam acts in an elastic way then the shear flow, T (shear force per unit length), between the concrete slab and the steel section may be calculated from: 6.7.1.1 (2)

$$T = \frac{V.S}{I} \qquad \text{Eq. 1-1}$$

where :

- V: is the applied vertical shear force at the point considered;
- I : is the second moment of area of the equivalent section (see Lecture 3);
- S : is the first moment of area of either the equivalent concrete slab or the steel section about the elastic neutral axis.

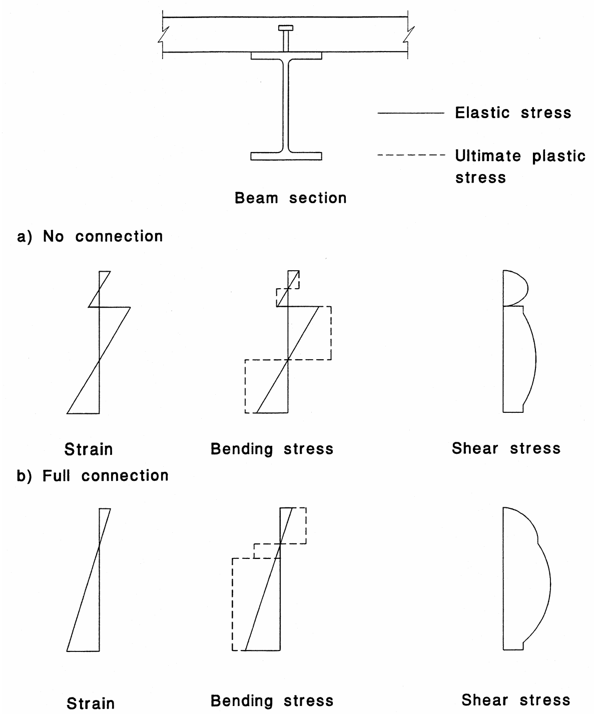


Figure 1 Strain, bending and shear stresses for a) no connection and b) full connection

Figure 1 also shows the elastic shear stress developed in the section for the conditions of both full and zero connection.

It can be seen, from the above equation, that the longitudinal shear forces that must be carried by the connection will vary depending upon the vertical shear present. The solid line of Figure 2a shows the distribution of longitudinal shear, the connector force along the interface between the steel section and the slab, along one half of a simply supported uniformly loaded beam that has a rigid full connection. It must be remembered, however, that this applies only when the beam is assumed to be behaving in an elastic manner. As the ultimate moment of resistance is reached, the steel section will yield or the concrete slab will crush and a plastic hinge will form at the critical section. The bending stresses in the beam are as shown by the dashed lines of Figure 1; the distribution of longitudinal shear in the beam also changes and the connectors close to the hinge are subject to higher loads. The dashed line in Figure 2a shows the distribution of shear force close to the fully plastic failure load.

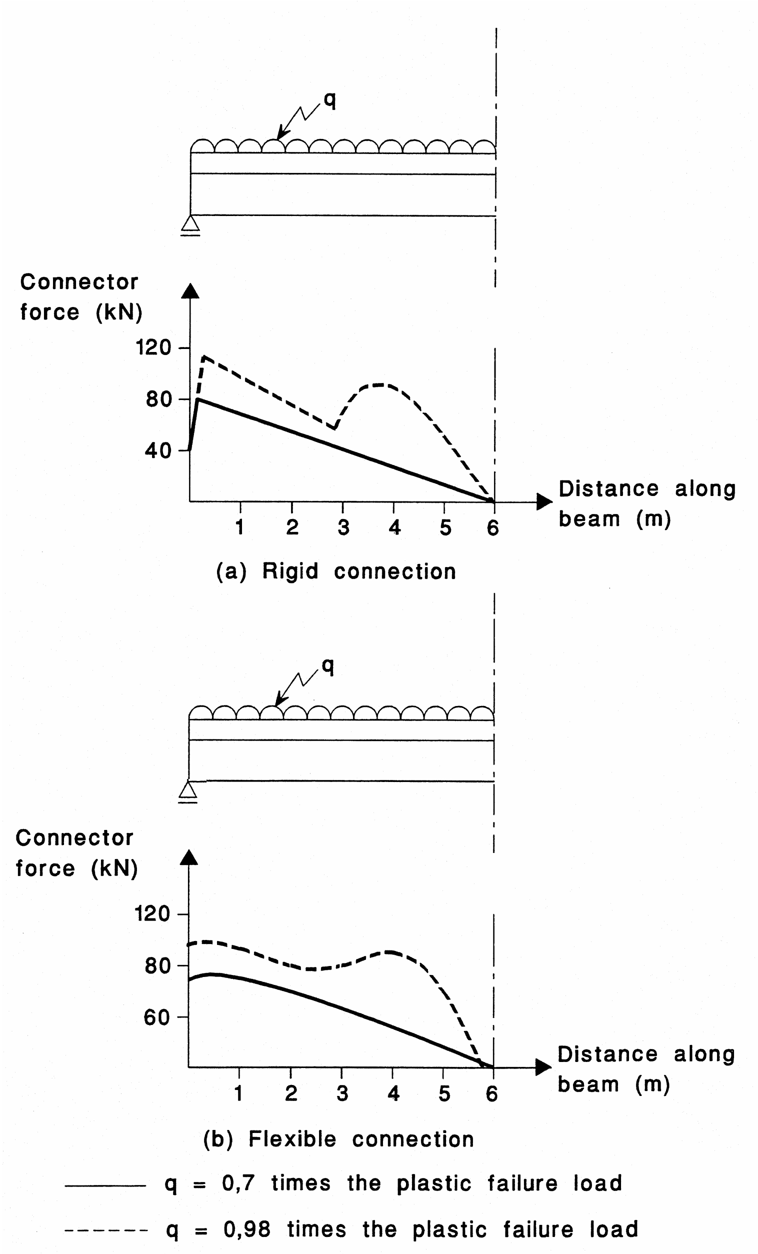


Figure 2 Connector loads for rigid and flexible connectors

In practice, connectors are never fully rigid, and there is always some slip between the slab and the steel section. The flexibility of the connectors allows more ductility and a variation in the distribution of longitudinal shear between slab and steel section. The corresponding longitudinal shear force distribution present in a composite beam with flexible connection is shown in Figure 2b.

At ultimate load, when the plastic hinge has formed, it is likely that the end connectors will have deformed to a considerable extent and yet still be expected to carry a high longitudinal shear load. Hence there is a requirement that shear connectors must have substantial ductility to perform adequately.

In determining the resistance of the beam, it is assumed that all the connectors, even when deformed, will be capable of resisting a longitudinal shear force. It is this shear resistance of the connectors that controls the resistance of the beam. If sufficient connectors are provided to withstand the longitudinal shear force generated when the full plastic resistance of the beam is developed, the beam is said to be "fully connected". It is also possible to reduce the amount of connection so that the moment resistance of the beam is limited accordingly; this is a resistance criterion and the beam is termed "partially connected".

The slip that occurs as the connectors deform has a profound effect upon the stiffness of the beam. Very flexible but strong connectors may allow high bending resistance but, because of the substantial slip, there will be a loss of stiffness. The stiffness of the shear connection, in relation to the stiffness of the steel section and concrete slab, defines the degree of interaction. Consequently, a beam where the connectors are infinitely stiff is said to have "full interaction" and one where the connection is relatively flexible is said to have "partial interaction". It may be deduced that the strength and stiffnesses of both connector and concrete will affect the shear connection.

The major force acting on the connector is one of direct shear. The shear force is generally assumed to be greatest at the level of the weld between the steel section and the connector itself. The area and shear strength of the connector and weld must, therefore, be adequate to carry the forces generated. It is unlikely that any substantial deformation will take place due to this shear.

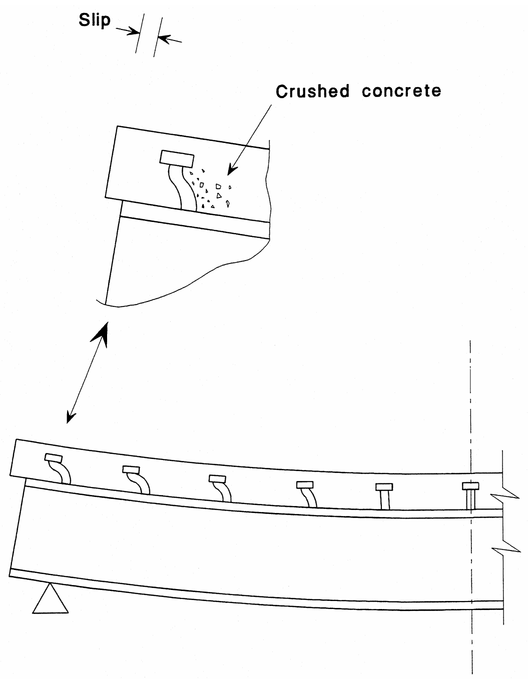


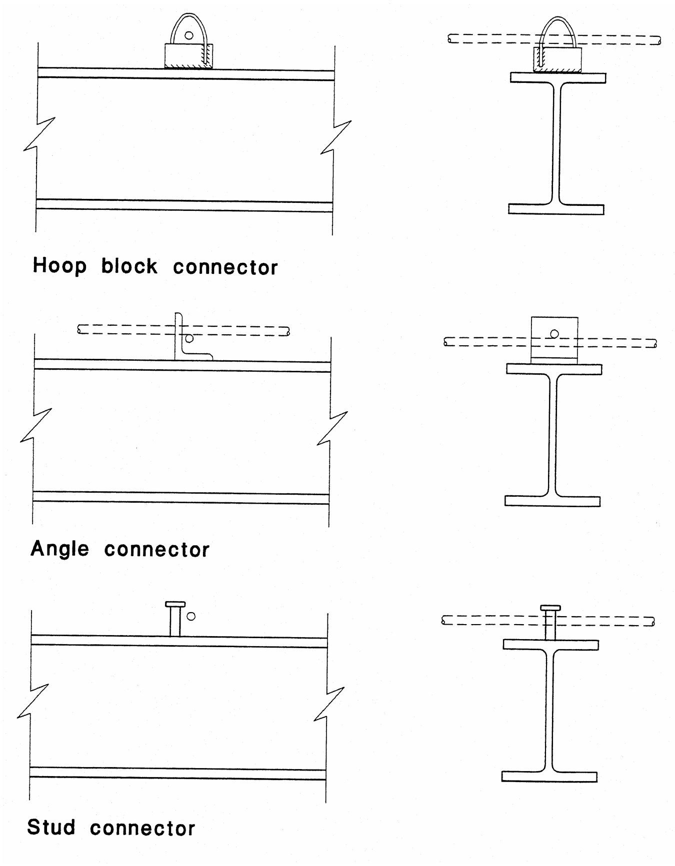
Figure 3 The deformation of flexible connectors

However, relative movement between the slab and steel section does occur. The mechanism for this movement can be seen in Figure 3. The concrete may crush at the connector base allowing some deformation of the connector itself. However, at the head of the connector the confining concrete is not so highly stressed and this part of the connector remains in its original position. The result is bending deformation in the connector, which can be seen clearly in Figure 3. Long slender connectors are more likely to deform into this characteristic "S" shaped pattern and therefore tend to be ductile. Short stocky connectors tend to be brittle and are therefore undesirable. Most codes of practice require stud connectors to be at least three or preferably four times longer than their diameter.

The major force resisted by the concrete is one of bearing against the leading edge of the connector. It has already been mentioned that the concrete in this region is likely to crush allowing bending deformation to occur in the connector. The bearing resistance of the concrete in this region is dependent upon its volume as well as strength and stiffness. In fact, where there is sufficient concrete around the connector, the bearing stress may reach several times the unconfined crushing strength of the concrete.

There is also likely to be direct tension in the connector. The different bending stiffnesses of the slab and the steel section, coupled with the deformed shape of the connectors, gives rise to the tendency for the slab to separate from the steel section. It is, therefore, usual for connectors to be designed to resist this tensile force.

In most composite beams the connectors are spaced along the steel section and, therefore, provide a resistance to longitudinal shear only locally to the top flange. The longitudinal shear force must, therefore, be transferred from the narrow steel section into the much wider slab. This transfer is normally achieved using bar reinforcement that runs transverse to the beam line. These bars are normally placed below the head of the stud (or other connector) and extend into the slab, as shown in Figure 4.



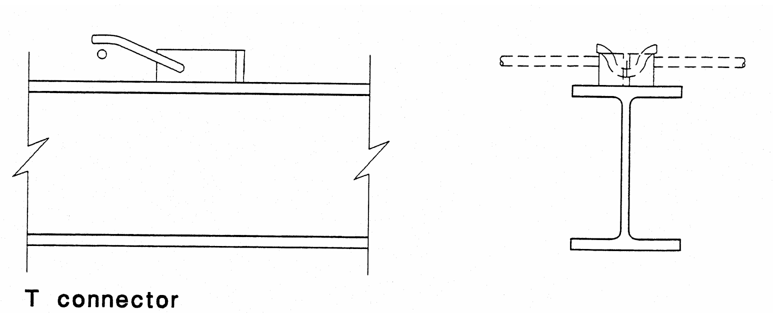


Figure 4 Types of connector

To summarise, the connector must be capable of transferring direct shear at its base, resisting bending forces and creating a tensile link into the concrete. The concrete must have sufficient volume around the connector and be of sufficient strength to allow a high bearing stress to be resisted; bar reinforcement is often provided to ensure adequate lateral distribution of longitudinal shear.

1.1.2 Basic forms of connection

Early forms of shear connector were shop welded, using conventional arc welding. Various forms of connector welded in this way are shown in Figure 4. The most common types are the hoop connector and the T connector, which serve to show the complexity of the forming and welding operation necessary. The popularity of composite beam construction has led manufacturers to develop very simple forms of shear connector.

Despite the plethora of connection types available, the shear stud connector has now become the primary method of connection for composite beams. The stud can be spot welded to the steel section in one operation, using microchip controlled welding equipment. The most advanced machines allow studs to be welded through galvanised steel sheeting. This ability has enabled the economic advantages of composite floor decks to be fully exploited.

1.1.3 Classification of the connectors

Shear connectors can be classified as *ductile* or *non-ductile*. Ductile connectors are those with sufficient deformation capacity to justify the simplifying assumption of plastic behaviour of the shear connection in the structure considered. Shear-slip (*P-s*) curves are obtained by push-out tests. Figure 5 shows examples of both ductile and non-ductile behaviour. A ductile connector has an elastic-plastic type of curve with a yield plateau corresponding to the connector characteristic resistance P_{Rk} and to a high ultimate slip capacity s_u. *Eurocode 4* considers that connectors having a characteristic slip capacity higher or equal to 6 mm can be assumed to be ductile, provided that the degree of shear connection is sufficient for the spans of the beam being considered. 6.7.1.1 (6)

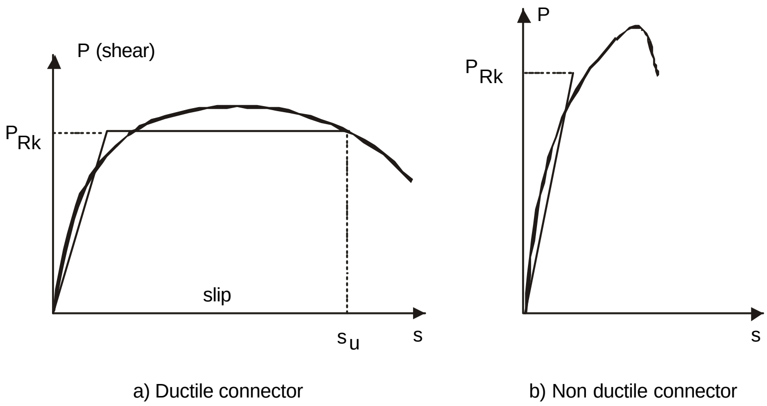


Figure 5 Simple web angle connection

Headed studs with an overall length after welding of not less than 4 times the stud diameter, and with a shank of diameter not less than 12 mm and not exceeding 22 mm may be considered as ductile according to experimentation. This statement is true for a plain concrete slab. For a composite slab with profiled steel sheeting, the slip capacity is much higher; more or less 10 to 15mm, provided that the studs extend sufficiently from the ribs. 6.7.1.2 (1)

Other connectors may also be considered as ductile: high strength bolts, slender welded angles and gun fixed cold-formed angles. However, block connectors should be classified as non-ductile as their slip capacity is only due to the deformation of the concrete surrounding the connectors.

1.2 Design resistance of common shear connectors

1.2.1 Headed studs in solid concrete slabs

For a plain concrete slab, the design shear resistance P_{Rd} of a welded stud with a normal weld collar should be determined from : 6.7.4.1 (1)

$$P_{Rd} = \min(P_{Rd}^{(1)}, P_{Rd}^{(2)}) \qquad \text{Eq. 1-2}$$

$$P_{Rd}^{(1)} = 0{,}8 \,.\, f_u \,.\, (\pi \,.\, d^2/4)/\gamma_v \qquad \text{Eq. 1-3}$$

$$P_{Rd}^{(2)} = 0{,}29 \,.\, \alpha \,.\, d^2 . \sqrt{f_{ck} \,.\, E_{cm}}\,/\gamma_v \qquad \text{Eq. 1-4}$$

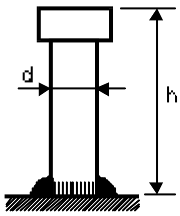


Figure 6 Welded stud

$P_{Rd}^{(1)}$ and $P_{Rd}^{(2)}$ represent respectively the shear failure of the stud and the local concrete crushing around the shear connector, where :

d :	is the diameter of the shank of the stud ($d \le 25$ mm); 6.7.4.1 (2)
h :	is the overall height of the stud;
f_u :	is the specified ultimate tensile strength of the material of the stud but not greater than 500N/mm²;
f_{ck} :	is the characteristic cylinder strength of the concrete at the age considered;
E_{cm} :	is the value of the secant modulus of the concrete (short term);
α :	is the corrective factor equal to: 1 if $h/d > 4$ $0{,}2\,[(h/d)+1]$ if $3 \le h/d \le 4$;
$\gamma_v = 1{,}25$:	is a partial safety factor for connectors.

1.2.2 Headed studs with profiled steel sheeting

For sheeting with ribs *transverse* to the supporting beams (Figure 7), the design shear resistance should be taken as their resistance in a solid slab multiplied by the reduction factor k_t given by the following expression: 6.7.5.2 (1)

$$k_t = \frac{0{,}70}{\sqrt{N_r}} \frac{b_0}{h_p} \left(\frac{h}{h_p} - 1 \right) \qquad \text{Eq. 1-5}$$

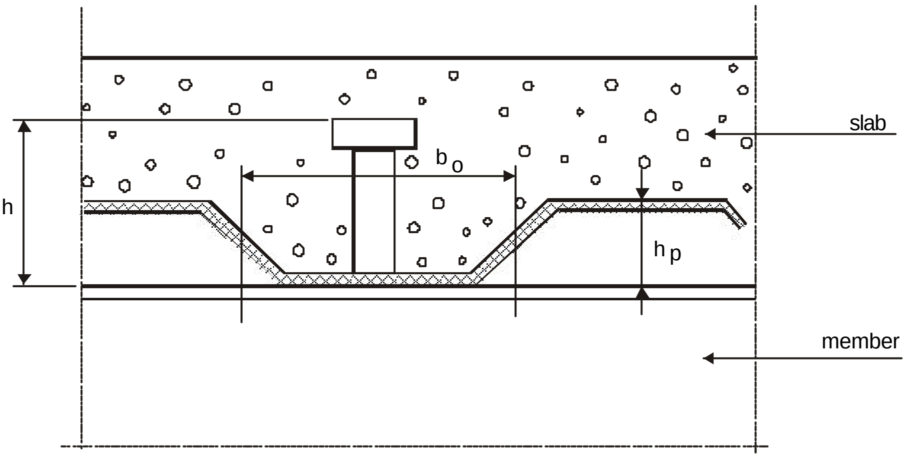


Figure 7 Welded stud with composite slab

This reduction formula is only valid if the following conditions are simultaneously fulfilled:

- *d £ 20 mm;*
- *h_p £ 85 mm;*
- $b_0 \geq h_p$.

N_r is the number of stud connectors in one rib at a beam intersection; N_r is limited to 2 in computations even if more than two studs are present in one rib.

And finally:

- For studs welded through the steel sheeting of thickness greater than 1,0 mm, k_t should not be taken greater than 1,0 when $N_r = 1$ and not greater than 0,8 when $N_r \geq 2$ in order to keep a homogeneous safety level. 6.7.5.2 (2)
- For sheeting with holes or for studs welded through sheeting with a thickness t less than or equal to 1,0 mm, the reduction factor k_t should not be taken greater than the values given in Table 1. 6.7.5.2 (3)

Number of stud connectors per rib	Studs not exceeding 20 mm in diameter and welded through profiled steel sheeting	Profiled sheeting with holes and studs 19 mm or 22 mm in diameter
$N_r = 1$	0,85	0,75
$N_r = 2$	0,70	0,60

Table 1 Upper limits for the reduction factor k_t

For sheeting with ribs *parallel* to the supporting beams (as for an edge beam for example), the reduction coefficient k_l is obtained as : 6.7.5.1 (2)

$$k_l \quad = \quad 0{,}6\,\frac{b_0}{h_p}\left(\frac{h}{h_p}-1\right) \;\leq\; 1 \qquad \text{Eq. 1-6}$$

These reduction factors k_t and k_l are due to the smaller encasement of the studs and the more difficult welding conditions; the welding being made through steel profiled sheeting.

1.2.3 Welded angle connectors

The design resistance of a welded angle connector (Figure 8) should be determined from the dimensionally inhomogeneous and experimental formula : 6.7.9 (1)

$$P_{Rd} \quad = \quad \frac{10\,.\,l\,.\,h^{3/4}\,.\,f_{ck}^{\;2/3}}{g_v} \qquad \text{Eq. 1-7}$$

where:

P_{Rd} :	is in Newton
l :	is the length of the angle (in mm);
h :	is the width of the upstanding leg of the angle (in mm);
f_{ck} :	is the characteristic strength of concrete (in N/mm²);
g_v = 1,25 :	is a partial safety factor.

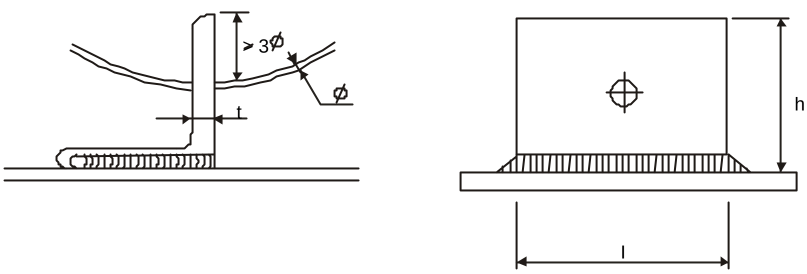


Figure 8 Angle connector with reinforcement

To prevent the uplift of the slab during the bending of the beam, a reinforcement bar should cross the leg of the angle and its diameter should satisfy the following condition: 6.7.9 (4)

$$A_e \, . \, f_{sk} \; / \, g_s \quad \geq \quad 0{,}1 \, . \, P_{Rd} \qquad \text{Eq. 1-8}$$

where:

A_e :	is the cross-sectional area of the bar, $p.f^2/4$;
f_{sk} :	is the characteristic yield strength of the reinforcement;
$g_v = 1{,}15$:	is a partial safety factor for the reinforcement.

1.2.4 Other connectors

Eurocode 4 gives advice for other less usual connectors such as block connectors, anchors, 6.7.7
hoops and block connectors with anchors or hoops... 6.7.8

2. Global analysis of a composite structure

2.1 Global analysis for ultimate limit states

The distribution of applied design bending moments M_{Sd} and shear forces V_{Sd} shall be determined for the different ultimate load combinations in order to check the cross-sectional resistance of a composite beam.

An *elasto-plastic analysis* of a continuous beam should consider the cracked and uncracked zones of the beam, the plastic local distributions, and the slip (and eventually the uplift of the slab) at the steel-concrete interface and also the rotation due to local buckling on intermediate supports. However, such analysis can reasonably only been made by means of numerical methods. Moreover, Eurocode 4 gives no application rules for elastic-plastic analysis. 5.1.2

For practical use, only two types of analyses are relevant. These are :

- A *rigid-plastic* analysis which may be used only for buildings where certain requirements are verified. This analysis, based on the plastic hinge concept, allows the determination of the collapse mode of the beam for an ultimate loading characterised by a load multiplier called the failure multiplier. It is performed using the kinematic or static theorem or the combined theorem of ultimate analysis.
- An *elastic* analysis based on the well-known elastic beam analysis may be used in all cases, provided homogeneous cross-sections are defined with the help of modular ratios between steel and concrete. Moreover, a distinction should be made between the elastic cracked analysis and the elastic uncracked analysis to take into account the important loss of rigidity due to the concrete cracking in hogging bending regions. 5.2.3

For the calculation of the modular ratio, the deformation of concrete due to creep should be taken into account. But for the design of buildings, it is accurate enough to take account of creep by replacing in analyses concrete areas A_c by effective equivalent steel areas equal to A_c/n. The modular ratio n is the ratio $E_a / E_c^{'}$, where E_a is the elastic modulus of structural steel, and $E_c^{'}$ is an "effective" modulus of the concrete.

Two nominal values $E_c^{'}$ should be used: one equal to E_{cm} for short term effects, the other equal to $E_{cm}/3$ for the long term effects. In the ESDEP lectures (Volume 10), three sets of values for short-term and long-term modular ratios are recommended. These values are listed in Table 2.

Option	Short-term effects	Long-term effects	Comments
(a)	Secant modulus E_{cm} (Table 3)	Various, depending on concrete grade	This method takes account of concrete grade and age.
(b)	6	18	Takes no account of concrete grade, but of concrete age.
(c)*	15	15	Takes no account of concrete grade or age

* Restricted to beams, the critical sections of which are in Class 1 or 2

Table 2 Value of modular ratio *n*

Strength Class of Concrete	20/25	25/30	30/37	35/45	40/50	45/55	50/60
E_{cm} (kN/mm^2)	29	30,5	32	33,5	35	36	37

Table 3 Secant modulus of elasticity E_{cm}

It will not usually be necessary to resort to method (a) in that table, which involves explicit calculation of the creep coefficient ϕ. The choice of method should take account of the purpose of the analysis and the accuracy required. The value of the modular ratio has much less influence on the accuracy of calculated action effects than on calculated stresses or deformations. Method (c), which adopts the same high value of the modular ratio for both short-term and long-term effects, could thus be used conveniently for the global analysis of structures; this would remove the need for separate analyses for these two conditions.

To accommodate the effects of imperfections, the internal forces and moments may generally be determined using:

- *First-order theory*: which may be used for a given load case if the elastic critical load ratio V_{Sd}/V_{cr} of the whole structure for that load case satisfies the criterion: 5.1.3

$$\frac{V_{Sd}}{V_{cr}} \leq 0,1 \qquad \text{Eq. 2-1}$$

where:

V_{Sd} : design value of the total vertical load;

V_{cr} : its elastic critical value.

- *Second-order theory*: the secondary effects may be included indirectly by using a first-order analysis with an appropriate amplification. Eurocode 4 gives a formula for this amplification factor k. 5.1.3

2.1.1 Rigid-plastic global analysis

In a *rigid-plastic analysis*, elastic deformations of the members, the connections and the foundations should be neglected and plastic deformations assumed to be concentrated at the plastic hinge locations. 5.1.5.2 (1)

In order to perform a rigid-plastic analysis, the critical cross-sections (i.e. those where a plastic hinge may appear) should be able to develop and also to maintain their plastic bending resistance until, under monotonically increasing loading, the plastic collapse mode mechanism occurs. This mechanism develops as a result of a re-distribution of the bending moments between the cross-sections; this redistribution progressing successively as each plastic hinge appears.

Rigid-plastic global analysis should not be used unless: 5.1.5.2 (2)

- The frame is non-sway or the frame, if unbraced, is of two storeys or less ;
- All the members and joints of the frame are steel or composite ;
- The cross-sections of steel members satisfy the principles of clauses 5.1.6.4 and 5.2.3 of EN 1993-1-1:20xx
- The steel material satisfies clause 3.2.3 of EN 1993-1-1:20xx ;
- At each plastic hinge location:
    - The cross-section of the structural steel member or component should be symmetrical about a plane parallel to the plane of the web or webs; 5.3.4 (1)
    - The proportions and restraints of steel components should be such that lateral-torsional buckling does not occur;
    - Lateral restraint to the compression flange should be provided at or within a small distance of all hinge locations at which plastic rotation may occur under any load case;
    - The rotation capacity should be sufficient to enable the required hinge rotation to develop;
    - Where rotation requirements are not calculated, all members containing plastic hinges should have effective cross-sections at plastic hinge locations that are in Class 1.

For composite beams in buildings, the rotation capacity may be assumed to be sufficient when: 5.3.4 (2)

- The grade of structural steel does not exceed S355;
- All effective cross-sections at plastic hinge locations are in Class 1; and all other effective cross-sections are in Class 1 or Class 2;
- Each beam-to-column joint has been shown to have sufficient rotation capacity, or to have a design moment resistance at least 1,2 times the design plastic moment resistance of the connected beam;
- Adjacent spans do not differ in length by more than 50% of the shorter span and end spans do not exceed 115% of the length of the adjacent span (figure 9);
- In any span in which more than half of the design load for that span is concentrated within a length of one-fifth of the span, then at any hinge location where the concrete slab is in compression, not more than 15% of overall depth of the member should be in compression (to avoid a premature collapse due to concrete crushing). This condition does not apply where it can be shown that the hinge will be the last to form in the span;
- The steel compression flange at a plastic hinge location is laterally restrained.

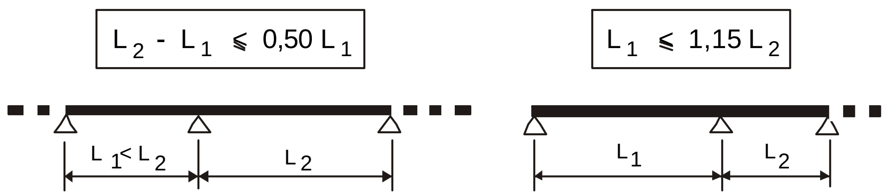


Figure 9 Length of span conditions for rigid-plastic analysis

2.1.2 Elastic global analysis

An *elastic analysis* may be used to advantage for all continuous beams whatever the class of the cross-sections. It is based on the assumption that the stress-strain relationships for material are linear, whatever the stress level. 5.1.4.1 (1)

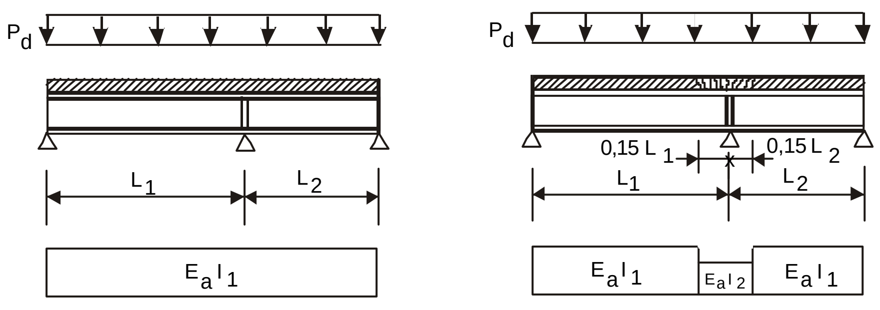
It is necessary however to remember that construction occurs in stages and this influences the stress history (and hence the final stress levels). In global analysis for buildings, the effects of staged construction may be neglected provided that all cross-sections of composite beams are in Class 1 or Class 2 (as redistribution of stresses can occur). 5.1.4.2

The main difficulty of this analysis is to know how to consider the loss of rigidity due to the cracking of the concrete in hogging bending regions. The influence of cracking on the bending moment redistribution occurs before service limit state and is more important for composite beams than for reinforced concrete beams (in which, there is also cracking in sagging bending regions and then the stiffness ratios are lower). Between the service limit state and the ultimate limit state, the more or less complete yielding of the critical cross-sections combined eventually with the local buckling phenomenon, contributes to the bending moment redistribution.

Eurocode 4 allows two types of elastic analysis as represented on Figure 10. 5.1.4.3

- For braced structures, the *uncracked elastic global analysis* is made using a constant flexural stiffness for each span. This stiffness is determined by considering the concrete in tension as uncracked and by using a homogeneous steel cross-section with the effective width b^{+}_{eff} defined at the mid-span. 5.2.2

- Provided that all ratios of the length of adjacent continuous spans between supports are at least 0,6, the *cracked elastic global analysis* is made using the second moment of area I_2 of the cracked cross-section over 15% of the span on each side of each internal support and keeping the second moment of area I_1 of the uncracked cross-section elsewhere. The second moment of area I_2 is determined by neglecting the concrete in tension but considering reinforcement within the effective width b^{-}_{eff} defined on support. The fixed ratio hypothesis for the cracked length greatly facilitates the analysis as it avoids any iterative methods to determine this length. This method gives sufficient accurate results. 5.2.2



a) "uncracked" method   a) "cracked" method

Figure 10 Elastic global analysis methods

The deformation of concrete due to creep should be taken into account using a modified modular ratio n_L or an effective modulus of elasticity for concrete E_c (see EC4 Part 5.1.4.4). 5.1.4.4

For composite structures for buildings, no consideration of temperature effects is necessary and 5.1.4.5
the effects of shrinkage of concrete may be neglected in verifications for ultimate limit states, 5.1.4.6
except in global analysis with members having cross-sections in Class 3 or Class 4.

The bending moment distribution given by an elastic global analysis may be modified in a way that satisfies equilibrium, and takes account of the effects of cracking of concrete, inelastic behaviour of materials and all types of buckling. 5.1.4.7

The calculation of the bending moment redistribution at the end of an elastic analysis cannot be defined precisely. This redistribution aims to decrease applied bending moment in the cross-sections where the ratio between applied moment and resistant moment is the highest (generally on internal supports) and which consequently increases the applied bending of the opposite sign, the internal forces and moment after redistribution still being in equilibrium with the loads.

If p is the maximum redistribution percentage, a negative peak moment M^-_{pic} may be reduced to the value of the design moment resistance M^-_{Rd} if the following condition is fulfilled :

$$M^-_{Rd} \quad \leq \quad \left| M^-_{pic} \right| \quad \leq \quad \frac{M^-_{Rd}}{(1-p/100)} \qquad \text{Eq. 2-2}$$

and if the resulting applied sagging bending moment is lower than the corresponding bending resistance. *Eurocode 4* gives values of that percentage p. These values depend on the type of the elastic analysis used and also on the class of the section on the support. These values are given in Table 4 here below. For sake of clarity, Table 4 is presented in a slight different way than Table 5.2 in prEN 1994-1-1:2001.

Table 5.2

Class of cross-section (hogging bending)	1	2	3	4
Uncracked elastic analysis	40 %	30 %	20 %	10 %
Cracked elastic analysis	25 %	15 %	10 %	0 %

Table 4 Limits to redistribution of bending moment on support

The difference between authorized redistribution values for both analyses for a same class of the support cross-section is equal to 10 to 15% (see Table 5 which is also valid for cases when elastic design is carried out at ULS). It considers the cracking of the concrete which is evaluated in a safe mode which reflects the influence of the casting mode, the temperature and shrinkage, the tension stiffening in the slab, the percentage of steel in the concrete, the ratio between permanent and variable loads, etc. The authorized reduction percentage for cracked elastic analysis recognises the ductile behaviour when the design moment resistance is reached; this ductility depends on the class of the section and is logically equal to zero for class 4 cross-sections.

2.2 Global analysis for serviceability limit states

2.2.1 Global analysis

Deformations and internal forces and moments shall be determined by elastic analysis taking account of cracking, creep and shrinkage of concrete. 5.4 (1)

For a braced structure, the flexural stiffness may be taken as the uncracked value E_aI_1 throughout the length of the member. The effect of cracking of the concrete in the hogging moment regions may be taken into account by adopting one of the following methods: 5.4 (3)

- Maximum hogging moments are reduced by the limiting values given in Table 5 for cracking concrete;

Class of cross-section (hogging bending)	1	2	3	4
For cracking of concrete	15 %	15 %	10 %	10 %

Table 5.2

Table 5 Limits to redistribution of bending moment on support

- The hogging bending moment at each internal support and the resulting top-fibre tensile stress in the concrete, σ_{ct}, are first calculated using the flexural stiffnesses E_aI_1. For each support at which σ_{ct} exceeds 0,15 f_{ctk}, the stiffness should be reduced to the value E_aI_2 over 15% of the length of the span on each side of the support. A new distribution of bending moments is then determined by re-analysing the beam. At every support where stiffnesses E_aI_2 are used for a particular loading, they should be used for all other loadings; 7.2.2 (5)(a)

- For beams with critical sections in Classes 1, 2 or 3 the following method may be used. At every internal support where σ_{ct} exceeds 0,15 f_{ctk}, the bending moment is multiplied by the reduction factor f_1 given in Figure 11, and corresponding increases are made to the bending moments in adjacent spans. 7.2.2 (5)(b)

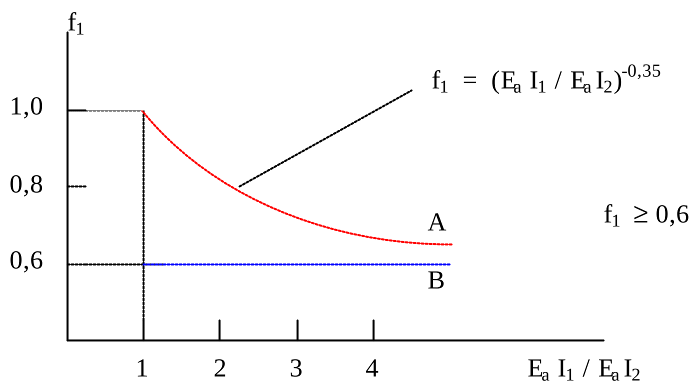


Figure 11 Reduction factor for the bending moment at supports

2.2.2 Calculation of deflections

7.2.2

The calculation of deflection for a simply supported beam is undertaken in the normal way. The inertia of the homogenised composite section I_1 is used. It must be noticed that the collaborating width b_{eff} is a relatively conservative concept here because the influence of shear lag is smaller on SLS (Service Limit States) deflections than on ULS (Ultimate Limit States) solicitations.

On the other hand, the cracking of the concrete in the hogging bending regions and the partial plastification of the steel on intermediate supports must be taken into account for the calculation of the deflections of a continuous beam.

About cracking, two methods of calculation are possible:

- Reduce by a multiplying factor f_1 the negative bending moments on supports, having calculated these using the inertia of non-cracked bending $E_a l_1$ along the whole span. Factor f_1 can conservatively be taken equal to 0,6 or more precisely as follows: $f_1 = (I_1 / I_2)^{-0,35} \geq 0,6$ when the load uniformly distributed is the same in all spans and the lengths of the spans do not differ by more than 25%.

- A more precise method consists in using a "cracked " global elastic analysis like that already used for checking ULS but in this case relating to the combination of forces SLS; no reduction of bending moments on support occurs in this case.

It is possible to represent the effect of the partial plastification of the steel in continuous composite beams caused by a combination of ELS solicitations (relatively frequent at the level of the inferior flange when the structure is not propped) on the deflection by using a second reduction factor f_2. Eurocode 4 indicates that f_2 should be taken as 0,7 when the plastification is caused by a combination of actions once the concrete is hardened (which is the current case), and 0,5 when the plastification has already been produced under the weight of concrete already poured.

The mid-span deflection of any beam can then be calculated with the following formula:

$$d_f = d_0\left[1 - C \,.\, f_1 \,.\, f_2 \,.\left(M_A^- + M_B^-\right) / M_0^+\right] \qquad \text{Eq. 2-3}$$

with :

M_A^- and M_B^-	:	are the bending moments at the supports (considered as absolute value) resulting from “non-cracked” elastic analysis;
C = 0,6	:	for a uniformly distributed load and C = 0,75 for load concentrated in mid-span;
d_0 and M_0^+	:	are the deflection and the sagging bending moment at mid-span respectively when the span is considered independent and is simply supported on its extremities.

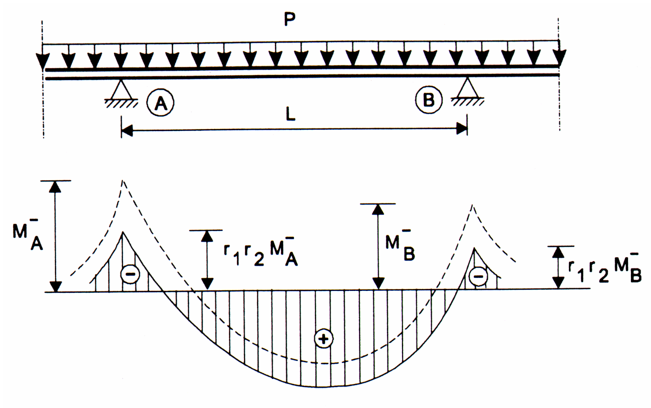


Figure 12 Simplified method for the calculation of deflection

Another parameter must be considered: the eventual influence on the deflection of slip at the steel-concrete interface. In complete connection (or more than complete), this influence is totally negligible (deflection denoted as d_f). For partial connections, the increase of the deflections cannot be neglected and depends on the method of construction. To calculate the increased deflection, the following simple formula can be proposed:

$$d = d_f \left[1 + b \, \frac{1 - N / N_f}{d_a / d_f - 1} \right] \qquad \text{Eq. 2-4}$$ 7.2.2 (6)

with:

$$N / N_f \geq 0{,}4$$

and:

d_f :	is the deflection of the composite beam in a complete connection;
d_a :	is the defection the steel beam alone with the same loads;
d :	is the real deflection;
β :	is the coefficient equal to 0,3 for an unpropped construction and 0,5 for a propped construction. The difference is due to the fact that the connectors are more solicited in the service phase for a propped structure than for an unpropped structure.
N and N_f :	Are the real number of shear connectors and the number of shear connectors necessary for a complete connection

2.2.3 Concrete cracking

Practically inevitable where concrete is in traction, the cracking of concrete is due to restrained deformations (shrinkage of concrete, displacements of support) and to direct service loads. It is necessary to control this cracking only in situations where it affects the function and durability of the structure. Appearance criteria can also be a factor.

When no measures are adopted to limit the width of the cracks in the concrete in the upper face 7.3.1 (9)
of the slab of a composite beam, it is convenient to provide in the interior of the collaboration width of the slab a percentage of longitudinal reinforcement at least equal to:

- 0,4 % of the area of concrete for a propped structure;
- 0,2 % of the area of concrete for an unpropped structure.

It is equally important to lengthen the reinforcing bars on a quarter of the span at both sides of 7.3.1 (9)
an intermediate support or on the half-span for a cantilever. Otherwise, for a composite slab, the contribution of the profiled sheet is not included in the precedent calculations. When it is judged necessary to limit the width of the cracks due to the only imposed restrained deformations, the minimum area of longitudinal reinforcement can be calculated with the following formula:

$$(A_s)_{\min} = k_s \,.\, k_c \,.\, k \,.\, f_{ct,eff} \,.\, A_{ct} \,/\, s_s \qquad \text{Eq. 2-5}$$ 7.3.2

where:

A_{ct} : is the area of the part of the slab which is in traction (for b^{-}_{eff}, the effective width);

f_{ct} : is the average resistance in traction of the concrete when the cracking is about to occur (beyond 28 days, $f_{ct} = 3N/mm^2$);

s_s : is the maximum permissible stress in the reinforcement immediately after cracking. On the one hand, it is convenient to have $s_s < f_{sk} / g_s$ in order that the reinforcement remains elastic at the time of the first cracking; on the other hand, it is even easier to limit the width of the cracks if high adherence reinforcing bars are used and if the diameter of these rods is smaller. The table related to the high adherence bars details the value to adopt for s_s as a function of the diameter of the bars and the width w_k allowed for the cracks (between 0,3 and 0,5mm);

k : is a coefficient which allows for the effect of non-uniform self equilibrating stresses which may be taken as $k=0,8$;

k_s : is a coefficient which allows for the effect of the reduction of the normal force of the concrete slab due to initial cracking and local slip of the shear connection, which may be taken as $k_s=0,9$;

k_c : is a coefficient which takes account of the stress distribution within the section immediately prior to cracking and is given by

$$k_c = \frac{1}{1+\dfrac{h_c}{2\,z_0}} + 0,3 \quad \leq \quad 1,0 \qquad \text{Eq. 2-6}$$

h_c : is the thickness of the concrete flange, excluding any haunch or ribs;

z_0 : is the vertical distance between the centroids of the uncracked and unreinforced composite section, calculated using the modular ratio for short term effects.

Diameter of the bar (mm)	6	8	10	12	16	20	25	32
Width of the crack	Maximum steel stress σ_s (N/mm^2)							
w_k = 0,3 mm	450	400	360	320	280	240	200	160
w_k = 0,4 mm	Still in discussion							

Table 7.1

Table 6 Maximum reinforcement stress

At least half of the required minimum reinforcement should be placed between middepth of the slab and the face subjected to the greater tensile strain. 7.3.2 (3)

Minimum longitudinal reinforcement for the concrete encasement of the web of a steel I-section should be determined from Eq. 2.5 with $k_c=0,6$ and $k=0,8$. 7.3.2 (4)

7.3.3

Finally, when the area of the longitudinal reinforcement imposed by the bending resistance of the composite beam at the ULS exceeds the minimum previously designated value $(A_s)_{min}$ and if the opening of cracks due to direct service stresses is limited, it is necessary to determine a maximum distance of the reinforcing bars as a function of w_k and s_s (the stress s_s must then be calculated taking account of the increase of stress along the line of a crack brought by the rigidity in traction of the slab between two cracks).

2.2.4 Vibrations

In service conditions, it can be important to limit the vibrations caused by machines as well as oscillations due to harmonic resonance, by ensuring that the eigenfrequencies of the structure or of parts of it are different from those at the source of the vibrations.
In order to analyse the frequencies and vibration modes of a composite floor in a building, it is allowed to use the characteristics of non-cracked composite sections, with the secant elasticity module E_{cm} for short-term load. In this analysis, the effects of the slip at the steel-concrete interface can be neglected. The fundamental eigenfrequency of a simply-supported composite beam can be evaluated with the help of the following simple formula:

$$f = \frac{1}{2p}\sqrt{\frac{g}{d}} \qquad \text{Eq. 2-7}$$

Taking account of the fact that $g = 9810$ mm/sec^2:

$$f = 15{,}8/\sqrt{d} \qquad \text{Eq. 2-8}$$

where f is expressed in Hz and d in mm; d is the instantaneous deflection of the beam produced by the application of its own weight and the load applied to the slab. This formula also works for an isolated beam or slab. In case of a classic floor, in the presence of slabs overlapping several parallel composite beams, the formula can still be applied with satisfactory approximation, as long as d is taken as the sum of the deflection d_s of the slab (with regard to the beams which support it) and the deflection d_b of the beams.

Concerning the floors subjected to normal usage are concerned (offices, dwellings ….) as well as car-parks, it is satisfactory if fundamental frequency f is not lower than 3Hz. In the case of gymnasium or dance floors, f should not be lower than 5Hz.

3. Concluding summary

The shear connection between the concrete slab and the steel profiles in composite construction is a key parameter for design.

In the present lecture, design rules and ductility requirements are given for usual stud connectors.

This knowledge appears as a pre-requisite for the design of simply supported and continuous beams (Lectures 6 and 7).

In the second part of the lecture, guidelines on how to perform elastic and plastic analyses of composite structures at ultimate limit state are expressed. The importance of the redistribution effects is particularly pointed out as well as the conditions in which these redistributions may take place.
In addition, emphasis is given to the verifications of the serviceability limit states (deflection, cracking, vibration).

Structural Steelwork Eurocodes

Development of a Trans-National Approach

Course: Eurocode 4

Lecture 6: Simply supported beams

Summary:

- The procedures for determining the section classification are similar to those for bare steel sections, although some modifications may be made.
- The moment resistance of class 1 and 2 sections is calculated using plastic analysis, the details depending on the neutral axis position.
- The moment resistance of class 3 sections is calculated using elastic analysis, with due account for creep and special consideration of buildings used mainly for storage.
- The vertical shear strength is based on that of the bare steel section.
- The details of the longitudinal shear connection (number and type of connector, and slab reinforcement) are determined on the basis of the longitudinal force transmitted between the steel section and concrete slab.
- Where insufficient connectors are provided, the beam may be designed on the basis of partial interaction, the moment resistance calculated on the basis of the longitudinal force transmitted between the steel section and concrete slab.
- Deflection limits are as stated in EC3 for bare steel sections.
- Concrete cracking can be controlled by ensuring a minimum amount of slab reinforcement and limiting bar size and spacing.

Pre-requisites:

Lecture 1: Introduction to Composite Construction - for those unfamiliar with composite construction

Lecture 2: Introduction to EC4 - outlines the concept of the code, and explains some of the principal terminology and notation.

Lecture 3: Structural Modelling - provides essential coverage of technical issues such as effective width, section classification in relation to global analysis, and analysis for serviceability.

Notes for tutors:

This material comprises one 60 minute lecture.

Objectives:

- To outline the design checks which are necessary for both ultimate and serviceability limit states.
- To describe the procedures for determining the section classification, and the modifications which may be made.
- To explain the procedures for calculating the plastic moment resistance of class 1 and 2 sections in relation to neutral axis position.
- To explain the procedures for calculating the elastic moment resistance of class 3 sections in relation to the method of construction, with special consideration of buildings used mainly for storage.
- To describe the simplified procedures for checking the vertical shear strength of composite beams.
- To explain how the longitudinal shear connection is designed.
- To introduce the concept of partial interaction, and describe how it affects the calculated moment of resistance.
- To outline the requirements for controlling deflections and concrete cracking at the serviceability limit state.

References:

- EC4: EN 1994-1-1: Eurocode 4 (Draft 2): Design of Composite Steel and Concrete Structures Part 1.1: General rules and rules for buildings.

Contents:

1. Introduction

A composite beam is formed by a reinforced (possibly prestressed) concrete slab attached to the upper flange of a hot-rolled or welded steel beam by shear connectors so that the two components act together as a single section.

As with steel beams, composite beams must be checked for both ultimate limit state and serviceability limit state. This lecture covers the principal checks to be applied to simply supported beams. This includes calculation procedures for the moment resistance, which depend on the section classification and position of the neutral axis. The treatment of elastic moment resistance depends on the construction sequence and whether the building is intended mainly for storage, in which case the loading is predominantly long term. Design checks for vertical shear are similar to those for bare steel beams. The design of the connectors is discussed in relation full and partial interaction, and the requirements for transverse reinforcement are described. Serviceability design is based on elastic analysis and concerns limiting deflections and controlling cracking in the concrete. The calculation procedures for these are outlined.

2. Ultimate limit state design

2.1 Design checks

The ultimate limit state design checks for simply supported beams relate to bending strength, the strength of the longitudinal shear connection, the longitudinal shear strength of the concrete slab, and the shear strength of the web including buckling. *cl. 6.2.1*

The composite beam section is based on an effective width of slab acting together with the steel beam. Details of how the effective width is determined are given in Lecture 3.

2.2. Section classification of composite beams

In the analysis of composite beams, it is important to consider the possibility of local buckling. This is done by defining the class of section, as for bare steel sections. Details of how these influence the analysis are included in Lecture 3, but a general description of the different classes and how they are determined for a particular composite section are given here for simply supported beams (in positive bending). The general description of the different classes is as follows: *cl. 5.3*

- Class 1 and 2: the section is capable of developing the full plastic bending moment $M^{+}_{pl.Rd}$; class 1 sections can also rotate after the formation of a plastic hinge, but this is not important for simply supported beams. *cl. 5.3.1*
- Class 3: because of local buckling in the part of the steel section in compression, the full plastic moment resistance cannot be achieved, although the stresses in the extreme fibres of the steel section can reach yield.
- Class 4: local buckling in the steel section occurs before yield is reached in the extreme fibres.

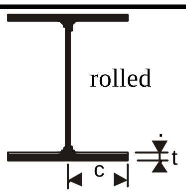
In Eurocode 4, the slenderness limits for the compression flange and web (c/t and d/t respectively) are identical to those in Eurocode 3. A section is classified according to the least favourable class of its steel elements in compression. For a simply supported composite beam this may be the top flange of the steel section, and possibly the web. *cl. 5.3.2* *cl. 5.3.3*

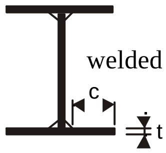
2.2.1 Classification of compression flange

For simply supported beams the compression flange is restrained from buckling by the concrete slab to which it is attached by shear connectors. Flange buckling can therefore be assumed to be effectively prevented, and the flange may be defined as Class 1. For partially encased beams (that is beams with concrete infill between the flanges, but without shear connection to the slab) the slenderness limits for the outstand of the compression flange are as shown below (EC4 *cl. 5.3.2*

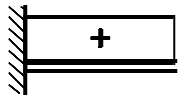
Table 5.4).

Table 5.4





Stress distribution
(compression positive)



Class	Type	Limit
1	rolled welded	c/t ≤ 10 ε c/t ≤ 9 ε
2	rolled welded	c/t ≤ 15 ε c/t ≤ 14 ε
3	rolled welded	c/t ≤ 21 ε c/t ≤ 20 ε

1	Rolled	c/t ≤ 10ε
	Welded	c/t ≤ 9ε
2	Rolled	c/t ≤ 15ε
	Welded	c/t ≤ 14ε
3	Rolled	c/t ≤ 21ε
	Welded	c/t ≤ 20ε

Table 1. Classification of steel outstand flanges in compression for partially encased sections (After EC4 Table 5.4)

c = outstand of flange (clear distance between web and toe of flange)
t = flange thickness
$\varepsilon = \sqrt{(235/f_y)}$

2.2.2 Classification of web

When the plastic neutral axis is in the slab or the upper flange, the composite section can be considered as class 1 since the web is in tension throughout. However, if the plastic neutral axis is in the web, the slenderness of the web should be checked in accordance with Table 5.2a of EC3 to determine the classification of the web, and hence the cross-section. This condition seldom applies for simply supported beams, but is described in lecture 7 for continuous beams.

EC3 Table 5.2a

Eurocode 4 allows some modifications to the classification of the web when the steel compression flange is class 1 or 2, as follows:

cl. 5.3.3.2

- A class 3 web encased in concrete can be taken as a class 2 web of the same section;
- A class 3 web not encased in concrete can be taken as an equivalent class 2 web by considering an effective height of the web in compression made up of two parts of the same

height 20εt, at the extremities of the compressed zone, where $\varepsilon = \sqrt{(235/f_y)}$. This provides a transition between classes 2 and 3, the classification of a web being very sensitive to slight changes in the area of the longitudinal reinforcement or the effective width of the slab.

2.3 Plastic moment resistance of class 1 or 2 sections

The bending resistance of a class 1 or 2 section is determined by plastic analysis. The following simplifications are assumed: *cl. 6.3.1.2*

- There is full interaction between the steel beam and concrete slab. (The case of reduced plastic resistance moment due to partial interaction is considered later).
- All the fibres of the steel beam, including those at the neutral axis, are yielded in tension or compression. The stresses in these fibres are therefore equal to the design yield strength f_{yd} ($= +/-f_y/\gamma_a$).
- The distribution of compression stresses in the concrete is uniform and equal to $0{,}85f_{ck}/\gamma_c$. The factor 0,85 allows for the difference between the test cylinder strength and the real strength observed in a structural element.
- The resistance of the concrete in tension is negligible and taken as zero.
- The slab reinforcement, when in tension, is yielded with a stress of f_{sk}/γ_s.
- Slab reinforcement (and the decking in the case of a composite slab) in compression have negligible effect on the resistance moment of the section and may be ignored. (EC4 does permit inclusion of reinforcement, excluding the profiled steel sheeting, in compression, in which case it is assumed to be stressed to its design strength.)

EC4 does not give explicit expressions for the resistance moment, but the following sections develop the analysis based on the above principles. In establishing the formulae below, the general case of a composite slab with decking profiles perpendicular to the beam is considered. The concrete in the ribs is ignored so the maximum depth of concrete in compression is limited to the thickness of the slab above the profiles h_c. The depth of the profiles is designated h_p. These formulae can be applied to solid slabs by setting $h_p = 0$. For the sake of simplicity, it is also assumed that the steel section is doubly symmetrical; for other cases the principles are the same but the formulae need modification. The value of the positive moment of plastic resistance $M^+_{pl.Rd}$ depends on the position of the plastic neutral axis; consequently, three cases are examined below.

2.3.1 Plastic neutral axis located in the slab depth

Let the plastic axial resistances of the steel beam (in tension) and of the concrete slab (in compression) be represented by N_{pla} and N_{cf}:

$$N_{pla} = A_a f_y/\gamma_a \tag{1}$$

$$N_{cf} = h_c\, b^+_{eff}(0{,}85 f_{ck}/\gamma_c) \tag{2}$$

where A_a is the area of the steel beam and b^+_{eff} the effective width of the slab in positive bending. Simply by considering the longitudinal equilibrium of the composite section it can be seen that the plastic neutral axis is located in the thickness h_c of the concrete of the slab if $N_{cf} > N_{pla}$.

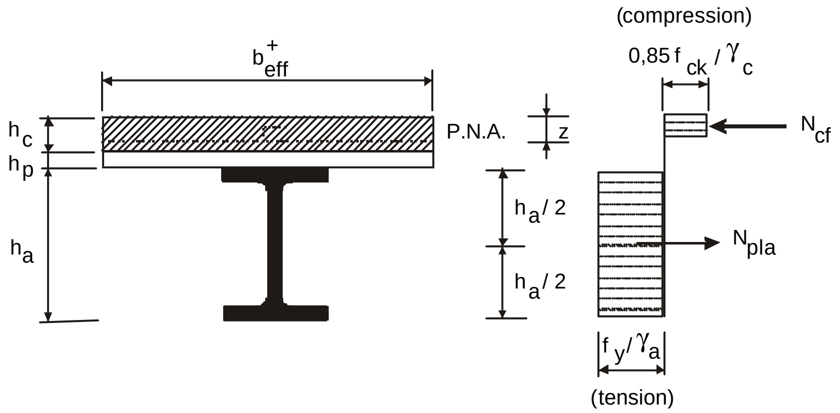


Figure 1. Plastic distribution of the normal stresses: Example of plastic neutral axis in the slab

The depth of the plastic neutral axis z measured from the upper surface of the slab (Figure 1), is given by:

$$z = N_{pla} / (b^+_{eff} 0,85 f_{ck} / \gamma_c) < h_c \qquad (3)$$

Taking moments about the resultant compression, the moment resistance is obtained:

$$M^+_{plRd} = N_{pla}\,(0,5h_a + h_c + h_p - 0,5z) \qquad (4)$$

2.3.2 Plastic neutral axis in the flange of the steel beam

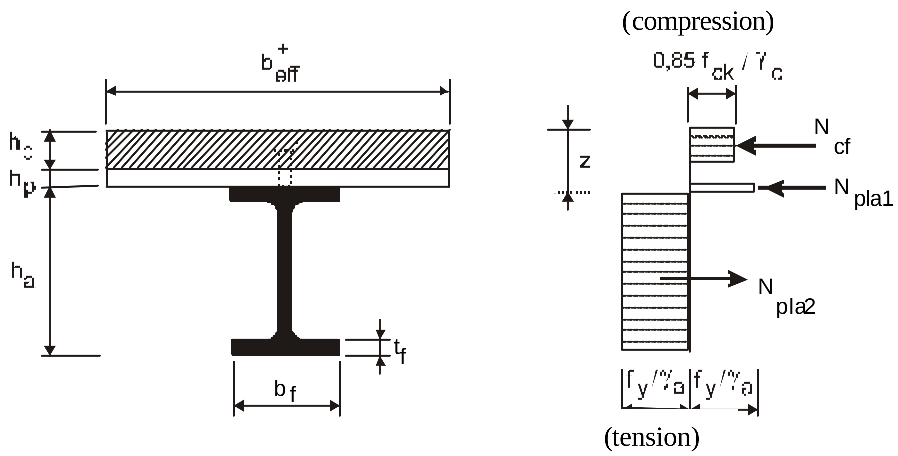


Figure 2. Plastic distribution of normal stresses: Example of the plastic neutral axis in the steel flange

If $N_{cf} < N_{pla}$ the plastic neutral axis is located in below the level of the interface - in practice within the upper flange of a symmetric steel beam for simply supported conditions. The depth of the plastic neutral axis z is greater than the total thickness of the slab ($h_c + h_p$). For the plastic neutral axis to be located in the flange of thickness t_f, and width b_f, it is also necessary that:

$$N_{pla1} < b_f\, t_f\, f_y / \gamma_a \qquad (5)$$

or

$$N_{pla} - N_{cf} < 2b_f t_f f_y / \gamma_a \quad (6)$$

The static equilibrium is unchanged if two equal and opposite forces, N_{pla1}, acting at the centre of gravity of that part of the flange in compression, are added to the forces in Figure 2: the longitudinal equilibrium can then be stated as:

$$N_{cf} + 2N_{pla1} - (N_{pla2} + N_{pla1}) = 0 \quad (7)$$

Noting that $N_{pla}=N_{pla1} + N_{pla2}$, it can be deduced that:

$$N_{pla1} = 0{,}5\,(N_{pla} - N_{cf}) \quad (8)$$

or

$$N_{pla} = N_{cf} + 2N_{pla1} \quad (9)$$

The neutral axis depth z is easily calculated by observing that the depth of the flange in compression is $[z - (h_c + h_p)]$, so that $N_{pla1} = b_1(z - h_c - h_p)f_y / \gamma_a$, and therefore:

$$N_{pla} = N_{cf} + 2b_1(z - h_c - h_p).f_y / \gamma_a \quad (10)$$

Taking moments about the centre of gravity of the concrete, the moment resistance is:

$$M^+_{pl..Rd} = N_{pla}(0{,}5h_a + 0{,}5h_c + h_p) - 0{,}5(N_{pla} - N_{cf})(z + h_p) \quad (11)$$

2.3.3 Plastic neutral axis in the web of the steel beam

The plastic neutral axis is located within the web of the steel beam if, simultaneously:

$$N_{cf} > N_{pla} \text{ and } N_{pla} - N_{cf} > 2b_f t_f f_y / \gamma_a \quad (12)$$

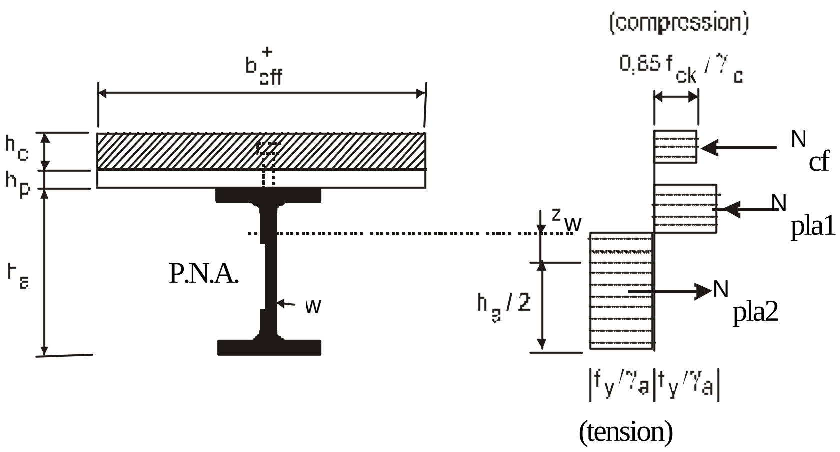


Figure 3. Plastic distribution of the normal stresses: Example of the plastic neutral axis in the web

For simplicity, the web-flange fillet is ignored. The tensile force N_{pla1} is balanced by an equal and opposite force acting in an equivalent position on the other side of the centre of gravity of the steel section. There is therefore an area of the web, of depth $2z_w$ and width t_w, at a stress of f_y / γ_a to balance the force N_{cf}. Consequently:

$$z_w = N_{cf} / (2t_w\, f_y / \gamma_a) \quad (13)$$

The moment of resistance, calculated in relation to the centre of gravity of the steel beam can then be written:

$$M^+_{pl.Rd} = M_{apl.Rd} + N_{cf}(0{,}5h_a + 0{,}5h_c + h_p) - 0{,}5N_{cf}\, z_w \quad (14)$$

The advantage of this expression is the use of the plastic moment resistance of the steel beam $M_{apl.Rd}$ which can be taken directly from standard tables for rolled steel sections.

2.4 Elastic moment resistance (class 3 sections)

Class 3 composite sections (positive bending), relate to slender webs since the compression flange is always class 1 or 2 if adequately connected to the concrete slab. Eurocode 4 provides two methods for calculating the bending strength in such cases: *cl. 6.3.1.5*

- The replacement of the class 3 web by an equivalent class 2 web. The plastic moment resistance $M^{+}_{pl.Rd}$ is then calculated according to the principles stated above on the basis of the equivalent section.
- The calculation of the elastic moment resistance $M^{+}_{el.Rd}$ considering the full steel section. No explicit expressions are given in EC4 but the following descriptions develop these in accordance with generally accepted principles and the rules specified in EC4.

In calculating the elastic moment of resistance, the effects of creep in the concrete should be taken into account. In doing so the duration of load is important, and so a distinction must be made between propped and unpropped construction, and special considerations apply to warehouses and other buildings intended mainly for storage. The approach is to use a transformed section; in general, an average value can be used for the modular ratio for steel-concrete (n_L) but for warehouses it is necessary to distinguish between short-term and long-term effects.

For non-sway structures for buildings not mainly intended for storage, a single, average value may be used for the modular ratio n, corresponding to an effective modulus of elasticity for concrete, E_c, taken as $E_{cm}/2$. E_{cm} is secant modulus of elasticity for concrete for short term loading according to Table 3.2 of EC2. Thus the average values of modular ratio, n_{av}, shown in Table 2 can be used. *cl. 5.1.4.4* *EC2 Table 3.2*

f_{ck} at 28 days (N/mm²)	25	30	35
Average modular ratio (n_{av})	13,8	13,1	12,5

Table 2. Modular ratios - nominal values, n_{av}, for non-sway structures for buildings not intended mainly for storage. (After Table 3.2 EC2)

For a warehouse, it is necessary to take account of the effects of concrete creep more precisely. The modular ratio depends on the type of loading, and is given by: *cl. 5.1.4.4*

$$n_L = n_0(1 + \psi_L \varphi_t) \qquad (15)$$

where

$n_0 = E_a / E_{cm}$ is the modular ratio for short term loading, E_a is the modulus of elasticity for structural steel, and E_{cm} is the secant modulus of elasticity of the concrete for short term loading according to Table 3.1 of EC2.

φ_t is the creep coefficient according to EC2

ψ_L is the creep multiplier which may be taken as 1,1 for permanent loads.

It is common to use the values corresponding to the strength of concrete at 28 days, in which case the values of the modular ratio, n, to be used are given in Table 3. In this table n_L is calculated according to equation 15 above.

f_{ck} at 28 days (N/mm²)	25	30	35
Short term modular ratio n_0	6,9	6,6	6,3
Long term modular ratio n_L	20,7	19,7	18,8

Table 3. Modular ratio to be used for a storage building

The elastic moment resistance is calculated on the basis of the elastic modulus of the transformed section, which is a function of the duration of the load. Note also that, because the composite beam cross-section is not symmetrical, it is necessary to consider two section moduli, relating to the extreme top and bottom of the section respectively.

EC4 does not provide explicit treatments for this, but the following sections are based on generally accepted procedures and the requirements set out in the Eurocode.

2.4.1 Propped composite beams in buildings in general

The propping of the steel beam means that it functions exclusively as a composite section.

If

- $W_{c.ab.el}$ ($W_{c.at.el}$) represent the elastic moduli of the composite section for the bottom (top) fibre of the steel beam, calculated on the basis of $n_{av.}$
- $W_{c.ct.el}$ is the elastic modulus of the composite section related to the upper fibre of the concrete slab calculated on the basis of $n_{av.}$

Then

$$M^{+}_{el.Rd} = \min \{W_{c.ab.el}\, f_y / \gamma_a;\ W_{c.at.el}\, f_y / \gamma_a;\ W_{c.ct.el}(0{,}85 f_{ck}) / \gamma_c\} \qquad (16)$$

2.4.2 Propped composite beams in storage buildings

Eurocode 4 does not provide an explicit method for this calculation which must take account of the different modular ratios.

Designating the following:

- σ_{ab} (σ_{at}) the total longitudinal stress in the lower (upper) fibre of the steel beam;
- σ_{ct} the total longitudinal stress in the upper fibre of the concrete slab;
- r the ratio of the total longitudinal stress to the permissible stress;
- $M_{0.Sd}$ ($M_{L.Sd}$) the design bending moment due to short term and long term loading respectively

the following can be calculated;

$$\begin{aligned} s_{ab} &= \frac{M_{0.Sd}}{W_{c,ab,el}\ (with\,\mathrm{n}_0)} + \frac{M_{L.Sd}}{W_{c,ab,el}\ (with\,\mathrm{n}_\mathrm{L})} \\ s_{at} &= \frac{M_{0.Sd}}{W_{c,at,el}\ (with\,\mathrm{n}_0)} + \frac{M_{L.Sd}}{W_{c,at,el}\ (with\,\mathrm{n}_\mathrm{L})} \\ s_{ct} &= \frac{M_{0.Sd}}{W_{c,ct,el}\ (with\,\mathrm{n}_0)} + \frac{M_{L.Sd}}{W_{c,ct,el}\ (with\,\mathrm{n}_L)} \end{aligned} \qquad (17)$$

with:

$$\begin{aligned} \mathrm{r}_{ab} &= \left| \frac{s_{ab}}{f_y / g_a} \right| < 1{,}0 \\ \mathrm{r}_{at} &= \left| \frac{s_{at}}{f_y / g_a} \right| < 1{,}0 \\ \mathrm{r}_{ct} &= \left| \frac{s_{cts}}{0{,}85.\, f_{ck} / g_c} \right| < 1{,}0 \end{aligned} \qquad (18)$$

Using the maximum factor r

$$r_{\max} = \max\{ r_{ab}; r_{at}; r_{ct} \} \tag{19}$$

The elastic moment resistance can then be calculated:

$$M^{+}_{el.Rd} = \frac{1}{r_{\max}} (M_{0,Sd} + M_{L,Sd}) \tag{20}$$

2.4.3 Unpropped composite beams in buildings in general

Before the development of composite action, the steel beam alone is subjected to a bending moment $M_{a.Sd}$. The composite beam is then subject to a bending moment $M_{c.Sd}$. It should be noted that the bare steel beam may be class 3 or 4, but following the development of composite action it can become class 1 or 2. It is therefore necessary to consider the possibility of local buckling in the top (compression) flange of the beam, and this can be done by following the procedures of EC3, limiting the effective slenderness of the flange, $\overline{\lambda}_p$, to 0,673. In EC3 this check is applied using the full design stress, f_y. In the case of composite beams during the construction stage, this can be replaced by the actual stress due to the self weight and construction loads, σ_{at}, multiplied by γ_a. Note that, in order to avoid yielding, the stress σ_{at} should not exceed f_y/γ_a. Thus the check becomes:

EC3 cl. 3.5.3(3)

$$\overline{\lambda}_p = \sqrt{\sigma_{at}\gamma_a / \sigma_{cr}} \le 0{,}673$$

$$\begin{aligned} \sigma_{ab} &= \frac{M_{a.Sd}}{W_{a,ai,el}} + \frac{M_{c.Sd}}{W_{c,ab,el}\ (with\ n_{av})} \\ \sigma_{at} &= \frac{M_{a.Sd}}{W_{a,at,el}} + \frac{M_{c.Sd}}{W_{c,at,el}\ (with\ n_{av})} \\ \sigma_{ct} &= \frac{M_{c.Sd}}{W_{c,ct,el}} \end{aligned} \tag{21}$$

with:

$$\begin{aligned} \rho_{ab} &= \left| \frac{\sigma_{ab}}{f_y/\gamma_a} \right| < 1{,}0 \\ \rho_{at} &= \left| \frac{\sigma_{at}}{f_y/\gamma_a} \right| < 1{,}0 \\ \rho_{ct} &= \left| \frac{\sigma_{ct}}{0{,}85.\,f_{ck}/\gamma_c} \right| < 1{,}0 \end{aligned} \tag{22}$$

$$r_{\max} = \max\{ r_{ab}; r_{at}; r_{ct} \} \tag{23}$$

$$M^{+}_{el.Rd} = \frac{1}{r_{max}}.(M_{a,Sd} + M_{c,Sd}) \tag{24}$$

2.4.4 Unpropped composite beams in storage buildings

The only difference from the preceding case is that two modular ratios must be used, one for short term effects, the other for long term effects. Therefore:

$$s_{ab} = \frac{M_{a.Sd}}{W_{a,ab,el}} + \frac{M_{0.Sd}}{W_{c,ab,el}\ (with\, \mathrm{n}_0)} + \frac{M_{L.Sd}}{W_{c,ab,el}\ (with\, \mathrm{n}_\mathrm{L})}$$

$$s_{at} = \frac{M_{a.Sd}}{W_{a,at,el}} + \frac{M_{0.Sd}}{W_{c,at,el}\ (with\, \mathrm{n}_0)} + \frac{M_{L.Sd}}{W_{c,at,el}\ (with\, \mathrm{n}_\mathrm{L})} \qquad (25)$$

$$s_{ct} = \frac{M_{0.Sd}}{W_{c,ct,el}\ (\text{with } \mathrm{n}_0)} + \frac{M_{L.Sd}}{W_{c,ct,el}\ (\text{with } \mathrm{n}_\mathrm{L})}$$

with:

$$\mathrm{r}_{\mathrm{ab}} = \left| \frac{s_{ab}}{f_y / g_a} \right| < 1{,}0$$

$$\mathrm{r}_{\mathrm{at}} = \left| \frac{s_{at}}{f_y / g_a} \right| < 1{,}0 \qquad (26)$$

$$\mathrm{r}_{\mathrm{ct}} = \left| \frac{s_{ct}}{0{,}85.f_{ck} / g_c} \right| < 1{,}0$$

$$r_a = \max\{ r_{ab}; r_{at} \} \qquad (27)$$

$$M^{+}_{el.Rd} = \min\{ \frac{M_{a,Sd} + M_{0,Sd} + M_{L,Sd}}{r_a}; M_{a,Sd} + \frac{M_{0,Sd} + M_{L,Sd}}{r_{ct}} \} \qquad (28)$$

2.5 Shear resistance

For composite beams, no simple method exists for estimating how much vertical shear is taken by the slab. This contribution is sensitive to the arrangement of connectors and to cracking of the slab at an internal support of a continuous beam. Therefore the shear stress is generally assumed to be carried by the steel web only, as for a non-composite section.

cl. 6.4.3.2
cl. 6.3.2.2

The condition for satisfactory performance when exposed to a shear force V_{Sd} can be stated as:

$$V_{Sd} < V_{plRd} \qquad (29)$$

The plastic resistance $V_{pl.Rd}$ is given by $A_v\ (f_y/\sqrt{3})/\gamma_a$ where A_v is the shear area of the bare steel beam. For a welded I or H beam, A_v is strictly the area of the web; for a rolled I or H section, part of the shear stress is transmitted by the flanges immediately adjacent to the web-flange fillets, so the following expression can be used for A_v:

EC3
cl. 5.4.6(1)

$$A_v = A_a - 2b_f t_f + (t_w + 2r)t_f \qquad (30)$$

where r is the radius of the fillets.

This simple check is only valid if the web is not subject to shear buckling. That is the case if:

EC3
cl. 5.4.6(7)

$d/t_w < 69\varepsilon$ for an unstiffened web which is not encased;

$d/t_w < 124\varepsilon$ for an unstiffened web which is encased in concrete suitably reinforced with longitudinal bars, stirrups or a welded mesh;

$d/t_w < 30\varepsilon\sqrt{k_\tau}$ for a stiffened web not encased, with:

$$t_{cr} = k_t \frac{p^2 . E_a}{12.(1-n^2)} \left(\frac{t_w}{d}\right)^2 \qquad (31)$$

and k_τ , the buckling coefficient, equal to:

EC3
cl. 5.6.3(3)

$k_t = 4+5{,}34/(a/d)^2$ if $a/d£1$

$k_t = 5{,}34+4/(a/d)^2$ if $a/d>1$ (32)

The coefficient a/d is the panel aspect ratio, where a is the stiffener spacing and *d* is the height of the web (in the case of a rolled profile, the depth of the section). For a web which is both stiffened and encased, the limiting value for d/t_w should be taken as the lower of the two limits.

If these conditions relating to the slenderness of the web are not satisfied, it is necessary to substitute the shear buckling resistance $V_{b.Rd}$ for the plastic resistance $V_{pl.Rd}$. This is quite common for composite bridges but less so for buildings. In this case case the check becomes:

$$V_{Sd} < V_{b.Rd} \qquad (33)$$

$V_{b.Rd}$ is determined in accordance with Eurocode 3.

cl. 6.3.2.3

For unstiffened webs or webs with transverse stiffeners only, the methods given in Eurocode 3 to calculate $V_{b.Rd}$ are used. For a uniformly loaded, simply supported, unstiffened composite beam, with full interaction (see below) an alternative method can be used. This is a modified form of the simple post-critical method of Eurocode 3 as follows:

$$V_{b.Rd} = d\, t_w\, \tau_{bd} / \gamma_{Rd} \qquad (34)$$

$$\begin{array}{ll} \bar{l}_w \le 1{,}5 & t_{bd} = f_{yd}/\sqrt{3} \\ 1{,}5 < \bar{l}_w \le 3{,}0 & t_{bd} = (f_{yd}/\sqrt{3})(3/\bar{l}_w + 0{,}2\,\bar{l}_w - 1{,}3) \\ 3{,}0 < \bar{l}_w \le 4{,}0 & t_{bd} = (f_{yd}/\sqrt{3})(0{,}9/\bar{l}_w) \end{array} \qquad (35)$$

With

$$\bar{l}_w = \frac{d/t_w}{37{,}4\, e \sqrt{k_t}} \qquad (36)$$

It should be noted that the reduced slenderness ratio $\bar{l}_w$ should not exceed 4,0. Furthermore, wherever $V_{sd} > V_{cr}$ the N connectors providing the complete connection in each span must be arranged with a concentration of connectors close to the supports as follows:

- N_2 connectors, determined as below, should be provided within a distance from the support of $1{,}5b_{eff}$
- The remainder of the connectors should be distributed evenly throughout the half span.

$$V_{cr} = d\, t_w\, t_{cr} \text{ where } t_{cr} \text{ is defined by } k_t . \frac{p^2 . E_a}{12.(1-n^2)} . \left(\frac{t_w}{d}\right)^2 \qquad (37)$$

$$N_2 = N.(1 - V_{cr}/V_{Sd})^2 \text{ and } N_1 = N - N_2$$

3. Design of connectors for simply supported composite beams class 1 or 2

3.1. Full interaction

Consider a simply supported beam (Figure 4), with either a uniformly distributed design load, P_d, or a concentrated design load, Q_d (the case of the two types of load acting together and more complex loads are described later). *cl. 6.7.1.3* *cl. 6.2.1*

The beam is considered as a series of 'critical lengths' representing the lengths between adjacent critical cross-sections, defined as: *cl. 6.2.1*

- point of maximum bending moment
- supports
- concentrated loads

Thus the critical lengths for Figure 4 are AB and BC.

If the plastic moment resistance is reached at the critical section B, the total longitudinal shear force V_{lN} exerted on each critical length depends on whether the tensile strength of the steel section is less than or greater than the compressive strength of the slab, and is given by:

$$V_{lN} = min\ (A_a\, f_y / g_a\ ;\ 0{,}85\ b_{eff}\, h_c\, f_{ck}/g_c) \qquad (38)$$

If the connectors are assumed to be ductile, plastic redistribution of the shear force results in them all operating at the same load, P_{Rd}, where P_{Rd} is the design strength of a single connector. The number of connectors for the critical length necessary to achieve complete connection is therefore: *cl. 6.7.1.1*

$$N_f^{(AB)} = N_f^{(BC)} = V_{lN} / P_{Rd} \qquad (39)$$

These connectors can generally be spaced uniformly over each critical length. *cl. 6.7.1.3*

3.2 Partial interaction

If the number of connectors provided is less than that calculated the interaction between the beam and slab is partial. However, if the connectors are 'ductile', and the cross-section classification is class 1 or 2, the principles of composite design can still be used. *cl. 6.7.1.1*

3.2.1 Ductile connectors

Ductile connectors are those which can provide sufficient slip at the steel-concrete interface, whilst maintaining their shear resistance. Headed studs may generally be considered ductile, subject to the following limitations: *cl. 6.7.1.1*

- overall length of stud should be not less than four times its diameter *cl. 6.7.1.2*
- stud diameter should be not less than 12mm and not greater than 25mm
- the degree of shear connection, defined by the ratio $\eta \geq N/N_f$ is within the following limits:
- For a solid slab and a steel profile with equal flanges

  for $L_e \leq 25$m $\quad \eta \geq 1-(355/f_y)\ (0{,}75-0{,}03\ L_e);$ $\quad \eta$ ³ $0{,}4$

  for $L_e > 25$m $\quad \eta \geq 1$ (40)

- For a solid slab and a steel profile in which the area of the lower flange does not exceed three times the area of the upper flange:

  for $L_e \leq 20$m $\quad \eta \geq 1-(355/f_y)\ (0{,}30-0{,}015\ L_e);$ $\quad \eta$ ³ $0{,}4$

  for $L_e > 20$m $\quad \eta \geq 1$ (41)

- For a composite slab (with b_o/h_p ³ 2 and h_p £ 60mm) connected with welded studs (d = 19 or 20mm and h ³ 76mm):

for $L_e \leq 25$m $\quad\eta \geq$ *1-(355/f_y) (1-0,04 L_e)* ³ *0,4*

for $L_e > 25$m $\quad\eta \geq$ *1* (42)

where L_e is the distance in the span between points of zero bending moment (metres). For simply supported beams this is therefore equal to the span L. *EC4 Fig. 5.1*

3.2.2 Partial composite design of beams

When the number of connectors N on a critical length is lower than N_f, this length, and hence the beam, are partially connected. As a result, the total longitudinal shear force transferred by the connectors over the critical length is reduced.

$$V_l^{(réd)} = N\,P_{Rd} < V_{lN} \tag{43}$$

In the same way, the moment resistance of the critical section B is reduced:

$$M^+{}_{Rd}{}^{(réd)} < M^+{}_{pl.Rd} \tag{44}$$

In effect the axial force in each component, steel and concrete, is limited to +/-$V^{(red)}$. The reduced moment resistance $M^+{}_{Rd}{}^{(red)}$ is determined in the same way as the plastic moment resistance $M^+{}_{pl.Rd}{}^{(red)}$, assuming rectangular stress blocks in the different materials. Two plastic 'neutral axes' are defined, one in the slab and the other in the steel beam. The compression in the slab and tension in the steel must be identical and equal to $V_l^{(red)}$. The expression for the reduced plastic moment resistance $M^+{}_{pl.Rd}{}^{(red)}$ can be calculated in a similar manner to the full plastic moment but replacing N_c by $V_l^{(red)}$.

The relation between the reduced moment resistance $M^+{}_{pl.Rd}$ and the number of connectors N on the critical length can be deduced analytically. Diagrammatically $M^+{}_{pl.Rd}{}^{(red)} = f(N/N_f)$, as shown by the curve ABC in Figure 5. The ratio N/N_f is designated the *degree of connection* of the critical length. It is evident that when $N^{(AB)}$ is different from $N^{(BC)}$, it is the weaker of the two which is significant for the beam.

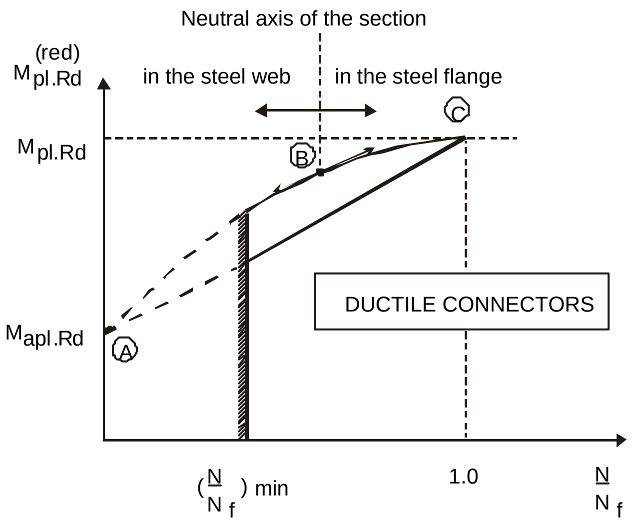


Figure 5. Resistant moment in terms of the degree of connection (ductile connectors)

For N/N_f=1 (complete connection) the resistant moment is not reduced and is equal to $M^+{}_{apt.Rd}$; for N/N_f=0 (no connectors), the reduced resistant moment is the moment of plastic resistance of the steel beam alone, $M_{pt.Rd}$. The point B on the curve corresponds to the condition where the neutral axis of the composite beam is located just at the level of the flange/web junction;

different calculation procedures for the moment resistance are therefore required either side of this point as described in section 2.3. The curve ABC is continuous at B and is always convex; consequently, one can use a simplified conservative method, replacing the curve ABC by the line straight line AC. The reduced moment resistance can then be calculated as follows:

$$M^{+}_{pl.Rd}{}^{(réd)} = M_{apl.Rd} + N/N_f \; (M^{+}_{pl.Rd} - M_{apl.Rd}) \qquad (45)$$

If the degree of connection is too low, the curve ABC (or its simplification AC) ceases to be valid because the collapse mechanism involves rupture of the connectors (the design method assumes overall ductility which they cannot provide) instead of a plastic hinge in the critical section.

3.3 More complex load cases

Until now, only simple load cases have been considered. When significant concentrated loads are applied at the same time as a distributed load, intermediate sections must be checked under these concentrated loads and the number of connectors must be sufficient for each critical length corresponding to the distance between loads. Thus, for Figure 6, the bending moment diagram can prove relatively flat and intermediate point B should be considered within the critical length AC (similar considerations apply to point D within CE). If the design bending moment at B is $M_{Sd}^{(B)}$ and using the linear approximation for reduced moment of resistance, the number of connectors $N^{(AB)}$ can be assumed to be equal to:

$$N^{(AB)} = N_f^{(AC)} \; (M_{Sd}^{(B)} - M_{apl.Rd})/(M_{pl.Rd} - M_{apl.Rd}) \qquad (46)$$

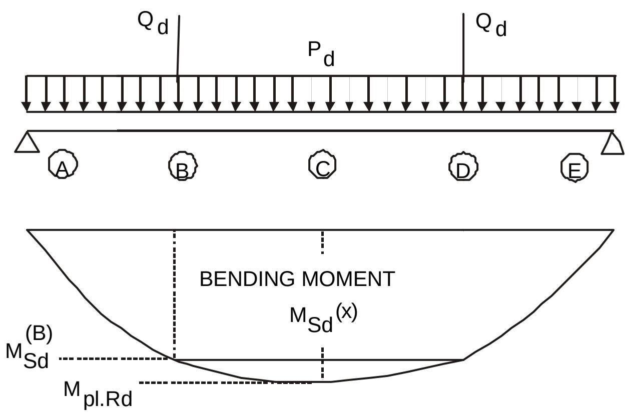


Figure 6. Internal section for checking

In practice, $N^{(AC)}$, the total number of connectors to achieve full connection along AC is calculated. Of these, $N^{(AB)}$ connectors will be distributed uniformly over the length AB and the rest of the connectors, $(N_1^{(AC)}-N^{(AB)})$ uniformly over BC.

4. Connector design for class 3 or 4 beams

The longitudinal shear check for class 3 and 4 sections is based on elastic behaviour. The longitudinal shear stress, V, is calculated as: *cl. 6.7.2.2*

$$V=T.S_1/I \qquad (47)$$

using the elastic properties of the section. The spacing of the connectors must be designed to ensure that the resistance to longitudinal shear is higher than the design longitudinal shear force. There will be a greater concentration of connectors near to the supports, where the shear stress, and therefore the longitudinal shear stress, are greatest.

5. Transverse reinforcement *cl. 6.7.17*

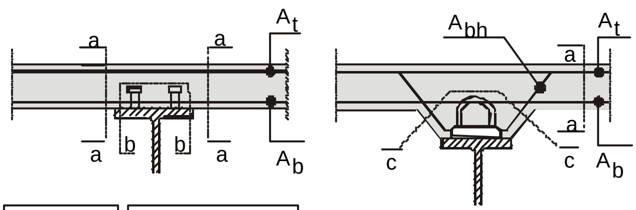
A slab must have adequate transverse reinforcement to transmit the stress from the connectors *cl. 6.7.17.1* and ensure that there is no risk of premature failure of the concrete due to longitudinal shear. A_e represents the total area of transverse reinforcement per unit length of the beam intersecting the potential shear failure surfaces in the slab (Figure 7). The value of A_e will depend on the arrangement of connectors and reinforcement, on the presence or otherwise of a haunch, and on the failure surface considered. L_s defines the length of this failure surface. Hence, for example, for failure along b-b (Figure 7):

$$L_s=2h+s+d_1 \tag{48}$$

where h is the total height of a stud, d_1 the diameter of its head and s the spacing (centre to centre) of the two studs.

For the same failure surface, the value of A_e is given by:

$$A_e = 2\,A_b \tag{49}$$



Plane	A_e
a - a	$A_b + A_t$
b - b	$2\,A_b$
c - c	$2\,(A_b + A_{bh})$
d - d	$2\,A_{bh}$

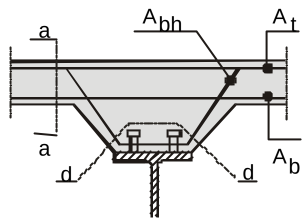


Figure 7. Definition of the transverse reinforcement section relative to different shear planes

The design shear per unit length, V_{Sc1} must not be greater than the shear resistance V_{Rd} of the failure surface (L_s x 1). By using a classic reinforced concrete truss analogy, relative to the reinforcement crossing a slip plane, the shear resistance V_{Rd} can be assumed to be equal to:

$$V_{Rd} = \min(V_{Rd}^{(1)}, V_{Rd}^{(2)}) \tag{50}$$

with

$$V_{Rd}^{(1)} = 2{,}5\tau_{Rd}\,L_s + A_e\,f_{sk} / \gamma_s \tag{51}$$

and

$$V_{Rd}^{(2)} = 0{,}2\,L_s\,f_{ck} / \gamma_{Rd} \tag{52}$$

The resistance $V_{Rd}^{(1)}$ can be interpreted as the resultant of the concrete in shear and the reinforcement in tension (functioning as truss members); $V_{Rd}^{(2)}$ is equivalent to the truss diagonals represented by the concrete in compression (the coefficient 0,2 in the formula, rather than 0,4, accounts for the 45 degree inclination of these 'diagonals' and is justified by the reduced compressive strength of the concrete f_{ck}/γ_c due to transverse stress in the concrete). The resistance τ_{Rd} corresponds to the design shear strength of the concrete; the factor 2,5 (classic for reinforced concrete) represents the effect of the longitudinal reinforcement; this is related to the compression strength f_{ck} as shown in Table 4 for normal weight concrete.

f_{ck} (N/mm²)	20	25	30	35	40	45	50
t_{Rd} (N/mm²)	0,26	0,30	0,34	0,38	0,42	0,46	0,50

Table 4. Characteristic strength of concrete in shear

The formulae also apply to lightweight concrete as long as the strengths f_{ck} and τ_{Rd} are multiplied by the factor

$$\eta = 0,3 + 0,7(\rho/2400) \qquad (53)$$

where ρ is the density of the concrete in kg/m^3.

Profiled steel sheet used for composite slabs can be treated as equivalent reinforcement. Thus, in the case of ribs perpendicular to and continuous over the steel beam, a third term can be added to the expression for $V_{Rd}^{(1)}$ which becomes *cl. 6.7.17.3*

$$V_{Rd}^{(1)} = 2,5\ \tau_{Rd}\ L_s + A_e\ f_{sk} /\ \gamma_s + A_p\ f_{yp} /\gamma_{ap} \qquad (54)$$

where

A_p is the area of the section of the sheet (crossing the potential failure surface) along the length of the beam

f_{yp} is the nominal elastic limit of the sheet

γ_{ap} is the appropriate partial safety factor taken as 1,1.

Furthermore, all transverse reinforcement can be included in A_e, including that provided for the transverse bending resistance of the slab, since the vertical shear stresses in this are generally small.

Finally, a minimum amount of transverse reinforcement is always necessary in order to carry the secondary shear stresses which cannot be calculated. For solid slabs Eurocode 4 recommends a minimum area of 0,2% of the area of concrete; the same proportion applies to composite slabs considering only the concrete above the ribs (when these are perpendicular to the beam), but possibly including the profiled sheet in this proportion.

6. Serviceability limit state design

6.1 General

Serviceability limit state design checks for composite beams concern the control of deflection, cracking of the concrete, and vibrations (for large spans). In conventional building applications a rigorous analysis can often be avoided. For example, the shrinkage effects of concrete on deflections need only be considered for simply supported beams with a span:depth ratio greater than 20 and the predicted free shrinkage of the concrete exceeds 4×10^{-4}. Similarly the elastic analyses may be simplified by using a single modular ratio, n, combining long-term creep effects and instantaneous elastic deformations. Eurocode 4 does not specify 'permissible stress' limits, thereby admitting partial plasticity at the serviceability limit state, whether in mid-span (which does not greatly influence the deflection), or on the intermediate supports in the case of a continuous beam (the effect on the deflection is taken into account in a prescriptive manner). Experience indicates that the risk of cumulative plastic deformation is negligible in view of the nature of the building loads and the high proportion of permanent loads. *cl. 7.1*

6.2 Calculation of deflections

Eurocode 4 adopts the same limits as Eurocode 3 for permissible deflections. In practical floor construction, these requirements are met (although not explicitly stated in EC4) if the span:depth ratio of the composite section is less than the following: *cl. 7.2.1*

- for simply supported beams: 15 to 18 for main beams, 18 to 20 for secondary beams (joists);

- for continuous beams : 18 to 22 for main beams, 22 to 25 for secondary beams (joists).

The calculation of deflections for a simply supported beam is undertaken in the normal way, using the second moment of area of the transformed composite section I_1. It should be recognised that, in this context, the effective width b_{eff} is relatively conservative since the influence of shear lag at serviceability limit state is less than at the ultimate limit state. *cl. 7.2.2*

Again no specific procedures are stated, but the effects of creep should be included. It is therefore appropriate to consider relevant values of the modular ratio in determining the equivalent second moment of area of the transformed section, distinguishing between propped and unpropped construction, and between normal buildings and those intended mainly for storage.

6.3 Concrete cracking

Cracking of concrete is almost inevitable when it is subject to tension. For simply supported beams, such tension is largely due to shrinkage of the concrete as it hardens. It is generally sufficient to limit crack widths to 0,3mm and to ensure compliance using the procedures of EC2. As a simplified, safe alternative, it is generally sufficient to ensure a minimum percentage of reinforcement and to limit the bar spacing or diameter. *cl. 7.3.1*

The simplified requirements for the minimum area of reinforcement, A_s is given by: *cl. 7.3.2*

$$A_s = k_s\, k_c\, k\, f_{ct,eff}\, A_{ct} / \sigma_s \qquad (55)$$ *(7.3)*

$f_{ct,eff}$ is the mean tensile strength of the concrete, which may often be taken as 3N/mm^2

k is generally taken as 0,8

k_s is generally taken as 0,9

k_c accounts for the stress distribution and is given by -

$$k_c = 1 / \{1 + h_c / (2\, z_o)\} + 0{,}3 \le 1{,}0 \qquad (56)$$ *(7.4)*

h_c is the thickness of the concrete flange, excluding any haunch or ribs

z_o is the distance between the centroids of the concrete flange and the composite section assuming the concrete is uncracked and ignoring any reinforcement.

A_{ct} may simply be taken as the area of the concrete section within the effective width

σ_s may simply be taken as the characteristic strength, f_{sk}, of the reinforcement, although a lower value may need to be adopted depending on bar size and design crack width (see Table 5).

Control of cracking due to direct loading is not relevant to simply supported beams.

steel stress σ_s N/mm^2	maximum bar diameter (mm) for design crack width		
	w_k = 0,4mm	w_k = 0,3mm	w_k = 0,2mm
160	40	32	25
200	32	25	16
240	20	16	12
280	16	12	8
320	12	10	6
360	10	8	5
400	8	6	4
450	6	5	-

Table 5. Maximum bar diameters for high bond bars - for crack control (after Table 7.1 EC4).

7. Concluding summary

- The procedures for determining the section classification are the same as for bare steel sections, although some modifications may be made for webs.
- The moment resistance of class 1 and 2 sections is calculated using plastic analysis, the details depending on the neutral axis position
- The moment resistance of class 3 sections is calculated using elastic analysis, with due account for creep and special consideration of buildings used mainly for storage
- The vertical shear strength is based on that of the bare steel section
- The details of the longitudinal shear connection (number and type of connector, and slab reinforcement) are determined on the basis of the longitudinal force transmitted between the steel section and concrete slab.
- Where insufficient connectors are provided, the beam may be designed on the basis of partial interaction, the moment resistance calculated on the basis of the longitudinal force transmitted between the steel section and concrete slab.
- Deflection limits are as stated in EC3 for bare steel sections
- Concrete cracking can be controlled by ensuring a minimum amount of slab reinforcement and limiting bar size and spacing.

Structural Steelwork Eurocodes
Development of
A Trans-national Approach

Course: Eurocode 4

Lecture 7 : Continuous Beams

Summary:

- Continuous beams are an alternative to simply supported beams and their use is justified by considerations of economy.
- In the hogging bending regions at supports, concrete will be in tension and the steel bottom flange in compression, leading to the possibility of onset of local buckling. This is taken up by the classification of cross sections.
- Rigid-plastic design may be performed for beams with Class 1 cross-sections. Plastic moment resistance of cross-sections can be used for Class 1 and 2 cross-sections.
- For Class 3 sections, elastic analysis and elastic cross-section resistance must be used.
- The principles of calculation of cross-section resistance, either plastic or elastic, are similar to the case of sagging bending. The tension resistance of concrete is neglected.
- Lateral-torsional buckling is a special phenomenon which can be prevented by conforming to certain detailing rules.
- The design of the shear connection in the case of continuous beams is more complex than for simply supported beams.
- Serviceability checks include deflection and vibration control as well as that of concrete cracking. This latter is specific to continuous beams because tension in concrete at the hogging moment regions may cause unacceptable cracks, while in simply supported beams cracking is only due to shrinkage of concrete and is therefore lower in magnitude.

Pre-requisites:

- Lecture 3 “Structural modelling”
- Lecture 5 “Shear connectors and structural analysis”
- Lecture 6 “Simply supported beams”

Notes for Tutors:

This material comprises one 45 minute lecture.
Unmarked references to Draft 2 of prEN 1994-1-1 (April 2000)
References marked by “EC3” to Revised Draft 2 of prEN 1993-1-1 (6th December 2000)

Objectives:

The student should:

- Appreciate the advantages of continuous composite beams and be aware of their disadvantages.
- Understand the methods of plastic and elastic design of continuous beams.
- Understand the methods of calculation of elastic and plastic cross-section resistance for hogging bending moment, shear resistance and resistance against lateral-torsional buckling.
- Understand the way shear connection is designed for class 1 and class 2 cross-sections.
- Be aware the need for serviceability checks for cracking in the hogging moment region.

References:

[1] Eurocode 4: Design of composite steel and concrete structures Part 1.1: General rules and rules for buildings, prEN 1994-1-1:2001 – draft 2, April 2000
[2] Eurocode 3: Design of steel structures. Part 1: General rules, prEN 1993-1-1:200x – revised draft 2, 6th December 2000
[3] ESDEP Group 10: Composite construction, Lectures 10.4.1 and 10.4.2: Continuous beams I-II
[4] R. P. Johnson, D. Anderson: Designer's handbook to Eurocode 4. Part 1.1: Design of composite steel and concrete structures, Thomas Tilford, London, 1993
[5] K. Roik, G. Hanswille, J. Kina: Solution for the lateral torsional buckling problem of composite beams, Stahlbau, 1990a, vol. 59, H11, pp. 327-332 (in German)
[6] K. Roik, G. Hanswille, J. Kina: Background to Eurocode 4 clause 4.6.2. and Annex B, Minister für Raumordnung, Bauwesen und Städtebau, Report RSII 2-674102-88.17, University of Bochum, 1990b
[7] R.P. Johnston, C.K.R. Fan: Distortional lateral buckling of continuous composite beams, Proc. Instn. Civ. Engrs., Part 2, 1991, vol. 91, March, pp. 131-161
[8] R.M. Lawson, J.W. Rackham: Design of haunched composite beams in buildings, Publication 060, The Steel Construction Institute, Ascot, 1989

Contents:

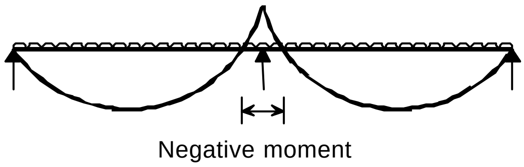
1. Introduction

The use of continuous beams in composite construction as an alternative to simply supported beams may be justified by their better load resistance and higher stiffness. If designed for the same loading, smaller sections may be sufficient than in the case of simply supported beams [3].

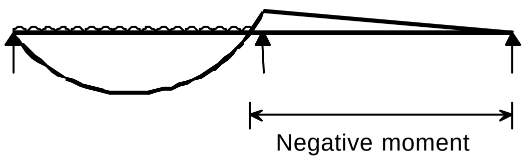
However, continuous beams are more complex to design as they are susceptible to buckling phenomena including local buckling and lateral-torsional buckling in the hogging (negative) moment regions. In the design situations related to the construction stage, these hogging moment regions may extend to a whole span of a continuous beam (Figure 1).

This lecture discusses the various phenomena specific to continuous beams and the relevant provisions for design according to EC4 [1]. Matters common to simply supported and continuous beams are discussed in Lecture 6.

Section 2 covers rigid-plastic design, relevant for composite beams in sections of Class 1 and 2, and discusses some aspects of analysis, the classification of the cross-section and presents methods to determine the design plastic moment resistance for hogging moment. Section 3 is devoted to elastic design, relevant for any class of composite beams, including the problem of the effective width in the hogging moment region, the calculation of the elastic moment resistance for hogging moment, and the question of plastic redistribution of internal moments obtained by elastic analysis. The subsequent chapters discuss various problems such as vertical shear resistance, lateral-torsional buckling, shear connection and serviceability requirements.



(a) Both spans loaded



(b) One span loaded

Figure 1. Bending moment distributions

2. Rigid-plastic design

2.1 Rigid-plastic analysis

Rigid-plastic design which has been introduced in Lecture 5 “Shear connectors and structural analysis” is used widely for continuous beams. It is recalled here that the analysis phase of the design process is based on the assumption that the plastic regions, extending in reality over finite lengths of the beam, are concentrated at discrete locations termed plastic hinges. These plastic hinges are capable of sustaining cross-section rotations which lead to a redistribution of the internal moments with respect to the elastic distribution. As a result, the structure fails through the occurrence of a plastic mechanism containing a sufficient number of plastic hinges for the structure to become statically over-determinate.

For this analysis to be valid, it is necessary that the cross-sections where the plastic hinges occur, are capable of developing and sustaining their plastic moment resistance during the process of moment redistribution. It is local buckling that may limit this capability, therefore limiting the slenderness of compression plate elements one may obtain a cross-section of appropriate behaviour.

2.2 Required plastic moment resistance of cross-sections

To design a suitable cross-section against flexure, one must determine the distribution of bending moments due to the applied load.

Let the ratio of the negative to the positive moments of resistance in a proposed cross-section be ψ, i.e.

$$\psi = \frac{M'_{pl}}{M_{pl}}. \tag{1}$$

Consider the end span of the continuous composite beam, subject to a uniformly distributed design load of w_f per unit length. The bending moment diagram at collapse is as shown in Figure 2. It can be shown by analysis of the collapse mechanism that:

$$\beta = \frac{\sqrt{1+\psi}-1}{\psi} \tag{2}$$

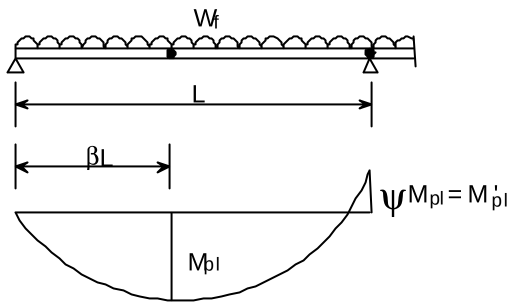


Figure 2. End span of a continuous beam

and the required value of M_{pl} is:

$$M_{pl} = \frac{w_f \beta^2 L^2}{2}. \tag{3}$$

For an internal span with equal support moments (Figure 3) it can similarly be shown that:

$$M_{pl} = \frac{w_f L^2}{8(1+\psi)}. \tag{4}$$

For other arrangements of loading and/or moment resistance ratios, the required resistance may be found from similar analyses.

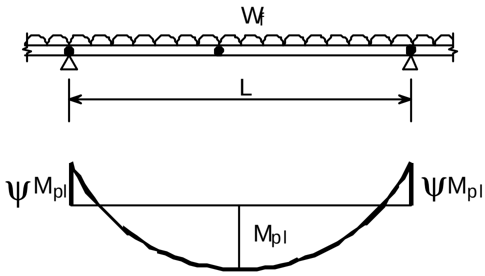


Figure 3. Internal span of a continuous beam

2.3 Cross-section classification according to Eurocode 4

Eurocodes 3 and 4 introduce the concept of cross-section classification to determine whether or not local buckling limits the ability of the cross-section to develop its plastic moment resistance and the rotations necessary for the redistribution of internal moments. The limitations included in the code recognise the fact that some loss of rotation capacity due to local buckling is offset by the beneficial effect of strain-hardening. 5.3 (1) 5.3 (2)

Class 1 cross-sections are those which may be considered capable of developing both the plastic moment resistance and the necessary rotation capacity before any local buckling. Class 2 cross-sections are able to develop their plastic moment resistance, but rotation capacity is limited by local buckling. The classification of a cross-section is determined by the classification of its constituent plate elements that are in compression, which in turn is determined by width-to-thickness ratios.

Tables 1 and 2 provide the appropriate limits of plate width-to-thickness ratios according to Eurocode 3 and 4 [1, 2]. For the sake of completeness, the tables also include the limits for Class 3 and 4, necessary for the case of elastic design. Table 5.4 EC3: Table 5.2

In Table 1 it can be seen that in the case of an encased web, the favourable restraint due to the concrete between the flanges of the steel cross-section is significant, especially for Classes 2 and 3.

Based on the observation that there is a discontinuity between Classes 2 and 3 which makes the classification of webs over-sensitive to small changes in the area of longitudinal reinforcement or in the effective width of the slab, Eurocode 4 allows modification of the classification of the web obtained from Table 2 as follows:

Provided that the compression flange is Class 1 or 2, it is possible–

- to represent Class 3 encased webs by an effective web of the same cross-section in Class 2; 5.3.3.2.(1)
- to represent Class 3 uncased webs by an effective web in Class 2, assuming that the depth of the web that resists compression is limited to 20 εt adjacent to the compression flange and 20 εt adjacent to the new plastic neutral axis (Figure 4). 5.3.3.2.(2) EC3: 5.5.4 (5)

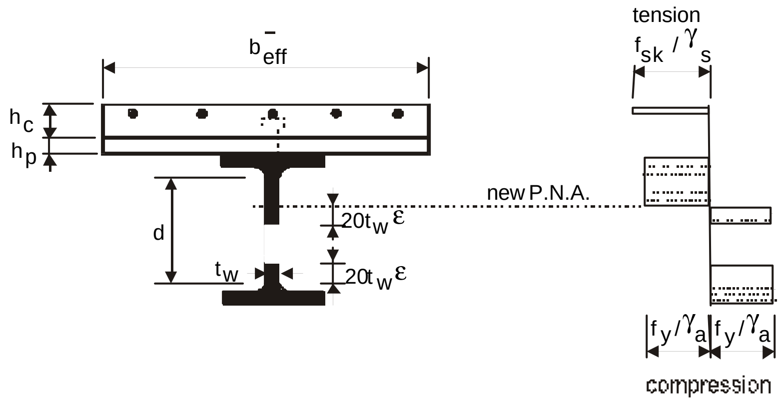


Figure 4. The use of an effective web in class 2 for a section in hogging bending with a web in class 3

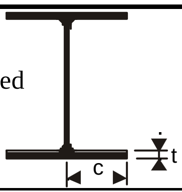
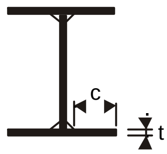
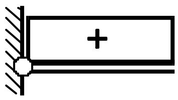
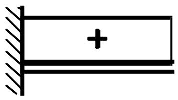
	rolled 	welded 	$\varepsilon = \sqrt{\dfrac{235\ \text{N/mm}^2}{f_y}}$
Class	Type	Web not encased	Web encased
Stress distribution (compression positive)			
1	rolled welded	c/t ≤ 9 ε c/t ≤ 9 ε	c/t ≤ 10 ε c/t ≤ 9 ε
2	rolled welded	c/t ≤ 10 ε c/t ≤ 10 ε	c/t ≤ 15 ε c/t ≤ 14 ε
3	rolled welded	c/t ≤ 14 ε c/t ≤ 14 ε	c/t ≤ 21 ε c/t ≤ 20 ε

Table 1. Classification for lower beam flanges in compression (negative bending)

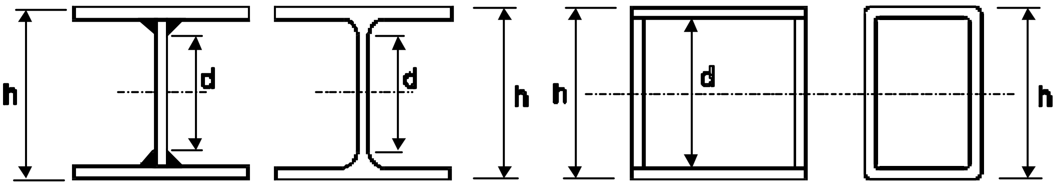
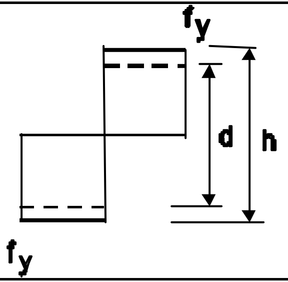
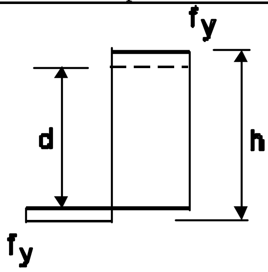
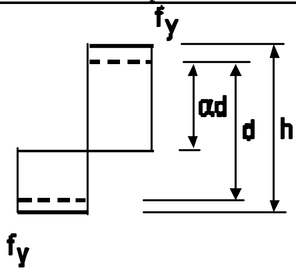
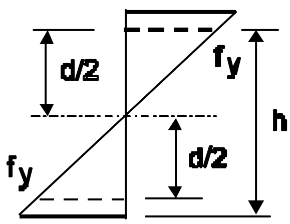
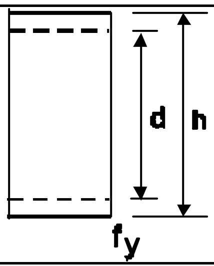
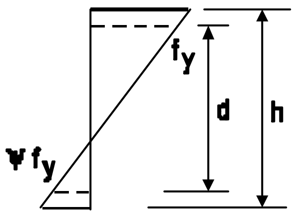
			
Class	Web subject to bending	Web subject to compression	Web subject to bending and compression
Stress distribution (compression positive)			
1	$d/t \le 72\ \varepsilon$	$d/t \le 33\ \varepsilon$	when $\alpha > 0,5$: $d/t \le 396\varepsilon/(13\alpha - 1)$ when $\alpha < 0,5$: $d/t \le 36\varepsilon/\alpha$
2	$d/t \le 83\ \varepsilon$	$d/t \le 38\ \varepsilon$	when $\alpha > 0,5$: $d/t \le 456\varepsilon/(13\alpha - 1)$ when $\alpha < 0,5$: $d/t \le 41,5\varepsilon/\alpha$
Stress distribution (compression positive)			
	$d/t \le 124\ \varepsilon$	$d/t \le 42\ \varepsilon$	when $\psi > -1$: $d/t \le 42\varepsilon/(0,67 + 0,33\psi)$ when ≤ -1: $d/t \le 62\varepsilon.(1-\psi).\sqrt{-\psi}$

Table 2. Classification for web (negative bending)

2.4 Plastic hogging bending resistance according to Eurocode 4

The plastic resistance of a composite cross-section against hogging bending is calculated considering the steel section and the effective anchored reinforcement located within the effective width b_{eff}^{-} (see section 3). This longitudinal reinforcement should be highly ductile so that the cross-section is able to develop its full plastic moment resistance. As in the case of general composite cross-section configurations, it is assumed that the concrete slab is cracked over the whole depth and the plastic neutral axis is located within the steel cross-section. Two cases should be distinguished according to the location of the plastic neutral axis within the steel section: 6.3.1.2 (1)

Case 1 – The plastic neutral axis is within the flange;
Case 2 – The plastic neutral axis is within the web.

EC4 does not give explicit expressions for the moment resistance, but the following sections develop the analysis based on the above principles and the basic assumptions mentioned in section 2.3 of Lecture 6.

The following notation is introduced:

- A_s is the total area of reinforcement located within the effective width b_{eff}^{-} ;
- h_s is the distance between the centroid of the reinforcement and the top of the upper flange of the steel section.

Case 1 – The plastic neutral axis is within the flange of the steel section

The design resistance F_s of the reinforcement is calculated as

$$F_s = A_s f_{sk} / \gamma_s \, . \tag{5}$$

The plastic neutral axis will be located in the flange of the steel section if both of the following conditions apply:

$$F_a > F_s \quad \text{and} \quad F_a - F_s \leq 2 b_f t_f f_y / \gamma_a \tag{6}$$

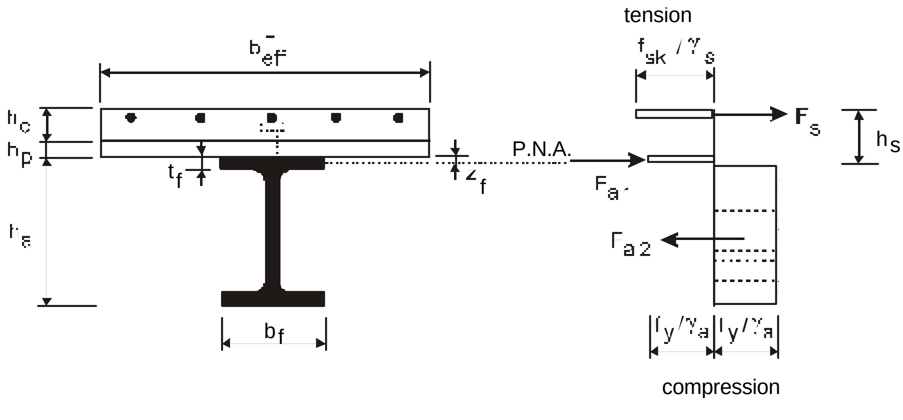


Figure 5. Plastic stress distribution: plastic neutral axis in the steel beam flange (hogging bending)

Similarly to case 2 of the sagging bending moment (see Lecture 6 "Simply supported beams"), the depth z_f of the flange of the steel section in tension is given by the following equilibrium equation (Figure 5):

$$F_a = F_s + 2b_f z_f f_y / \gamma_a \, , \tag{7}$$

and the design moment resistance with respect to the centroid of the reinforcement is:

$$M^-_{pl.Rd} = F_a(0{,}5h_a + h_s) - (F_a - F_s)(0{,}5z_f + h_s) \, . \tag{8}$$

Case 2 – The plastic neutral axis is within the web of the steel section

The plastic neutral axis will be located in the web of the steel section if both following conditions apply:

$$F_a > F_s \quad \text{and} \quad F_a - F_s > 2b_f t_f f_y / \gamma_a \, . \tag{9}$$

Similarly to case 3 of the sagging bending moment (see Lecture 6 "Simply supported beams"), the distance z_w between the plastic neutral axis and the centroid of the steel cross-section is given as (Figure 6):

$$z_w = \frac{\gamma_a \cdot F_s}{2t_w f_y} \, , \tag{10}$$

and the design moment resistance with respect to the centroid of the steel section is:

$$M^-_{pl.Rd} = M_{apl.Rd} + F_c(0{,}5h_a + 0{,}5h_c + h_p) - 0{,}5F_c z_w \, , \tag{11}$$

where $M_{apl.Rd}$ is the plastic moment resistance of the steel section alone.

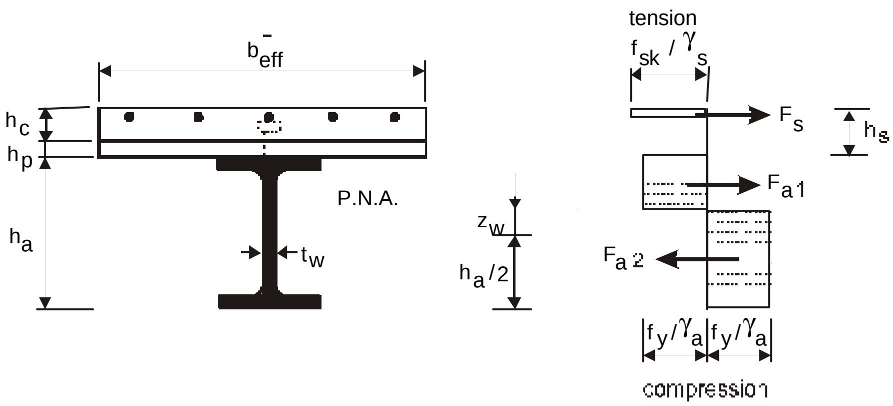


Figure 6. Plastic stress distribution: plastic neutral axis in the steel beam web (hogging bending)

For the cross-section classification, the depth of the web which is in compression may be calculated as αd where d is the depth of the web (for rolled sections, measured between the toes of the radii between the webs and the flanges), and

$$\alpha = 0{,}5 + \frac{z_w}{d} \quad \text{but} \quad \alpha \le 1 \, , \tag{12}$$

where z_w is as given above.

The expressions given for cases 1 and 2 are only applicable if the web slenderness d/t_w is such that the web can be classified as Class 1 or 2 (see Table 2). If the web is Class 3 and the compression flange is Class 1 or 2, then an effective web can be determined as discussed in section 2.3 above (Figure 4). This situation can be treated similarly to cases 1 or 2, but the resulting expressions will be more complex than those above.

3. Elastic design

3.1 Effective width of concrete flange

As discussed in Lecture 3 "Structural modelling", the in-plane shear flexibility (shear lag) of the concrete flange is taken into account through the introduction of the concept of effective widths. In general, the ratio of the effective width to the real flange width depends on many factors including the type of loading, the support conditions, the cross-section considered and the ratio of beam spacing to span. In Eurocode 4 however, very simple formulae are given for the effective width, related to the spans of the beam and expressed in terms of a length l_0 between points of contraflexure (see Lecture 3).

In the case of an end span, $l_0 = 0{,}8L$; for internal spans, $l_0 = 0{,}7L$ (Figure 7). Figure 5.1

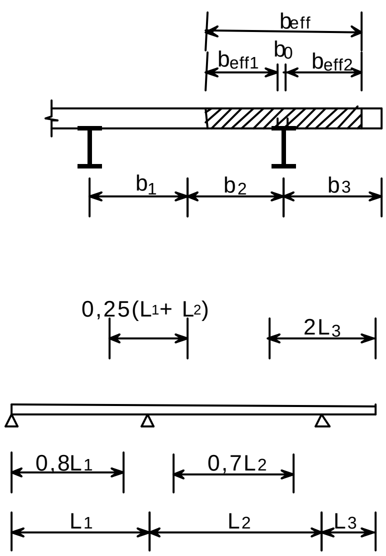


Figure 7. Effective width of concrete flange

Research on shear lag in negative moment regions indicates that when transverse reinforcement appropriate to the shear connector spacing is provided (i.e. they are spaced according to the principles of section 5 of Lecture 6 so as to provide sufficient resistance against the longitudinal shear force), the cracked slab is able to transfer shear to longitudinal reinforcement at a distance of several slab thicknesses on either side of the steel member. Over an internal support, Eurocode 4 gives $l_0 = 0{,}25(L_1 + L_2)$ where L_1 and L_2 are the spans adjacent to the support considered (Figure 7). Figure 5.1

For global analysis, however, it has been found that shear lag has little effect on the distribution of internal moments. Therefore a constant effective width may be assumed for the whole of each span, which greatly simplifies the analysis. As the greater part of the span of a beam is usually subject to sagging bending moment, it is appropriate that the constant effective width be taken as the value at mid-span. For a cantilever, however, the width should be that applicable at the support. 5.2.2.(1)

3.2 Elastic hogging bending resistance of cross-sections

The calculation of the elastic hogging bending resistance of a composite cross-section is more simple than that of the sagging bending resistance (see Lecture 6), because the concrete is considered cracked and only the steel section and the reinforcement will withstand the bending moment.

EC4 does not give explicit expressions for the moment resistance, but the following sections develop the analysis based on the above principles and the basic assumptions mentioned in section 2.4 of Lecture 6. 6.3.1.5.

Case 1 – Propped construction

The hogging bending moment resistance (elastic) of Class 3 cross-sections for propped composite beams is calculated according to the following formula (expressing that we limit the stress in either the steel section or the reinforcement to the yield stress):

$$M^{-}_{el.Rd} = \min\left(\frac{W_{c.ab.el}\, f_y}{\gamma_a};\frac{W_{c.ss.el}\, f_y}{\gamma_s}\right) \tag{13}$$

where

- $W_{c.ab.el}$ is the elastic modulus of the composite cross-section with respect to the bottom fibre of the steel section;
- $W_{c.ss.el}$ is the elastic modulus of the composite cross-section with respect to the reinforcement.

Case 2 – Unpropped construction

When calculating the hogging bending moment resistance (elastic) of a Class 3 cross-section in an unpropped composite beam, the first step is to calculate the stresses in the bottom and top fibres of the steel section respectively:

$$\sigma_{ab} = \frac{M_{a.Sd}}{W_{a.ab.el}} + \frac{M_{c.Sd}}{W_{c.ab.el}}\ ; \tag{14a}$$

$$\sigma_{at} = \frac{M_{a.Sd}}{W_{a.at.el}} + \frac{M_{c.Sd}}{W_{c.at.el}}\ . \tag{14b}$$

where

- $W_{a.ab.el}$ is the elastic modulus of the steel section alone with respect to its bottom fibre;
- $W_{c.ab.el}$ is the elastic modulus of the composite cross-section with respect to the bottom fibre of the steel section, calculated using a mean modular ratio;
- $W_{a.at.el}$ is the elastic modulus of the steel section alone with respect to its top fibre;
- $W_{c.at.el}$ is the elastic modulus of the composite cross-section with respect to the top fibre of the steel section, calculated using a mean modular ratio.

Then the stress utilisation ratio of the steel section is determined from:

$$r_a = \max\left(\left|\frac{\gamma_a \sigma_{ab}}{f_y}\right|;\left|\frac{\gamma_a \sigma_{at}}{f_y}\right|\right) \quad \text{with} \quad r_a \le 1\ . \tag{15}$$

Then the stress and the stress utilisation ratio of the top reinforcement is calculated:

$$\sigma_{ss} = \frac{M_{c.Sd}}{W_{c.ss.el}}\ ; \tag{16}$$

$$r_s = \left| \frac{\gamma_s \sigma_{ss}}{f_{sk}} \right| \quad \text{with} \quad r_s \leq 1. \tag{17}$$

where $W_{\mathrm{c.ss.el}}$ is the elastic modulus of the composite cross-section with respect to the reinforcement.

Finally, the negative moment resistance is determined from the following expression:

$$M^{-}_{el.Rd} = \min\left(\frac{M_{a.Sd} + M_{c.Sd}}{r_a}; M_{a.Sd} + \frac{M_{c.Sd}}{r_s} \right). \tag{18}$$

3.3 Distribution and redistribution of bending moments

Loss of stiffness due to cracking of concrete in negative moment regions has more effect on the distribution of bending moment in continuous composite beams than in continuous reinforced concrete members. This is because in the latter loss of stiffness also occurs due to cracking in the mid-span regions. It has been found that in continuous composite beams the bending moment at an internal support at the serviceability limit state may be 15 to 30% lower than that given by an elastic analysis in which no account is taken of cracking. At the ultimate limit state yielding of steel will also influence the distribution of moments.

The redistribution of moments cannot be predicted accurately because the longitudinal tensile stress in the concrete slab, in negative moment regions, is influenced by the sequence of casting and the effects of temperature and shrinkage, as well as by the proportions of the composite member and the dead and imposed loading. A wide variation in flexural rigidity can occur along a composite beam of uniform cross-section, leading to uncertainty in the distribution of bending moments and hence the amount of cracking to be expected.

Eurocode 4 for the ultimate limit state permits two methods of elastic global analysis: the *cracked section method* and the *uncracked section method*. Both may be used in conjunction with redistribution of support moments, the degree of redistribution being dependent on the susceptibility of the steel section to local buckling. 5.1.4.3. 5.2.3.

Design codes commonly permit negative (hogging) moments at supports to be reduced, except at cantilevers, by redistribution to mid-span. The extent of the redistribution is dependent, in part, on the method of analysis, as shown in Table 3. This table also shows that the degree of redistribution depends on the classification of the cross-section at the supports. 5.1.4.7. Table 5.2

Consider first a Class 4 section, i.e. one in which local buckling may prevent the design resistance from being attained. If redistribution is less than the designer assumes, the steel web or the compression flange at the support may buckle prematurely. For safety therefore, the maximum amount of redistribution to mid-span must be no greater than the minimum redistribution likely to occur in practice. Redistribution is therefore not permitted if the “cracked section method” of analysis has been used.

Studies on composite beams with critical sections in Class 3 or Class 4 have shown that provided at least 10% of the span is cracked, as is likely in practice, the reduction in support moment due to cracking will exceed 8%. It is reasonable to assume therefore that in round terms the difference between an “uncracked” and a “cracked” analysis with such beams is equivalent to 10% redistribution of the “uncracked” support moments, as shown in Table 3 for Class 3 and Class 4 sections.

There is no need to be so cautious for Class 3 sections as these can reach the design resistance, with local buckling only preventing the development of the full plastic moment. Numerical analysis, using experimental data on the falling branch of moment-rotation relationships for locally-buckling Class 3 cantilevers, confirms that up to 20% redistribution can be allowed, as given in Table 3.

In a Class 2 section the full plastic moment resistance can be developed. It has been proposed that a redistribution of 30% be permitted from an "uncracked" analysis to allow for local yielding at the supports and cracking of concrete. Comparisons with test results made during the assessment of Eurocode 4 confirm the latter figure as appropriate for sections which can attain the plastic resistance moment at the supports.

A beam with Class 2 (or Class 1) sections at supports will typically have a relatively low neutral axis, in order to meet the restrictions on the depth of the web in compression required in such sections. Hence only light tensile reinforcement can be provided and the ratio of "uncracked" to "cracked" flexural stiffness (I_1/I_2) can exceed 3.0. For such beams, the bending moment at the internal support from "cracked" analysis may then be less than 70% of the value from "uncracked" analysis and is almost always less than 85% of the "uncracked" value. This contrasts with the studies referred to above and summarised in Figure 2, for which the ratio I_1/I_2 was nearer 2 than 3. It follows that for Class 2 and Class 1 sections a 15% difference between "uncracked" and "cracked" analysis is more appropriate than the 10% difference adopted for beams with sections in Class 3 or Class 4. A 15% difference is given in Table 3 for Class 2 and Class 1 sections.

Finally, a Class 1 section is one which cannot only attain the plastic resistance moment but also sustain this level of moment whilst rotation occurs. In steel structures, the limits on flange and web slenderness which define a 'plastic' section are sufficiently restrictive to permit plastic global analysis without further checks on rotation capacity. This is not true for composite beams, partly because the degree of redistribution needed to attain a plastic hinge mechanism will be higher due to the greater relative moment resistance at mid-span. The conditions required for plastic global analysis are discussed in Section 2.1.1 of Lecture 5 "Shear connectors and structural analysis". The redistribution of elastic support moments permitted in Table 3 for Class 1 sections is based on the recognition that some rotation capacity exists for such sections.

Class of cross section in hogging moment region	1	2	3	4
For "uncracked" elastic analysis	40	30	20	10
For "cracked" elastic analysis	25	15	10	0

Table 3. Limits to redistribution of moments, per cent of the initial value of the bending moment to be reduced

4. Shear resistance in continuous beams

At internal supports of continuous composite beams, the cross-sections are subject to combined bending (M_{Sd}) and vertical shear (V_{Sd}). Experience shows that there is no significant reduction in the bending moment resistance M^{-}_{Rd} due to shear as long as the design vertical shear force V_{Sd} does not exceed half of the shear resistance V_{Rd}. If, however, the design vertical shear force exceeds this limit, allowance should be made for its effect on the design moment resistance. 6.3.2.4 (1)

If shear resistance is not limited by buckling, then the interaction between vertical shear and bending is expressed by the curve given in Figure 8. It is observed in the figure that

- where shear is low the moment capacity is not reduced;
- where the web is fully utilised in resisting shear (section CB), its contribution to resisting moment is deducted from the composite cross-section;
- in between these extremes (section AB) the following equation describes the interaction: 6.3.2.4.(2)

$$M^{-}_{v.Rd} = M^{-}_{f.Rd} + (M^{-}_{Rd} - M^{-}_{f.Rd}) \cdot \left[1 - \left(\frac{2V_{Sd}}{V_{pl.Rd}} - 1 \right)^2 \right]. \tag{19}$$

In this equation $M^{-}_{f.Rd}$ is the design bending resistance of a cross-section consisting of the flanges only (including the flanges of the steel beam and the slab reinforcement).

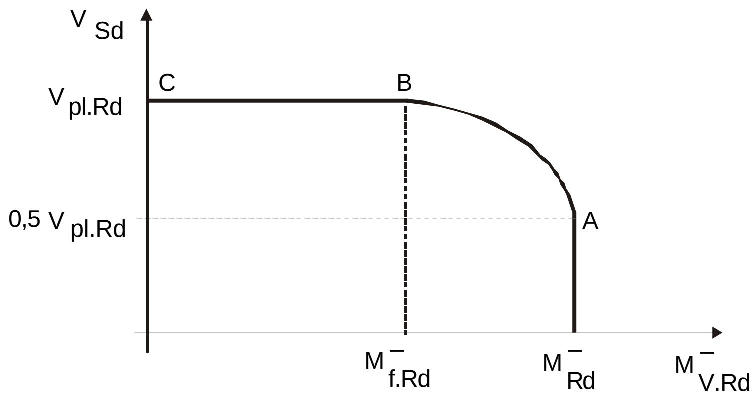


Figure 8. Interaction between vertical shear and bending moment if shear buckling is not relevant

5. Lateral-torsional buckling

5.1. Introduction

A steel flange attached to a concrete or composite slab by shear connection, may be assumed to be laterally stable, provided that the connection is appropriate and the overall width of the slab is not less than the depth of the steel section.

In the design situation of construction, it is normally assumed that decking prevents any lateral-torsional buckling, thus the beam can be considered fully restrained during and after concreting. Lateral-torsional buckling can, however, occur before fixing the deck, but this condition is normally not decisive. Lateral-torsional buckling may also be relevant at the construction stage when there is no decking applied.

In hogging moment regions of continuous composite beams, it is the bottom flange that is compressed. The extent of the hogging moment region at the internal supports depends on the conditions of loading. This region may be rather large when the variable loads act only on one of the spans (Figure 9). Thus there is a risk of lateral-torsional buckling of the lower flange at internal supports, during which the rigid-body rotation of the steel section around its centre of torsion is restricted, and the lateral support provided by the slab may lead to a beam distortion.

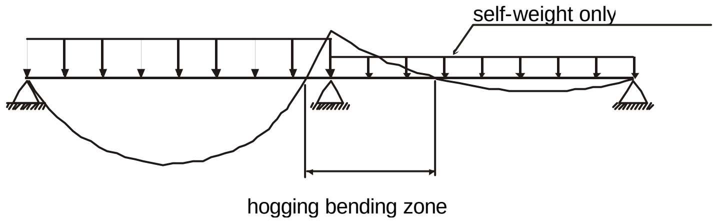


Figure 9. Loading that increase the extent of the hogging bending region

When calculating the beam buckling resistance, account may be taken of the torsional rigidity as it contributes to the reduction of the lateral-torsional buckling length, and thus the lateral-torsional slenderness. Furthermore, the lateral deflection of the lower flange consists of a half-wave on both sides of the internal supports as these supports are laterally restrained. These half-waves extend further than the respective hogging moment regions, and do not present a classical sine shape. The maximum lateral deflection occurs at a distance approximately equal to two to three times the beam depth.

The phenomenon of lateral-torsional buckling, and the theory describing it, is therefore fairly complex [4-8]. For this reason, only certain basic considerations and minimum detailing requirements are provided here. This minimum detailing is normally sufficient for many of the building frames.

5.2 Check without direct calculation

For Class 1 and 2 cross-sections, the lateral-torsional buckling slenderness ratio over supports $\bar{\lambda}_{LT}$ is defined as follows:

$$\bar{\lambda}_{LT} = \sqrt{\frac{M_{pl}^{-}}{M_{cr}^{-}}}\,, \qquad (20)$$ 6.5.3.(3)

where M_{cr}^{-} is the elastic critical (equilibrium bifurcation) moment over the support for lateral-torsional buckling, and M_{pl}^{-} is the value $M_{pl.Rd}^{-}$ calculated with partial safety factors equal to unity.

In the case of Class 3 or 4 cross-sections, the lateral-torsional buckling slenderness ratio is determined on the basis of the elastic moment resistance.

The lateral-torsional buckling resistance is given by: 6.5.3.(1)

$$M_{b.Rd}^{-} = \chi_{LT} M_{Rd}^{-}\,, \qquad (21)$$

where with γ_{Rd} being the partial safety factor for stability in EC3 (γ_{M1}),

- for Class 1 and 2 cross-sections, $M_{Rd}^{-} = M_{pl.Rd}^{-} \cdot \gamma_a / \gamma_{Rd}$;
- for Class 3 cross-sections, $M_{Rd}^{-} = M_{el.Rd}^{-} \cdot \gamma_a / \gamma_{Rd}$;
- for Class 4 cross-sections, $M_{Rd}^{-} = M_{el.Rd}^{-}$,

and where the lateral-torsional buckling reduction factor χ_{LT} is determined from the following formulae:

EC3: 6.3.2.2

$$\chi_{LT} = \frac{1}{\phi_{LT} + \sqrt{\phi_{LT}^2 - \bar{\lambda}_{LT}}} \le 1 \qquad (21a)$$

$$\phi_{LT} = 0,5\left[1 + \alpha_{LT}(\bar{\lambda}_{LT} - 0,2) + \bar{\lambda}_{LT}^2\right] \qquad (21b)$$

with $\alpha_{LT} = 0,21$ for rolled sections (buckling curve “a” of EC3), and $\alpha_{LT} = 0,49$ for welded sections (buckling curve “c” of EC3). (Note: these buckling curves are from the ENV version of EC3. The formulae to be used in the EN version have not yet been finalised.)

For the calculation of the elastic critical moment M_{cr}^{-}, EC4 provides a rather complex approach called “inverted U-frame model”. If the beam does not meet the requirements of this model, then to determine the value of M_{cr}^{-}, EC4 suggests to use specialised literature [7, 8], numerical analysis or the elastic critical moment of the steel section alone. 6.5.3.(4)

No allowance for lateral-torsional buckling is necessary if the lateral-torsional buckling slenderness ratio $\bar{\lambda}_{LT}$ is below 0,4.

The check of lateral-torsional buckling may also be omitted if certain conditions apply. These conditions are related to the following aspects of the structure: 6.5.2.(1)

- the regularity of adjacent span lengths;
- the loading of the spans and the share of permanent loads;
- the shear connection between top flange and concrete slab;
- the neighbouring member supporting the slab;
- the lateral restraints and web stiffeners of the steel member at its supports;
- the cross-sectional dimensions of the steel member;
- the depth of the steel member (Table 4).

Member	Steel grade S235	Steel grade S275	Steel grade S355	Steel grade S420 or S460
Not partially encased IPE or similar	600	550	400	270
Not partially encased HE or similar	800	700	650	500
Partially encased IPE or similar	750	700	550	420
Partially encased HE or similar	950	850	800	650

Table 4. Maximum depth h_a (mm) of steel member to avoid lateral-torsional buckling checks

6. Shear connection in continuous beams in Class 1 or 2 cross-sections

The design of the shear connection in the case of continuous beams is more complex than for simply supported beams because of the hogging moment regions that occur at the internal supports. The cross-sections over these supports may be critical for design and thus they need to be checked. These checks should take into account the loss of rigidity due to cracking of the slab and the position of the neutral axis being higher in the steel web due to the reinforcement of the slab. This may lead to a change in the classification of the steel cross-section. In these conditions, the application of plastic analysis is more limited. As for continuous beams, sufficient ductility is necessary for plastic moment redistribution. Only the case of a ductile connection will be discussed below.

Three remarks must be made.

- Even if an elastic analysis is used for a continuous beam (possibly with the redistribution of moments over the supports), the plastic calculation of the connection may be considered as long as cross-sections located at ends of the critical lengths are at least in Class 2.
- In hogging bending regions, partial shear connection design is normally difficult because of the required rotations of sections at internal supports, even when these cross-sections are in Class 1. As a result, partial shear connection is not allowed in hogging bending regions to eliminate the possibility of the local buckling of steel elements.
- In sagging bending regions, even if the cross-sections at the internal supports are in Class 2, 3 or 4, partial shear connection is generally sufficient as the design maximum sagging moment may be lower than the plastic moment resistance. In this case elastic global analysis is necessary.

6.1 Beams in Class 1 cross-sections

Consider the continuous beam shown in Figure 10. Design may be according to either the full or the partial shear connection method. As cross-sections are in Class 1, a complete plastic mechanism will occur at failure. At each plastic hinge, the bending moment is equal to the plastic moment resistance of the cross-section. The ultimate load is calculated as follows:

$$Q = \frac{M_u^{(red)} L + M'_u d}{d(L-d)} \tag{22}$$

where $M_u^{(red)}$ and M'_u are the positive and negative moment resistance respectively.

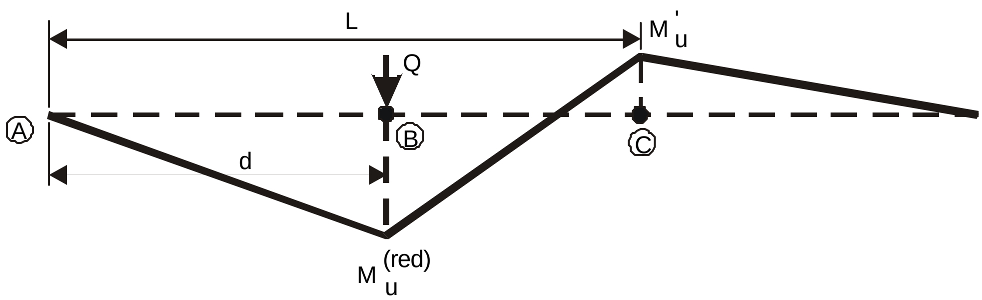


Figure 10. Bending diagram of a continuous beam with concentrated load

The value of $M_u^{(red)}$ depends on the shear connection. Let us fix the number, $N^{(BC)}$, of ductile connectors uniformly distributed over the critical length BC. From the equilibrium of the slab (Figure 11), one can write:

$$V_l^{(BC)} = N^{(BC)} P_{Rd} = F_u^{(red)} + F_s \,, \tag{23}$$

where $V_l^{(BC)}$ is the longitudinal shear force in the critical length considered, $F_u^{(red)}$ is the compression force in the slab over internal support B, and $F_s = A_s f_{sk} / \gamma_s$.

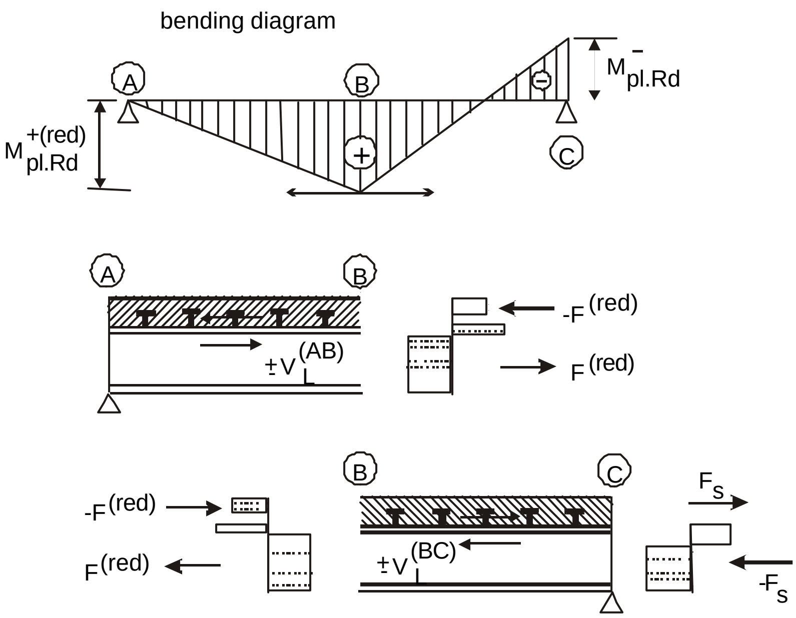


Figure 11. Equilibrium of the slab

This relation provides the value $F_u^{(red)}$ as $F_u^{(red)} = N^{(BC)} P_{Rd} - F_s$, and also, the positive reduced moment resistance $M_u^{(red)}$. Considering now the critical length AB where the bending is positive, the required number $N^{(AB)}$ of connectors comes from the equation

$$V_l^{(AB)} = N^{(AB)} P_{Rd} = F_u^{(red)} \,. \tag{24}$$

Thus the total number of connectors, N, will be:

$$N = N^{(AB)} + N^{(BC)} = 2N^{(BC)} - F_S / P_{Rd} \,, \tag{25}$$

and the ultimate load of the beam may be calculated as a function of the total number N of connectors over the span where the plastic mechanism occurs.

For full shear connection, the number of connectors on BC, $N_f^{(BC)}$, is given by the following equation:

$$N_f^{(BC)} = \frac{1}{P_{Rd}} \cdot \left[\min\left(\frac{A_a f_y}{\gamma_a} ; \frac{0{,}85 b_{eff} h_c f_{ck}}{\gamma_c} \right) + \frac{A_s f_{sk}}{\gamma_s} \right] . \tag{26}$$

The relation between Q and N (Figure 12) is similar to the variation of the moment resistance with the degree of shear connection for simply supported beams. As for simply supported beams, the entire curve cannot be used. A minimum degree of shear connection should always be considered. The previous formulae used for simply supported beams may be used to evaluate this minimum degree. These formulae are conservative as the partial connection only concerns the sagging bending region whose length is smaller than span *L*.

The curve of $M_u^{(red)}$ depending on *N* is also convex. A simplified method may be generally considered to evaluate the load *Q* as a function of the degree of shear connection, of the ultimate load of the steel beam alone and the ultimate load Q_u. We have then:

$$Q = Q_{apl} + \frac{N}{N_f}(Q_u - Q_{apl}) \,. \tag{27}$$

As already seen, *Q* may be calculated by fixing the number and the distribution of the connectors. Alternatively, *Q* being fixed, the number of connectors N may be calculated.

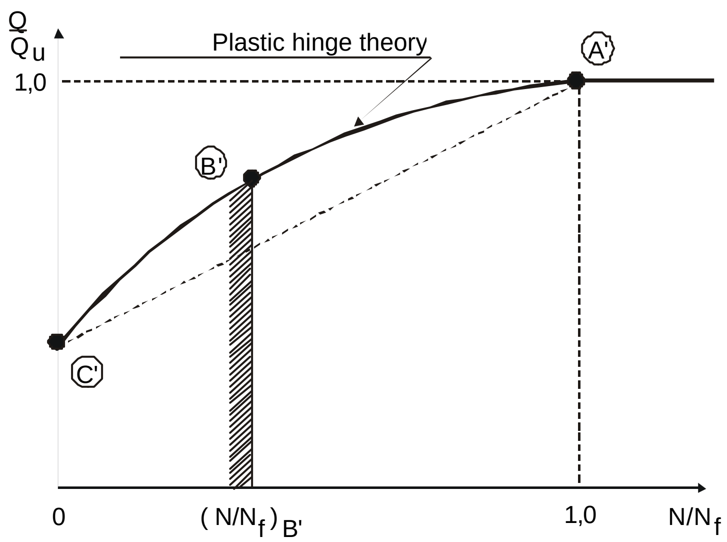


Figure 12. Ultimate load depending on the degree of shear connection

The calculations presented relate to a simple case. Similar principles may be used for more complex cases. The relation used to calculate the ultimate load will be different and the plastic mechanism defining the critical lengths less evident.

6.2 Beams in Class 2 cross-sections

An elastic analysis with redistribution of bending moments should be used for beams in Class 2 cross-sections. The ultimate bending should only be obtained along the span and partial connection should be used necessarily. The curve of *Q* in the function of *N* differs from the one obtained for class 1 cross-sections which results from plastic analysis, but generally, a line similar to *A'C'* may be used, that line being conservative. The degree of shear connection is also given by the relation

$$\frac{N}{N_f} = \frac{Q_d - Q_{apl}}{Q_u - Q_{apl}} \tag{28}$$

where Q_u and Q_{apl} are calculated using global elastic analysis.

7. The serviceability limit state of cracking of concrete

A special type of serviceability limit state, relevant for continuous beams, is the cracking of concrete, which must be checked along with other serviceability limit states of deflection and vibration discussed in Lecture 3 "Structural modelling".

The limit state of cracking of concrete may also be relevant for simply supported beams because during hardening of concrete shrinkage may cause the appearance of cracks even when the concrete is in compression. This problem and the related provisions are discussed in Lecture 6 "Simply supported beams".

In the hogging moment regions of continuous composite beams, cracking occurs due to tension in the concrete. To limit this cracking, it is necessary to meet the requirements of minimum reinforcement as described in Lecture 6. The limitation of crack width may be achieved by limiting bar spacing or bar diameters according to Table 5. 7.3.3

In Table 5, the maximum bar spacing values depend on the stress in the reinforcement σ_s and the design crack width w_s. This stress is determined from the quasi-permanent combination of actions, by elastic analysis, taking into account of the cracks of the concrete ("cracked section method") and the tension stiffening of concrete between cracks. Table 7.2

Unless calculated by a more precise method, this stress σ_s can be calculated by adding a term $\Delta\sigma_s$ to the stress in the reinforcement σ_s calculated neglecting the concrete in tension. This term $\Delta\sigma_s$ accounting for tension stiffening is calculated as

$$\Delta\sigma_s = \frac{0.4 f_{ctm}}{\alpha_{st}\rho_s} \tag{29}$$

where:

- f_{ctm} is the mean tensile strength of concrete;
- ρ_s is the "reinforcement ratio" expressed as $\alpha_{st} = A_s / A_{ct}$
- A_{ct} is the area of concrete flange in tension within the effective width
- A_s is the total area of reinforcement within the area A_{ct}
- α_{st} is the ratio $\dfrac{AI}{A_a I_a}$ where A and I are the area and second moment of area of the composite section neglecting concrete in tension and any sheeting, and A_a and I_a are the same properties for the bare steel section.

Table 5. Maximum bar spacing (high bond bars)

stress in reinforcement σ_s, N/mm^2	maximum bar spacing for w_k = 0,4 mm	maximum bar spacing for w_k = 0,3 mm	maximum bar spacing for w_k = 0,2 mm
160	300	300	200
200	300	250	150
240	250	200	100
280	250	150	50
320	150	100	–
360	100	50	–

8. Concluding summary

- Continuous beams are an alternative to simply supported beams and their use is justified by considerations of economy.

- In the hogging bending regions at supports, concrete will be in tension and the steel bottom flange in compression, leading to the possibility of onset of local buckling. This is taken up by the classification of cross sections.
- Rigid-plastic design may be performed for beams with Class 1 cross-sections. Plastic moment resistance of cross-sections can be used for Class 1 and 2 cross-sections.
- For Class 3 sections, elastic analysis and elastic cross-section resistance must be used
- The principles of calculation of cross-section resistance, either plastic or elastic, are similar to the case of sagging bending. The tension resistance of concrete is neglected.
- Lateral-torsional buckling is a special phenomenon which can be prevented by conforming to certain detailing rules.
- The design of the shear connection in the case of continuous beams is more complex than for simply supported beams.
- Serviceability checks include deflection and vibration control as well as that of concrete cracking. This latter is specific to continuous beams because tension in concrete at the hogging moment regions may cause unacceptable cracks, while in simply supported beams cracking is only due to shrinkage of concrete and is therefore lower in magnitude.

Structural Steelwork Eurocodes
Development of a Trans-National Approach

Course: Eurocode 4

Lecture 8: Composite Columns

Summary:

- Composite columns may take the form of open sections partially or fully encased in concrete, or concrete-filled hollow steel sections.
- Both types require longitudinal reinforcing bars, and shear studs may be used to provide additional bond between steel and concrete.
- The confinement provided by a closed steel section allows higher strengths to be attained by the concrete. Circular concrete-filled tubes develop hoop-tension which further increases the overall load-carrying capacity of the concrete.
- Complete encasement of a steel section usually provides enough fire protection to satisfy the most stringent practical requirements.
- *Eurocode 4* provides two methods for calculation of the resistance of composite columns; The General Method and the Simplified Method. The Simplified Method is covered in this article.
- Fully encased sections do not require any check on the thickness of the walls of the steel section. Other types of composite column are subject to a minimum thickness requirement.
- Loads from beams must be transmitted within a limited development length to both the steel and concrete parts of the column section. In some cases this may require the use of shear studs.
- In order to use the Simplified Method of design the column cross-section must be constant and doubly symmetric over its whole height.
- The bending stiffness of a column used in the calculation of its critical elastic resistance includes the steel section, reinforcement and concrete, which are assumed to interact perfectly. For short-term loading the concrete modulus is reduced by 20%, and for long-term loading it is further reduced to account for the effects of creep.
- The reduction of buckling resistance due to imperfections may be taken from the EC3 buckling curves. Equivalent imperfections are provided, which can be used directly as an eccentricity of the axial force in calculating the design moment.
- It is permissible to construct simply a linearised version of the interaction diagram for cross-sectional resistance to combinations of axial compression and uniaxial bending moment on a composite section.
- Second-order "*P-d*" effects can be taken into consideration approximately by applying an amplification factor to the maximum first-order bending moment.
- It can be assumed that transverse shear force is completely resisted by the steel section.
- The resistance of a column under axial compression and uniaxial bending may be found using the interaction diagram for cross-section resistance, subject to a reduction factor, provided that the design moment includes the effect of member imperfection and amplification by second-order effects.
- The resistance of a column under axial compression and biaxial bending is calculated using its uniaxial resistance about both axes, plus an interaction check between the moments at the specified axial force.

Pre-requisites:

- Stress distributions in cross-sections composed of different materials and under combinations of axial force and bending moments about the principal axes.
- Euler buckling theory for columns in compression.
- The reduction of perfect elastic buckling resistance due to the presence of initial imperfections in columns.
- The difference between first-order and second-order bending moments in members which are subject to *"P-d"* effects.
- Framing systems currently used in steel-framed construction, including composite systems.

Notes for Tutors:

- This material comprises one 60-minute lecture.
- The lecturer can break up the session with formative exercises at appropriate stages.
- The lecturer may apply a summative assessment at the end of the session requiring that students consider the consequences of adopting composite columns rather than steel H-section columns at the design stage for part of an example building.

Objectives:

The student should:

- be aware of the types of composite columns which may be used in building structures.
- appreciate the main characteristics of concrete-filled hollow sections and concrete-encased open sections when used as columns.
- understand how beam reaction forces must be transmitted through a predetermined load path into both the steel and concrete parts of a column section close to a beam-column connection.
- understand the principles of the Eurocode 4 "Simplified Method" of design for composite columns.
- know that buckling resistance under axial load is given by the normal EC3 column buckling curves.
- know how to construct the simple linearised version of the interaction diagram for cross-sectional resistance to combinations of axial compression and uniaxial bending moment on a composite section.
- understand that, when designing for axial force and bending, the design moment must take account of *"P-d"* effects, either by performing a second-order analysis or by applying an amplification factor to the maximum first-order bending moment.
- know the Eurocode 4 process for designing a column to resist axial compression and biaxial bending.

References:

- **ENV 1991-1: 1996** *Eurocode 1: Basis of Design and Actions on Structures. Part 1: Basis of Design.*
- **ENV 1992-1-1: 1991** *Eurocode 2: Design of Concrete Structures. Part 1.1: General Rules and Rules for Buildings.*
- **ENV 1993-1-1: 1992** *Eurocode 3: Design of Steel Structures. Part 1.1: General Rules and Rules for Buildings.*
- **Draft prEN 1994-1-1: 2001** *Eurocode 4: Design of Composite Steel and Concrete Structures. Part 1.1: General Rules and Rules for Buildings. (Draft 2)*
- **D J Oehlers and M A Bradford,** *Composite Steel and Concrete Structural Members – Fundamental Behaviour*, Pergamon, Oxford, 1995.

Contents:

1 Introduction

Composite columns may be classified into two principal types:

- Open sections partially or fully encased in concrete,
- Concrete-filled hollow steel sections.

Figure 1 shows different types of composite columns, and defines symbols used in this lecture.

Partially encased columns (Figs. 1b and 1c) are based on steel I- or H-sections, with the void between the flanges filled with concrete. In **fully encased columns** (Figure 1a) the whole of the steel section is embedded within a minimum cover-depth of concrete.

Concrete-filled hollow sections (Figs. 1d to 1f) may be circular or rectangular. The concrete fills the section, and its compressive strength is enhanced by its confinement. This is an additional advantage for the compression resistance of the column.

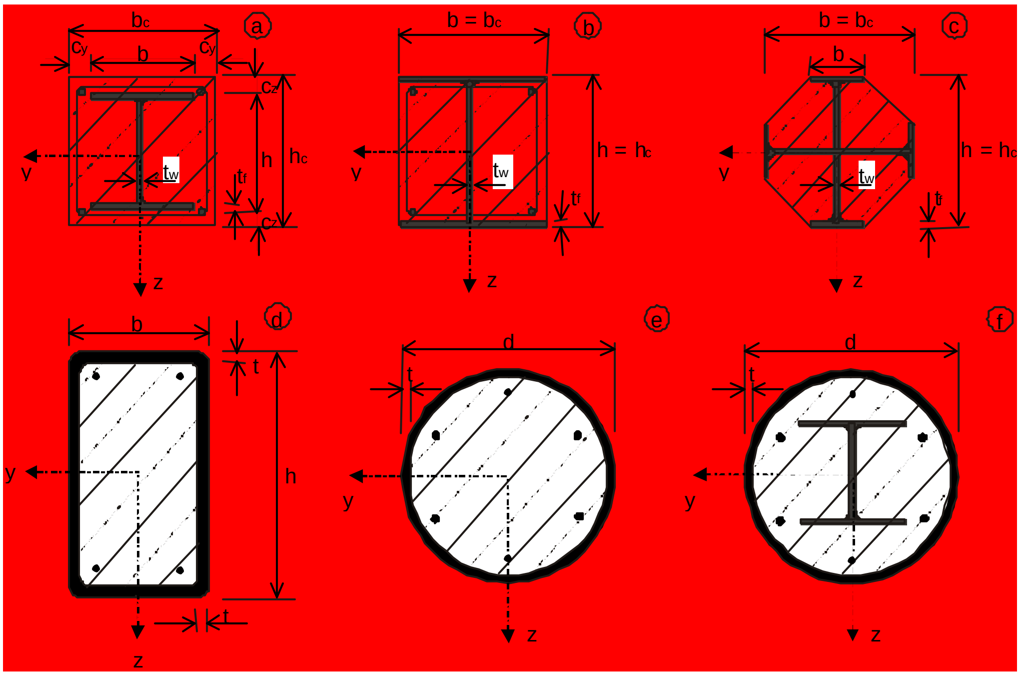


Figure 1 Typical cross-sections of composite columns

Composite columns can provide considerable advantages compared to open steel columns. For example, a cross-section of slender exterior dimensions can resist high axial loads. Different cross-sections of the same exterior dimensions can carry very different loads, depending on the thickness of the steel section, the strength of the concrete and the area of reinforcement used. It is possible to keep the same column dimensions over several storeys of a building, which provides both functional and architectural advantages.

In the case of concrete-filled hollow sections, the steel provides a permanent formwork to the concrete core. This allows, for example, the steel frame to be erected and the hollow column sections subsequently to be filled with pumped concrete. This leads to appreciable savings in the time and cost of erection. In addition, the confinement provided by the closed steel section allows higher strengths to be attained by the concrete. In the case of circular concrete-filled tubes the steel, in providing confinement to the concrete, develops a hoop-tension which increases the overall load-carrying capacity, although this is often ignored in design. Creep and shrinkage of concrete, are also generally neglected in the design of concrete-filled tubes, which is not the case for concrete-encased sections. On the other hand, complete encasement of a steel section usually provides enough fire protection to satisfy the most stringent requirements without resorting to other protection systems. For partially encased sections, and for concrete-filled hollow sections, codes of practice on fire resistance require additional reinforcement. Partially encased sections have the advantage of acting as permanent formwork; the concrete is placed in two stages with the section aligned horizontally, turning the member 24 hours after the first pour. In order to ensure adequate force transfer between the steel and concrete it is sometimes necessary to use stud connectors or reinforcement connected directly or indirectly to the metal profile. Another significant advantage of partially encased sections is the fact that, after concreting, some of the steel surfaces remain exposed and can be used for connection to other beams.

2 Calculation Methods

Eurocode 4 provides two methods for calculation of the resistance of composite columns. *EC4 Part 1.1*

The first is a **General Method** which takes explicit account of both second-order effects and imperfections. This method can in particular be applied to columns of asymmetric cross-section as well as to columns whose section varies with height. It requires the use of numerical computational tools, and can be considered only if suitable software is available. *6.8.2*

The second is a **Simplified Method** which makes use of the European buckling curves for steel columns, which implicitly take account of imperfections. This method is limited in application to composite columns of bisymmetric cross-section which does not vary with height. *6.8.3*

These two methods are both based on the following assumptions:

- There is full interaction between the steel and concrete sections until failure occurs;
- Geometric imperfections and residual stresses are taken into account in the calculation, although this is usually done by using an equivalent initial out-of-straightness, or member imperfection;
- Plane sections remain plane whilst the column deforms.

Only the **Simplified Method** will be considered further in this lecture, because it is applicable to the majority of practical cases.

3 Local buckling of steel elements

The presence of concrete firmly held in place prevents local buckling of the walls of completely encased steel sections, provided that the concrete cover thickness is adequate. This thickness should not be less than the larger of the two following values: *6.8.1(9)*

- 40 mm;
- One sixth of the width *b* of the flange of the steel section. *6.8.5.1(2)*

This cover, which is intended to prevent premature separation of the concrete, must be laterally reinforced, to protect the encasement against damage from accidental impact and to provide adequate restraint against buckling of the flanges. *6.8.4.2*

For partially encased sections and concrete-filled closed sections, the slendernesses of the elements of the steel section must satisfy the following conditions:

- $d/t \leq 90\, e^2$ (concrete-filled circular hollow sections of diameter *d* and wall thickness *t*);
- $d/t \leq 52\, e$ (concrete-filled rectangular hollow sections of wall depth *d* and thickness *t*); *Table 6.3*
- $b/t_f \leq 44\, e$ (partially encased H-sections of flange width *b* and thickness t_f);

in which $e = \sqrt{235/f_{y.k}}$, where $f_{y.k}$ is the characteristic yield strength of the steel section.

4 Force transfer between steel and concrete at beam-column connections

The forces transmitted from a beam through the beam-column connection must be distributed between the steel and concrete parts of the composite column. The nature of this force transfer from the steel to the concrete depends on the structural details and follows a load path which must be clearly identified. The introduction length *p*, necessary for full development of the compressive force in the concrete part of the column, is usually less than twice the appropriate transverse dimension *d* (see Figure 2) , and should not in any event exceed *2,5d*.

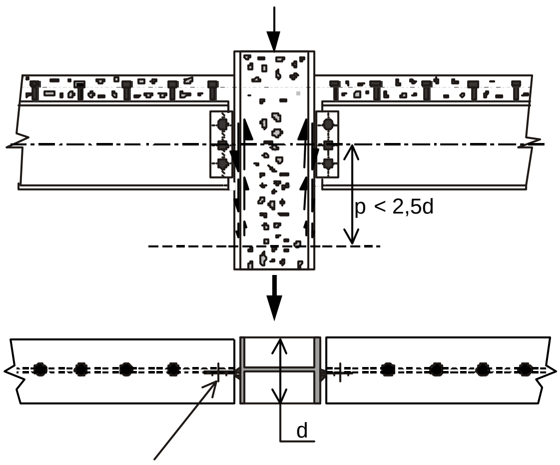


Figure 2 Force transfer in a composite beam-column connection

For the purposes of calculation it is recommended that the shear resistance at the interface between steel and concrete is not assumed to be greater than the following (indicative) values: *6.8.4.2*

Table 6.6

- 0,3 N/mm^2 for sections completely encased in concrete;
- 0,4 N/mm^2 for concrete-filled hollow sections;
- 0,2 N/mm^2 for the flanges of partially encased sections;
- 0 N/mm^2 for the webs of partially encased sections.

The detailed design of the beam-column connection has a considerable influence on this shear resistance, and the effects of hoop-stress, confinement and friction are intimately linked to the connection layout used. Figure 2 shows a typical beam-column connection, and defines the introduction length *p*. The force to be transferred within this length is not the total reaction force on the connection, but only the part which is to be transferred to the concrete of the composite column section. A part of the reaction force must always be carried by the concrete in order for the composite column to work properly.

In the particular case of a concrete-encased composite column for which the bond strength between steel and concrete is insufficient for the transfer to the concrete part to take place within the allowable length, it is possible to use shear connector studs welded to the web of the steel section. It is then possible to take account of the shear resistance P_{Rd} of the studs as an enhancement to the bond between the steel and concrete. This additional bond strength, acting only on the internal faces of the flanges, can be taken as $mP_{Rd}/2$ on each flange. The coefficient *m* can initially be taken as 0,50, although its real value depends on the degree of confinement of the concrete between the flanges of the section. This assumption is only valid if the distance between the flanges is less than the values in millimetres shown on Figure 3.

6.8.4.2(6)
6.7.4.1

6.7.16.2

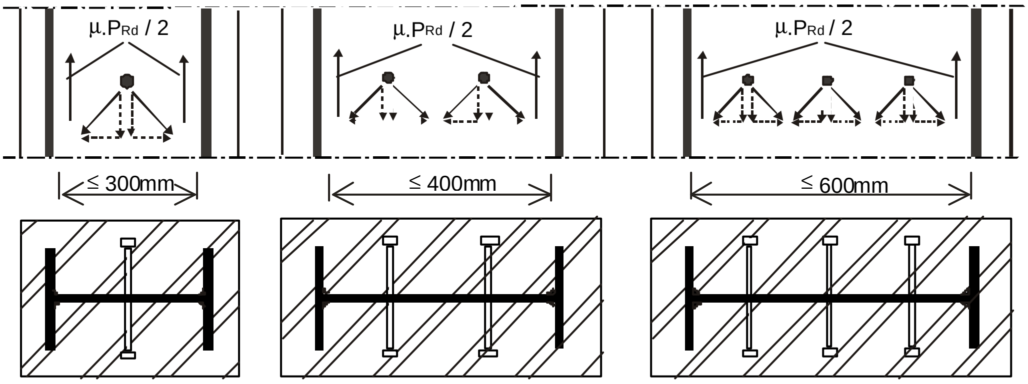


Figure 3 Use of studs to enhance force transfer in composite columns

5 Use of the simplified calculation method

The *Simplified Method* is subject to the following limitations: *6.8.3.1*

- The column cross-section must be prismatic and symmetric about both axes over its whole height, with its ratio of cross-sectional dimensions in the range 5,0 > h_c/b_c > 0,2.
- The relative contribution of the steel section to the design resistance of the composite section, given by $\delta = (A_a f_y / \gamma_a) / N_{pl.Rd}$, must be between 0,2 and 0,9;
- The relative slenderness $\bar{\lambda}$ of the composite column must be less than 2,0;
- For concrete-encased sections, the area of longitudinal reinforcement must be at least 0,3% of the concrete cross-section area, and the concrete cover must satisfy the following limits: *6.8.5.2*
  - In the y-direction: 40 mm ≤ c_y ≤ 0,4 b_c ;
  - In the z-direction: 40 mm ≤ c_z ≤ 0,3 h_c, where b_c and h_c are defined in Figure 1(a).

It is often necessary to specify concrete cover on the basis of a more significant criterion, for example to ensure sufficient fire resistance, but even in such cases it is expedient to be aware of how the cover thickness specified compares to the maximum values given above.

The cross-sectional area of the longitudinal reinforcement can only be included in the calculation of cross-sectional resistance if it is less than 6% of the area of the concrete. To ensure sufficient fire resistance it is sometimes necessary to use more reinforcement than this, but the area of reinforcement considered in the calculation of the resistance of the composite section is limited to 6% of the concrete area.

6 Composite columns subject to axial compression

6.1 Resistance of the cross-section

The cross-sectional resistance of a composite column to axial compression is the aggregate of the plastic compression resistances of each of its constituent elements as follows: *6.8.3.2*

For fully or partially concrete-encased steel sections :

$$N_{pl.Rd} = A_a \frac{f_y}{\gamma_{Ma}} + A_c .0{,}85 \frac{f_{ck}}{\gamma_c} + A_s \frac{f_{sk}}{\gamma_s} \qquad (1)$$ *(6.49)*

For concrete-filled hollow sections:

$$N_{pl.Rd} = A_a \frac{f_y}{g_{Ma}} + A_c \frac{f_{ck}}{g_c} + A_s \frac{f_{sk}}{g_s} \qquad (2)$$

(6.49)

in which A_a, A_c et A_s are respectively the cross-sectional areas of the steel profile, the concrete and the reinforcement. The increase of concrete resistance from $0{,}85f_{ck}$ to f_{ck} for concrete-filled hollow sections is due to the effect of confinement

For a concrete-filled circular hollow section, a further increase in concrete compressive resistance is caused by hoop stress in the steel section. This happens only if the hollow steel profile is sufficiently rigid to prevent most of the lateral expansion of the concrete under axial compression. This enhanced concrete strength may be used in design when the relative slenderness $\bar{l}$ of a composite column composed of a concrete-filled circular tube does not exceed 0,5 and the greatest bending moment $M_{max.Sd}$ (calculated using first-order theory) does not exceed $0{,}1N_{Sd}d$, where d is the external diameter of the column and N_{Sd} the applied design compressive force. The plastic compression resistance of a concrete-filled circular section can then be calculated as: 6.8.3.2 (5)

$$N_{pl.Rd} = A_a h_a \frac{f_y}{g_{Ma}} + A_c \frac{f_{ck}}{g_c}\left[1 + h_c \frac{t}{d}\frac{f_y}{f_{ck}}\right] + A_s \frac{f_{sk}}{g_s} \qquad (3)$$

(6.52)

in which t represents the wall thickness of the steel tube. The coefficients h_a and h_c are defined as follows for $0 < e \le d/10$, where $e = M_{max.Sd} / N_{Sd}$ is the effective eccentricity of the axial compressive force:

$$h_a = h_{a0} + (1 - h_{a0})\left(10\frac{e}{d}\right) \qquad (4)$$

(6.56**)

$$h_c = h_{c0}(1 - 10\frac{e}{d}) \qquad (5)$$

(6.55**)

When $e > d/10$ it is necessary to use $h_a = 1{,}0$ and $h_c = 0$. In equations (4) and (5) above the terms h_{a0} and h_{c0} are the values of h_a and h_c for zero eccentricity e. They are expressed as functions of the relative slenderness $\bar{l}$ as follows:

$$h_{a0} = 0{,}25(3 + 2\bar{l}\,) \qquad \le 1 \qquad (6)$$

(6.54)

$$h_{c0} = 4{,}9 - 18{,}5\bar{l} + 17\bar{l}^2 \qquad \ge 0 \qquad (7)$$

(6.53)

The presence of a bending moment M_{Sd} has the effect of reducing the average compressive stress in the column at failure, thus reducing the favourable effect of hoop compression on its resistance. The limits imposed on the values of h_a and h_c, and on h_{a0} and h_{c0}, represent the effects of eccentricity and slenderness respectively on the load-carrying capacity.

The increase in strength due to hoop stress cannot be utilised for a rectangular hollow section because its plane faces deform when the concrete expands.

6.2 Relative slenderness of a composite column

The elastic critical load N_{cr} of a composite column is calculated using the usual Euler buckling equation

$$N_{cr} = \frac{p^2 (EI)_{eff.k}}{L_{fl}^2} \qquad (8)$$

in which $(EI)_{eff.k}$ is the bending stiffness of the composite section about the buckling axis considered, and L_{fl} is the buckling length of the column. If the column forms part of a rigid frame this buckling length can conservatively be taken equal to the system length L.

For short-term loading the effective elastic bending stiffness $(El)_{eff.k}$ of the composite section is given by:

$$(EI)_{eff.k} = E_a I_a + K_e E_{cm} I_c + E_s I_s \qquad (9)$$ *(6.59)*

in which : *6.8.3.3*

I_a, I_c and I_s are the respective second moments of area, for the bending plane considered, of the steel section, the uncracked concrete section and the reinforcement;

E_a and E_s are the respective elastic moduli of the steel of the structural section and of the reinforcement;

E_{cm} is the elastic secant modulus of the concrete;

K_e a correction factor for cracking of concrete, which may be taken as 0,6.

For long-term loading the bending stiffness of the concrete is determined by replacing the elastic modulus E_{cd} with a lower value E_c which allows for the effect of creep and is calculated as follows:

$$E_c = E_{cm} \frac{1}{1 + \frac{N_{G.Sd}}{N_{Sd}} j_t} \qquad (10)$$ *(6.60)*

where $N_{G.Sd}$ is the permanent part of the axial design loading N_{Sd}. The term j_t is a creep coefficient defined in Eurocode 2, which depends on the age of the concrete at loading and at the time considered; for a practical building column it should normally be sufficient to consider the column at "infinite" time. This modification of the concrete modulus is only necessary if : *EC2 Part 1.1 3.1.3*

- the relative slenderness $\bar{l}$, for the plane of bending considered, is greater than 0,8 for concrete-encased sections or *0,8/(1-d)* for concrete-filled hollow sections, where $d = \frac{A_a f_y}{g_{Ma} N_{pl.Rd}}$ is the relative contribution of the steel section to the overall axial plastic resistance. It should be noted that the calculation of $\bar{l}$ requires knowledge of an initial value of the elastic modulus E_c of concrete. For checks against the limits given above it is permissible to calculate $\bar{l}$ without considering the influence of long-term loads. *EC4 Part 1.1 6.8.3.3 (6.57)*
- the relative eccentricity *e/d* (*d* being the depth of the section in the plane of bending considered) is less than 2.

These limiting values apply in the case of braced non-sway frames. They are replaced respectively by *0,5* and *0,5/(1-d)* in the case of sway frames or unbraced frames.

The relative slenderness $\bar{l}$ of a composite column in the plane of bending considered is given by

$$\bar{l} = \sqrt{\frac{N_{pl.Rk}}{N_{cr}}} \qquad (11)$$ *(6.58)*

in which $N_{pl.Rk}$ is the value of the plastic resistance $N_{pl.Rd}$ calculated using material partial safety factors g_a, g_c and g_y set equal to 1,0 (or, using the characteristic material strengths).

6.3 Member buckling resistance

A composite column has sufficient resistance to buckling if, for each of the planes of buckling, the design axial loading N_{Sd} satisfies the inequality: *6.8.3.5*

$$N_{Sd} \leq c N_{pl.Rd} \qquad (12)$$ *EC3 Part 1.1 6.2.1.2*

in which the value of *c*, the strength reduction factor in the plane of buckling considered, is a function of the relative slenderness $\bar{l}$ and the appropriate European buckling curve. The buckling curves which apply to composite columns are given in Table 1.

EC4 Part 1.1

Table 6.5

Buckling curve	Cross-section type	Imperfection
Curve *a* (*a* = 0,21)	Concrete-filled, reinforced (A_s/A_c<3%) or unreinforced, hollow sections without additional steel I-section.	L/300
Curve *b* (*a* = 0,34)	H-sections completely or partially encased in concrete, buckling about the major (y-y) axis of the steel section; Concrete-filled hollow sections either reinforced (3%<A_s/A_c <6%) or with additional steel I-section.	L/210
Curve *c* (*a* = 0,49)	H-sections completely or partially encased in concrete, buckling about the minor (z-z) axis of the steel section.	L/170

Table 1. Buckling curves and member imperfections

It is possible to calculate the value of the strength reduction factor *c* using:

$$c = \frac{1}{f + [f^2 - \overline{l}^2]^{1/2}} \leq 1 \qquad (13)$$

in which

$$f = 0{,}5[1 + a(\overline{l} - 0{,}2) + \overline{l}^2]. \qquad (14)$$

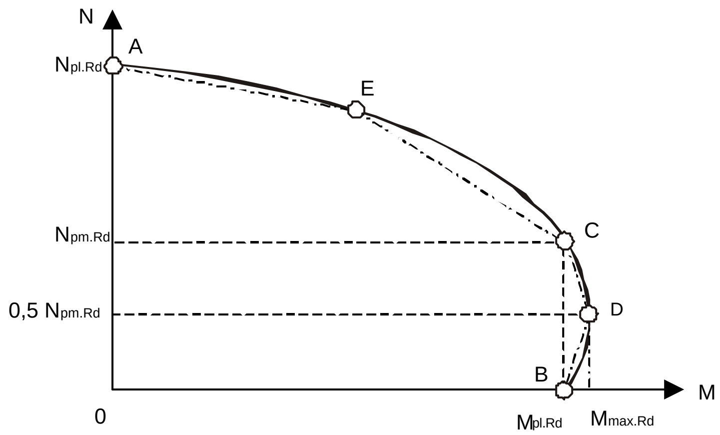
where *a* is a generalised imperfection parameter which allows for the unfavourable effects of initial out-of-straightness and residual stresses.

In some cases, particularly when considering slender columns under axial load and bending, it may be appropriate to use the imperfection values given in Table 1 to calculate an additional first-order bending moment caused by this eccentricity of the axial load.

7 Resistance to compression and bending

7.1 Cross-section resistance under moment and axial force

It is necessary to satisfy the resistance requirements in each of the principal planes, taking account of the slenderness, the bending moment diagram and the bending resistance in the plane under consideration. The cross-sectional resistance of a composite column under axial compression and **uniaxial bending** is given by an *M-N* interaction curve as shown in Figure 4.



EC4 Part 1.1 6.8.3.2

Figure 4 *M-N* interaction curve for uniaxial bending

The point *D* on this interaction curve corresponds to the maximum moment resistance $M_{max,Rd}$ that can be achieved by the section. This is greater than $M_{pl.Rd}$ because the compressive axial force inhibits tensile cracking of the concrete, thus enhancing its flexural resistance.

The above interaction curve can be determined point by point, by considering different plastic neutral axis positions in the principal plane under consideration. The concurrent values of moment and axial resistance are then found from the stress blocks, together with the two equilibrium equations for moments and axial forces.

Figure 5 illustrates this process for the example of a concrete-encased section, for four particular positions of the plastic neutral axis corresponding respectively to the points *A, B, C, D* marked on Figure 4.

Fig. 6.26

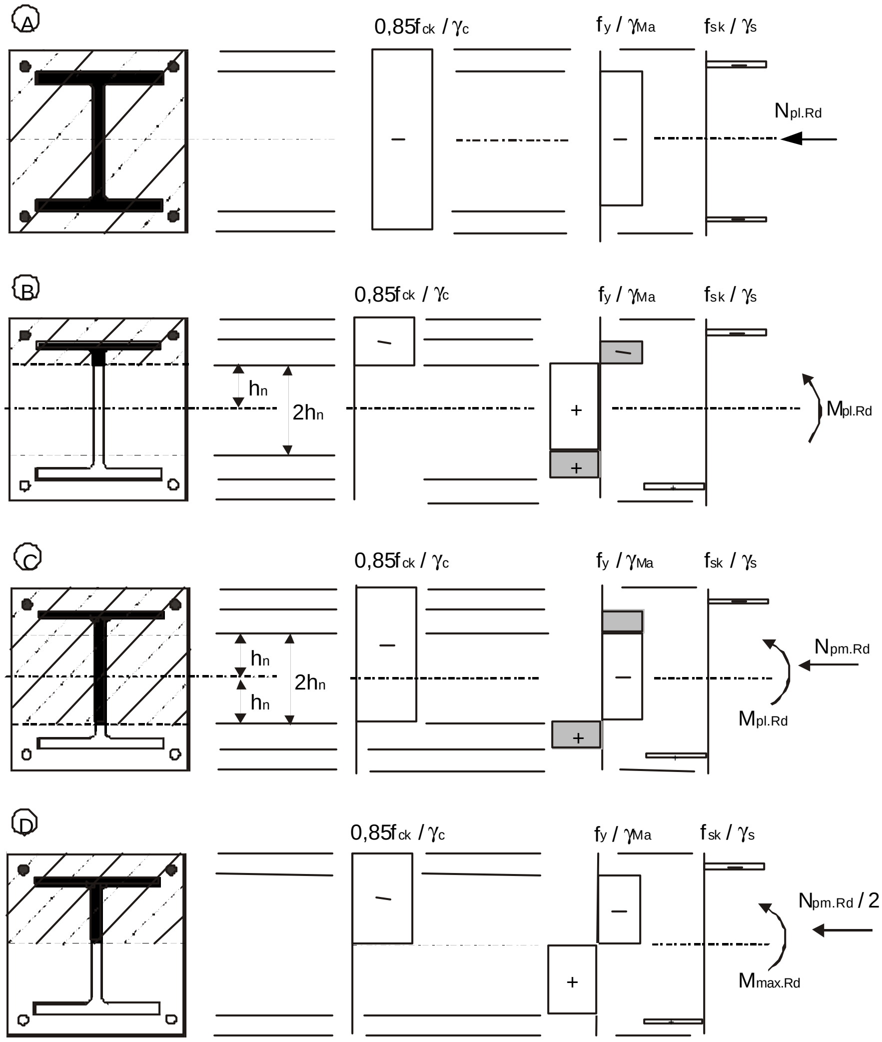


Figure 5 Development of stress blocks at different points on the interaction curve (concrete-encased section)

- **Point *A* :** Axial compression resistance alone:

$N_A = N_{pl.Rd}$ $\qquad$ $M_A = 0$

- **Point *B* :** Uniaxial bending resistance alone:

  $N_B = 0$ $\qquad\qquad\qquad\qquad$ $M_B = M_{pl.Rd}$

- **Point *C* :** Uniaxial bending resistance identical to that at point *B*, but with non-zero resultant axial compression force:

  $$N_C = N_{pm.Rd} = A_C .0{,}85 \frac{f_{ck}}{g_C} \quad \textit{(concrete-encased section)}$$

  $$= A_C \frac{f_{ck}}{g_C} \textit{(concrete- filled hollow section)}$$

  $$M_C = M_{pl.Rd}$$

  Note: f_{ck} may be factored by $[1 + h_c \frac{t}{d} \frac{f_y}{f_{ck}}]$ for a circular concrete-filled hollow section.

- **Point *D* :** Maximum moment resistance

  $$N_D = \frac{1}{2} N_{pm.Rd} = \frac{1}{2} A_c \; 0{,}85 \frac{f_{ck}}{g_c} \textit{(concrete-encased section)}$$

  $$= \frac{1}{2} A_c \frac{f_{ck}}{g_c} \textit{(concrete- filled hollow section)}$$

  Again f_{ck} may be factored by $[1 + h_c \frac{t}{d} \frac{f_y}{f_{ck}}]$ for a circular concrete-filled hollow section.

  $$M_D = W_{pa} . \frac{f_y}{g_a} + W_{ps} \frac{f_s}{g_s} + \frac{1}{2} W_{pc} \, 0{,}85 \frac{f_{ck}}{g_c}$$

  in which W_{pa}, W_{ps}, *and* W_{pc} are the plastic moduli respectively of the steel section, the reinforcement and the concrete.

- **Point *E* :** Situated midway between *A* and *C*.

  The enhancement of the resistance at point *E* is little more than that given by direct linear interpolation between *A* and *C*, and the calculation can therefore be omitted.

It is usual to substitute the linearised version AECDB (or the simpler ACDB) shown in Figure 4 for the more exact interaction curve, after doing the calculation to determine these points.

7.2 Second-order amplification of bending moments

It is necessary to consider the local influence of geometrically second-order effects on an individual member, in particular the amplification of the first-order moments which exist in the column due to the increased eccentricity at which the axial force acts. These can however be neglected in checking isolated columns within rigid frames if $N_{Sd} / N_{cr} \leq 0{,}1$ or if *6.8.3.4*
$\bar{l} < 0{,}2(2 - r)$, where *r* is the ratio of the end-moments on the ends of the column ($-1 \leq r \leq +1$). *(6.62)*

Second-order effects on the behaviour of an isolated column forming part of a non-sway frame can be taken into consideration approximately by applying an amplification factor *k* to the *5.1.3*
maximum first-order bending moment M_{Sd}. The factor *k* is given by:

$$k = \frac{b}{1 - N_{Sd} / N_{cr}} \geq 1{,}0 \qquad (15)$$

(5.2)

in which:

$b = 0{,}66 + 0{,}44r$ $\qquad$ for a column subjected to end-moments;
$b = 1{,}0$ $\qquad$ when bending is caused by lateral loading on the column.

Table 5.1

When axial loading and end-moments are both present, *b* should never be taken as less than 1,0 unless it is calculated by a more exact method.

7.3 The influence of shear force

It is permissible to assume for simplicity that the design transverse shear force V_{Sd} is completely resisted by the steel section. Alternatively it is possible to distribute it between the steel section and the concrete; in this case the shear force carried by the concrete is determined by the method given in Eurocode 2.

The interaction between the bending moment and shear force in the steel section can be taken into account by reducing the limiting bending stresses in the zones which are affected by significant shear force. This reduction of yield strength in the sheared zones can be represented, for ease of calculation, by a reduction in the thickness of the element(s) of the steel section which carries the shear force. This influence need only be taken into consideration if the shear force carried by the steel section, $V_{a.Sd}$, is greater than 50% of its plastic shear resistance: *6.8.3.2*

$$V_{pl.a.Rd} = A_v f_{yd} / \sqrt{3}$$

where A_v is the sheared area of the steel section. The reduction factor which may need to be applied to this area is:

$$r_w = \left[1 - \left(\frac{2V_{a.Sd}}{V_{a.Rd}} - 1 \right)^2 \right] \qquad (16)$$ *(6.14)*

6.3.2.4

For a concrete-encased H-section subjected to bending about the major axis the reduced shear area is therefore:

$$r_w t_w h$$

When the reduced thickness $r_w t_w$ is used, the method described in Section 7.1 for determination of the resistance interaction curve for the cross-section can be applied freely.

7.4 Member resistance under axial compression and uniaxial bending

The principle of the EC4 calculation method for member resistance under axial load and uniaxial moment is demonstrated schematically in Figure 6, which is a normalised version of the interaction diagram of cross-sectional resistance in Figure 4. For a design axial compression N_{Sd} the plastic section resistance M_{Rd}, which is a proportion m_d of the fully plastic resistance $M_{pl.Rd}$, is indicated by the interaction curve.

fig. (6.27)

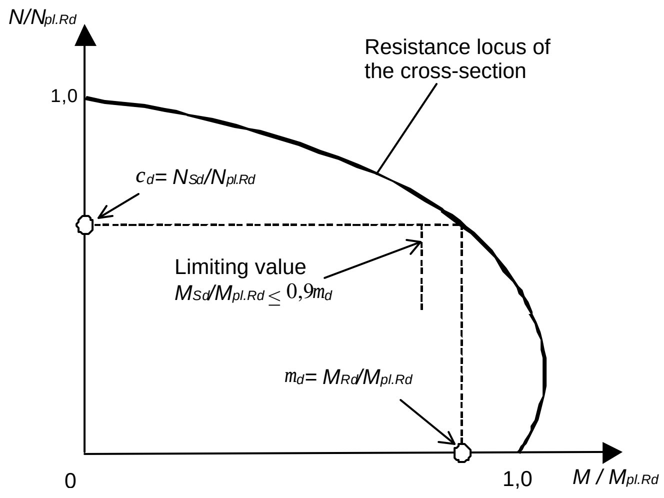


Figure 6 Resistance to axial compression and uniaxial bending

The design moment M_{Sd} is the maximum moment occurring within the length of the column, including any enhancement caused by the column imperfections and amplification of the total first-order moments due to the second-order "*P-d*" effect. Under the design axial force N_{Sd}, a composite column has sufficient resistance if *6.8.3.6*

$$M_{Sd} \leq 0{,}9 m_d M_{pl.Rd} \qquad (17)$$

(6.64)

The 10% reduction in resistance indicated by the introduction of the factor 0,9 compensates for the underlying simplifications in the calculation method. For example, the interaction curve has been established without considering any limits on the deformations of concrete. Consequently, the bending moments, including the second-order effects, are calculated using the effective bending stiffness $(EI)_e$ determined on the basis of the complete concrete cross-sectional area.

It is evident from Figure 4 that values of m_d taken from the interaction diagram may be in excess of 1,0 in the region around point D, where a certain level of axial compression increases the moment capacity of the section. In practice, values of m_d above 1,0 should not be used unless the moment M_{Sd} is directly caused by the axial force N_{Sd}, acting at a fixed eccentricity on a statically determinate column.

7.5 Member resistance under axial compression and biaxial bending

When a composite column is subjected to axial compression together with biaxial bending, it is first necessary to check its resistance under compression and uniaxial bending individually in each of the planes of bending. This is not however sufficient, and it is necessary also to check its biaxial bending behaviour. In doing so it is only necessary to take account of imperfections in the plane (for example Case (a) in Figure 7) in which failure is likely to take place. For the other plane of bending (for example Case (b) in Figure 7) the effect of imperfections is neglected. *6.8.3.7*

fig. (6.27)

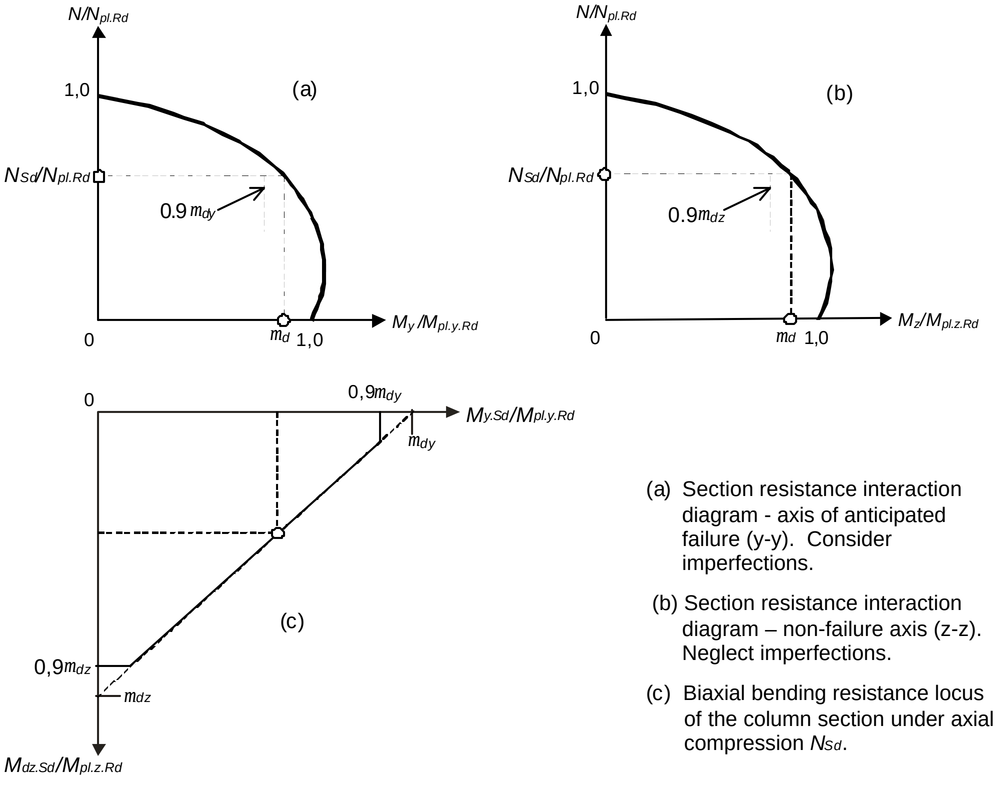


Figure 7 Member resistance under compression and biaxial bending

This can be represented by the two simultaneous conditions:

$$M_{y.Sd} \leq 0{,}9 m_{dy} M_{pl.y.Rd} \quad (18)$$

$$M_{z.Sd} \leq 0{,}9 m_{dz} M_{pl.z.Rd} \quad (19)$$ *(6.65)*

If there is any doubt about the plane of failure the designer is recommended to consider the effect of imperfections in both planes. *(6.65)*

To take account of the peak stresses caused by moments between the limits given by the inequalities (18) and (19), acting about two orthogonal axes, a linear interaction formula must also be satisfied between the two design moments. The design moments are calculated including both imperfections and the amplification due to second-order "*P-d*" effects.

$$\frac{M_{y.Sd}}{m_{dy} M_{pl.y.Rd}} + \frac{M_{z.Sd}}{m_{dz} M_{pl.z.Rd}} \leq 1{,}0 \quad (20)$$

(6.66)

These three conditions (18)-(20) together define the ultimate strength locus in terms of the orthogonal design moments at the design axial compression value N_{Sd} as shown in Figure 7(c).

8 Concluding Summary

Only the simplified method of composite column design has been addressed in this article. Its scope is limited to bisymmetric cross-sections containing only one steel section; it does not apply if two or more unconnected sections are used. The more general calculation method given in EC4 for asymmetric sections will often involve advanced analytical modelling, particularly where no axis of symmetry exists. This is only likely to be encountered in very specialised situations, such as corner columns where high biaxial moments are anticipated. The method described here will undoubtedly apply to the great majority of columns in practical composite construction. *6.8.3.1*

Composite columns are not a common feature in buildings which are currently described as "composite". The more usual framing scheme in routine multi-storey construction is to use composite flooring together with steel H-section columns. This tends to happen because of practical difficulties in connecting beams to composite columns on site. The solutions to this problem generally increase the cost of fabrication considerably, and may make an all-composite building uneconomic. In the case of hollow sections a connection method must be devised which avoids the need for access to both sides of the wall of the steel section. Where encased sections are used, at least part of the concrete encasement must usually be cast on site, again because of the need to connect members in a practical way. The use of composite columns becomes much more attractive where the need for high strength within a small "footprint" and good intrinsic fire resistance are considered more important than the basic price of the structural frame. For these reasons, while they are unlikely to become commonplace, composite columns are likely to find an increasing role in supporting the very long-span floors which are becoming more usual in commercial construction and in tall buildings.

Structural Steelwork Eurocodes
Development of
a Trans-National Approach

Course: Eurocode 4

Lecture 9 : Composite joints

Summary:

- Traditionally structural joints are considered as rigid or pinned
- The lecture is intended to introduce the concept of advanced, semi-rigid joints
- The requirements relating to stiffness, strength and rotation capacity are explained
- The steps of joint representation are shown
- The principles of the component method (including component identification, component characterisation and component assembly) for joint characterisation are presented
- Design provisions for composite joints

Pre-requisites:

- Basic knowledge about frame analysis and design
- Basic definitions and concepts on semi-rigid joints
- Plastic response of frames
- Knowledge of SSEDTA-1 Module 5

Notes for Tutors:

- This material comprises two 60 minute lectures
- The tutor should update the national changes of the Eurocodes
- All additional modifications of the training packages are under responsibility of the tutor
- For further details concerning composite joints see Annex A and Annex B

Objectives:

The student should:

- Understand the conceptual assumptions of simple joints
- Know, how to handle the response of structural joint in view of a global frame analysis
- Be able to characterise and idealise the behaviour of beam-to-column joints

References:

- [1] EN 1994-1-1: Design of composite steel and concrete structures Part 1.1
  General rules and rules for buildings; Draft No. 2; April 2000
- [2] prEN 1993-1-8: Eurocode 3: Design of steel structures
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- [3] SSEDTA (1): Structural Steelwork Eurocodes Development of A Trans-national Approach
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- [4] Gerald Huber: Non linear calculations of composite sections and semi-continuous joints
  Doctoral Thesis, Verlag Ernst & Sohn, December1999
- [5] COST C1: Composite steel-concrete joints in frames for buildings: Design provisions
  Brussels, Luxembourg 1999
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  Proceedings of the international conference Liège, 17 to 19 September 1998
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Contents:

1 Introduction

This lecture concerns building frames with composite steel-concrete beams. The primary aim is to explain how to design joints under hogging bending moment as composite elements.

1.1 Definition and terminology

[1] 1.4.2

- **Composite joint**
  A joint between composite members, in which reinforcement is intended to contribute to the resistance and the stiffness of the joint
- **Basic component** (of a joint)
  specific part of a joint that makes an identified contribution to one or more of its structural properties
- **Connected member**
  member that is supported by the member to which it is connected
- **Connection**
  location at which two members are interconnected, and the means of interconnection
- **Joint**
  assembly of basic components that enables members to be connected together in such a way that the relevant internal forces and moments can be transferred between them
- **Joint configuration**
  type or layout of the joint or joints in a zone within which the axes of two or more inter-connected members intersect
- **Structural properties** (of a joint)
  its resistance to internal forces and moments in the connected members, its rotational stiffness and its rotation capacity

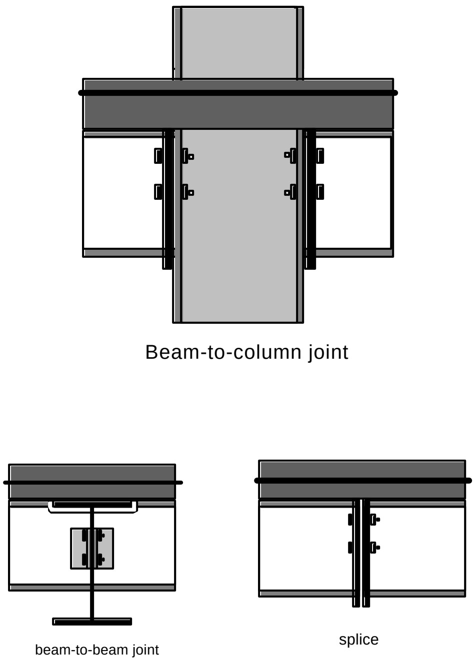


Figure 1 Types of joints

The key feature is the provision of continuous slab reinforcement to act in tension across the joint. This increases substantially both resistance and stiffness for little increase in work on site.

Generally there is a lack of full continuity between the floor system and the column. Frames with composite joints are therefore semi-continuous in nature. This type of framing enables advantage to be taken of the stiffness and moment resistance inherent in many forms of connection whilst avoiding the expense of rigid and full-strength steelwork connections. The recognition of this approach is widely regarded as one of the advances in prEN 1993-1-8 [2], compared to earlier standards.

Conventionally, joints have been treated as nominally pinned without any strength or stiffness (simple joints) or as rigid with full strength (continuous joints). The differences between conventional design and semi-continuous construction can be seen from Figure 2 to Figure 4. The types of framing depend on the joint characteristics, particularly on the initial stiffness and the moment resistance relative to the connected members. The semi-continuous approach provides greater freedom, enabling the designer to choose connections to meet the particular requirements of the structure.

1.1.1 Types of joints

The construction of composite joints used to be based on the already well-known conventional steel joints (see Figure 1 and Figure 5). The composite floor system was connected to the columns by hinged or semi-rigid steel connections, as e.g. welded, angle cleats and flush endplate joints. However a gap separated the concrete res. composite slab from the column to prevent an interaction with the joint. This constructional separation was necessary due to a lack of knowledge in view of modelling the interaction. As the gap had to be provided by expensive constructional means and furthermore the joint's stiffness and strength were relatively low in comparison to the adjacent composite members this solution was not economic.

Nowadays in advanced composite joints the floor system is integrated into the joint (Figure 5), leading to much higher values of stiffness and strength. This very efficient solution became possible, because the interaction problem between the slab and the column has been solved in comprehensive studies.

Generally one has to distinguish between beams located below the slab (conventional floors) and beams which are already integrated into the slab (slim floors). A further distinction concerns the type of connection before concreting of the slab. In that cases where already a semi-rigid steel connection is provided (e.g. welded, angle cleats, flush or partial depth endplates) the slab supplies additional stiffness and strength after hardening of the concrete. In the very economic case of hinged steel joints during erection, by adding contact pieces and due to the integration of the slab into the joint a high bearing capacity and stiffness can be gained without any further bolting or welding on site (joints with brackets and additional contact plates for the compression transfer or hinged steel joints with fins or angle cleats, see Figure 5). For slim floor decks the authors in any case recommend to design hinged steel joints, which then by concreting are automatically converted to semi-rigid composite joints with considerable resistance and rotation ability.

A further increase of joint stiffness and strength can be obtained by concreting the column sections (encased composite columns). Figure 5 gives an overview of all mentioned joint types using H-shaped column sections. Regarding the fire resistance and also the appearance, hollow column sections, which can be filled with concrete, turned out to be forward-looking.

1.1.1.1 Conventional joints:

- Simple joints

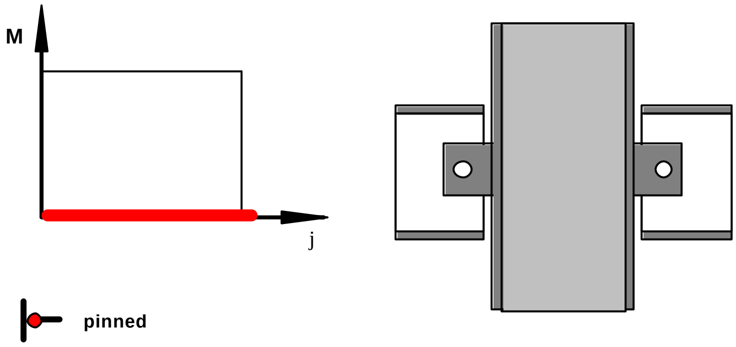


Figure 2 Simple joint (pinned)

- Continuous joints

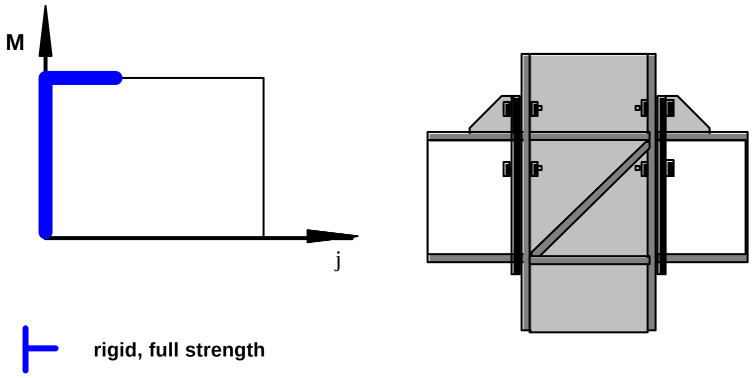


Figure 3 Rigid joint (continuous)

1.1.1.2 Advanced joints

- Semi continuous joints

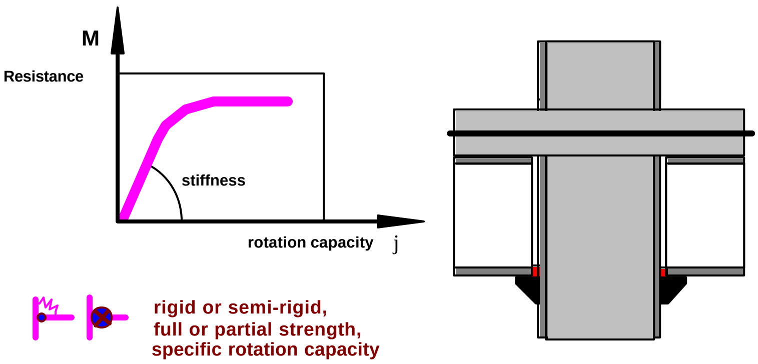


Figure 4 Advanced composite joint (semi continuous)

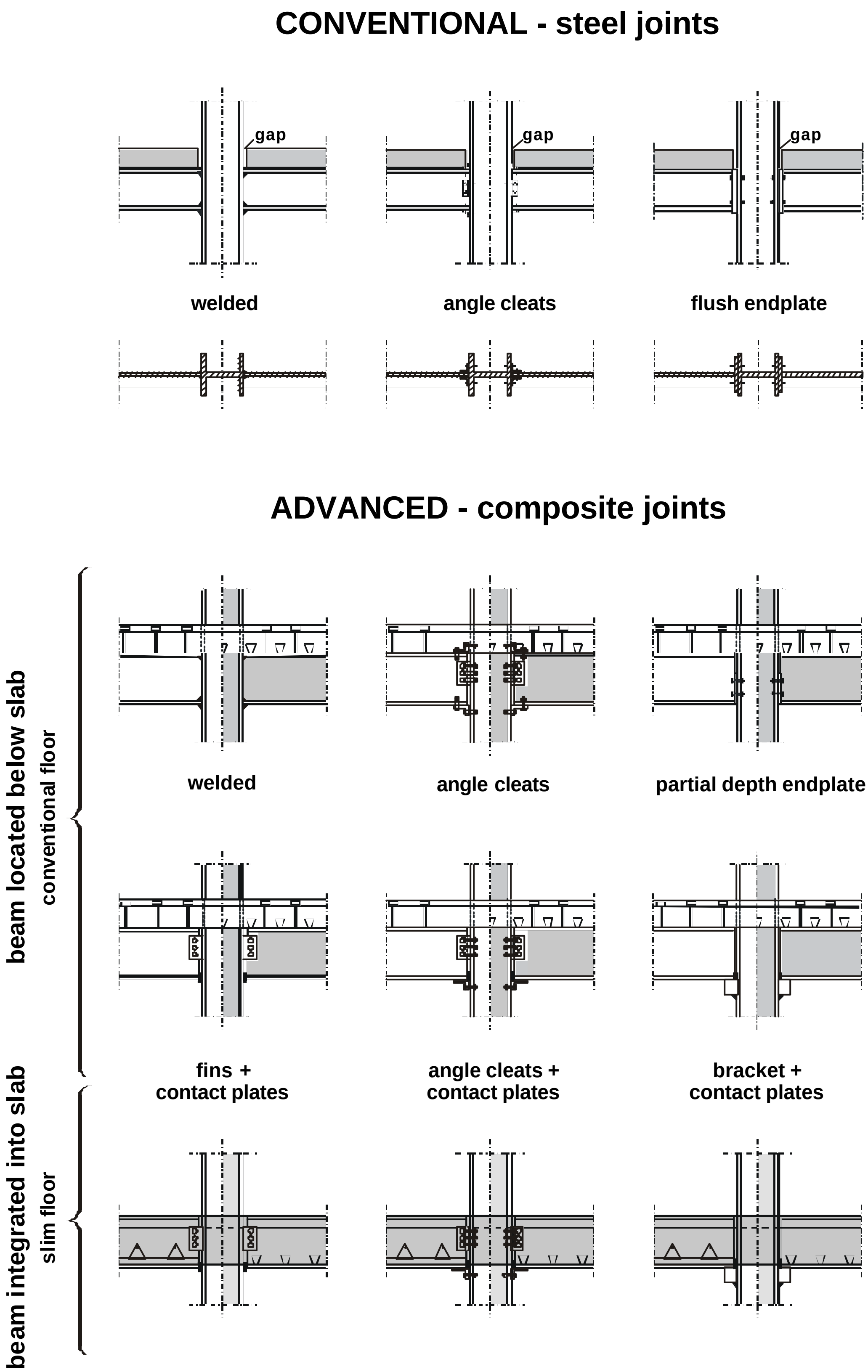


Figure 5 Types of joints- H- shaped column

As semi continuous joints influence the response of the frame to load, they should be modelled in the global frame analysis.
Traditionally the joints simply have not been considered in the global calculation as a separate element. However as a joint strictly consists of parts of the column, parts of the beams and parts of the slabs, connecting elements and sometimes also includes stiffening elements, so the real behaviour can only be taken into consideration by defining the joint as a separate element (Figure 6) within the structure, additional to the beams and column elements.

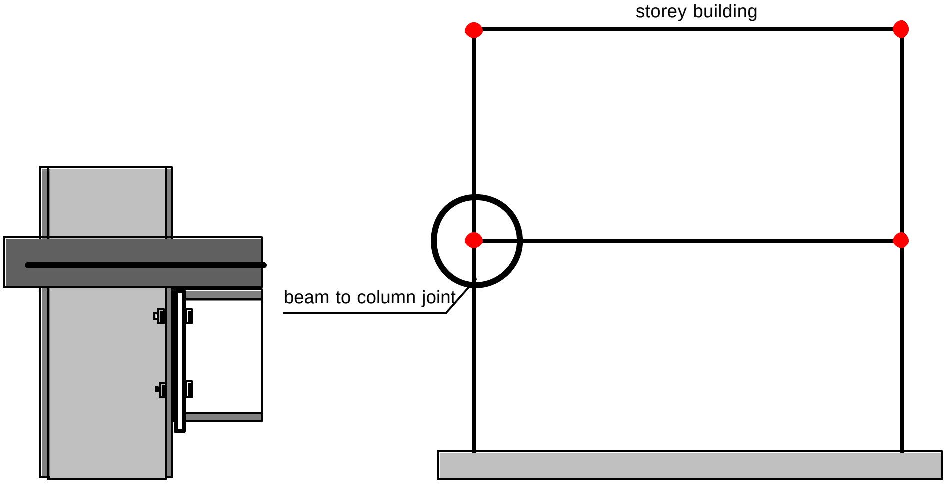


Figure 6 Joint as a separate element within the structure

This enables more efficient constructions but the influence of the joints on the global behaviour is so important that the old-fashioned philosophy of perfect hinges or fully continuous restraints does not describe the real behaviour of a semi continuous joint. (See Figure 7)

It is conventional to simplify the framing as simple or continuous framing, which has the advantage of straight-forward calculation. However with modern design approaches it is appropriate to replace the conventional calculation methods by more advanced ones which treat the joints in a realistic manner. As already noted, semi-continuous construction enables advantage to be taken of the stiffness and moment resistance inherent in many forms of connection, without the expense of forming the rigid and/or full-strength joints necessary for continuous construction.

Although structural behaviour is three-dimensional, the usual presence of stiff floor slabs normally allows the designer to neglect out-of-plane and torsional deformations of the joint. The joint characterisation is therefore usually in the form of a moment rotation relationship
(Figure 7).

According to this new approach, joints may be assessed with regard to the following three main characteristics.

- **The initial rotational stiffness $S_{j,ini}$:**

A joint with a very small rotational stiffness and which therefore carries no bending moment is called a hinge. A rigid joint is one whose rigidity under flexure is more or less infinite and which thus ensures a perfect continuity of rotations. In between these two extreme boundaries we speak about semi-rigid joints.

- **The design moment resistance $M_{j,Rd}$:**

In contrast to a hinge, a joint whose ultimate strength is greater than the ultimate resistance (ultimate strength) of the parts whose linkage it ensures is called a full strength joint. Again a partial strength joint represents a middle course between these extremes. (For simplicity from now on “resistance” will mostly be used for the ultimate resistance value; the terms “resistance” and “strength” are used in the Eurocodes with an identical meaning.)

- **The design rotation capacity f_{Cd}:**

Brittle behaviour is characterised by fracture under slight rotation, usually without plastic deformations. Ductile behaviour is characterised by a clear non-linearity of the moment-rotation curve with a large plateau before fracture. It usually indicates the appearance of plastic deformations. The ductility coefficient is the ratio between the ultimate rotation and the elastic rotation limit. Semi-ductility falls in between brittle and ductile behaviour.

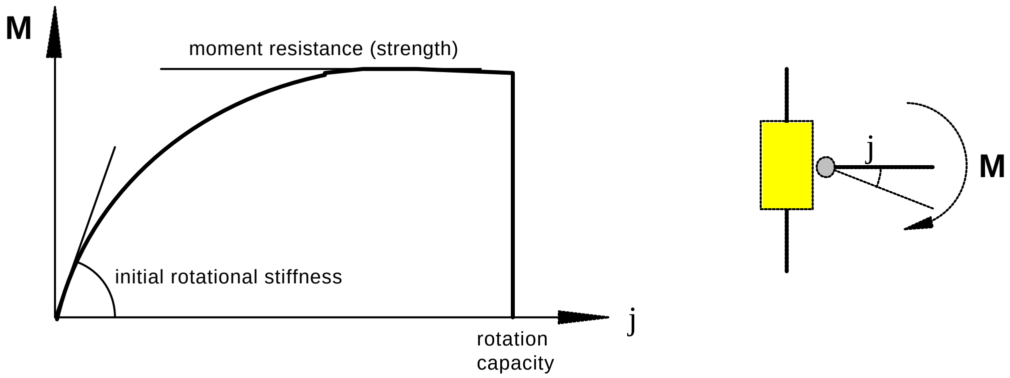


Figure 7 Joint response [4]

Chapter 3 and the extended annex A describes how these can be calculated. As is usual in the Eurocodes, partial safety factors are applied to loads and strengths during ultimate limit state verifications, but not to the modulus of elasticity nor to limiting strains. Annex A

By analysing full-scale joints it could be seen very quickly that the number of influencing parameters is too large. So world-wide the so-called component method is accepted universally as the best method to describe the joint behaviour analytically. In contrast to the common finite element method (FEM), which often fails to consider local load introduction problems, the joint here is divided into logical parts exposed to internal forces. So while the FEM works on the level of strains and stresses, the component method concentrates on internal forces and deformations of the component springs.

In recent years extensive testing programs have been performed world-wide for studying the non-linear behaviour of individual components and their assembly to gain the non-linear moment-rotation characteristic of the whole joint formed by these components.

Traditionally, engineering judgement has been used to ensure that joint behaviour approximates to that required for simple and continuous construction. However, the concept of semi-continuous construction requires a more precise statement of what constitutes each joint. In prEN 1993-1-8 [2] this is provided by a classification system based on joint resistance and stiffness. This is extended to composite constructions. Advice is also given on types of connections suitable for each type of construction and on detailing rules.

1.2 Composite joints for simple framing

Most of the joints in composite frames are currently treated as nominally pinned and the frames are therefore of “simple” construction. This provides the advantage that the global analysis is straightforward because the structure is easier to calculate.
Nominally pinned joints are cheap to fabricate and easy to erect. They may be the most appropriate solution when significant ground settlement is expected. However, deeper beam sections will be required compared to other forms of construction, leading to an increased height of the structure and greater cladding costs. Due to the beam end rotations, substantial cracking can occur in the joint area if the slab is continuous over internal supports.

The joint has to be designed to transfer safely the vertical shear and any axial forces. The resistance of the slab to vertical shear is small and is neglected. As any continuity in the slab is also neglected, a nominally pinned joint for a composite beam is therefore designed in the same way as a simple steelwork connection.

If the slab is constructed as continuous, uncontrolled cracking is permitted by Eurocode 2 if the exposure conditions are Class 1. According to EN 1992-1 [10], most interiors of buildings for offices or normal habitation are in this class, which implies that there is no risk of corrosion of reinforcement. Appearance requires a floor finish with ductile behaviour or provision of a covering. Even so, minimum areas of reinforcement are specified to prevent fracture of the bars or the formation of very wide cracks under service loading. Where cracks are avoided by measures such as the provision of joints in the slab, these should not impair the proper functioning of the structure or cause its appearance to be unacceptable.

In industrial buildings, no additional finish or covering is included and the slab itself provides the floor surface. In such cases continuous or semi-continuous construction is preferable, so that crack widths can be adequately controlled.
Although joint ductility is essential in simple construction, it is not usual for designers to calculate either the required or available rotation capacity. For steel connections it is sufficient to rely on the observed behaviour of joint components.

Rotation capacity can be provided by bolt slip and by designing the components in such a way that they behave in a ductile manner. Local yielding of thin steel end-plates and the use of a wide transverse bolt spacing are examples of this approach. Fracture of bolts and welds should not be the failure mode. However, as the slab reinforcement is assumed to have no influence on the behaviour at the ultimate limit state, fracture of this does not limit the rotation capacity of the joint.

1.3 Joints in semi-continuous construction

Composite moment-resisting joints provide the opportunity to improve further the economy of composite construction. Several possible arrangements for composite joints are shown in Figure 5, demonstrating the wide variety of steelwork connection that may be used. In the interest of economy though, it is desirable that the steelwork connection is not significantly more complicated than that used for simple construction of steel frames. With conventional unpropped construction, the joints are nominally pinned at the construction phase. Later the steelwork connection combines with the slab reinforcement to form a composite joint of substantial resistance and stiffness.

A particularly straightforward arrangement arises with "boltless" steelwork connections. At the composite stage all of the tensile resistance is provided by the slab reinforcement, and no bolts act in tension. The balancing compression acts in bearing through plates or shims, inserted between the end of the lower flange of the beam and the face of the column to make up for construction tolerances. Additional means to resist vertical shear must also be provided, for example by a seating for the beam.

The benefits which result from composite joints include reduced beam depths and weight, improved service performance, including control of cracking, and greater robustness. Besides the need for a more advanced calculation method and the placement of reinforcement, the principal disadvantage is the possible need for transverse stiffeners to the column web, placed opposite the lower beam flange. These are required if the compression arising from the action of the joint exceeds the resistance of the unstiffened column web. Alternatively, stiffeners can be replaced by concrete encasement.

1.4 Scope

In the following, detailed provisions are given for:

- Joints with flush or partial-depth end-plates;
- Joints with boltless connections and contact plates.

The joints are intended for conventional composite beams in which the structural steel section is beneath the slab. It is assumed that the connections are subject to hogging bending moment due to static or quasi-static loading. Profiled steel decking may be used both to form the in-situ concrete and to act as tensile reinforcement, creating a composite slab; alternatively, the slab may be of in-situ reinforced-concrete construction or may use precast units.
Steel sections for columns may be H- or I-shaped and may act compositely with concrete encasement.

2 Joint Representation (General)

Conventionally beam-to-column joints have been treated either as pinned without any strength or stiffness or as fully rigid with full strength due to lack of more realistic guidance in view of joint representation. In reality both assumptions may be inaccurate and uneconomic and do only represent the limiting cases of the real moment-rotation behaviour. They may lead to a wrong interpretation of the structural behaviour in terms of load resistance and deflections. So whereas up to now the joint construction expensively has been adopted to the possibilities of calculation, the new approach is to develop efficient joint types first and to take their realistic behaviour into consideration within the frame analysis afterwards. Due to the interaction between joints and members an overall cost-optimisation is only possible if both design tasks are taken over by the same party. So the designer at least should be able to bring in a first good guess of the joint characteristics depending on the chosen joint configuration.

First discoveries on the importance of joint representation released a real innovation boom, which can be seen by a tremendous number of research activities all over the world.
So e.g. comprehensive research projects on representation and design of structural steel and composite connections and their effect on the frame response have been co-ordinated by the "European Co-operation in the Field of Scientific and Technical Research (COST) [5],[6],[7].

Moment resisting joints have to transfer moments and forces between members with an adequate margin of safety. Their behaviour obviously influences the distribution of moments and forces within the structure. Therefore the list of construction elements as beams, slabs and columns has to be extended by the joints. An overall account of the behaviour would need to recognise its three-dimensional nature. However the presence of rather stiff continuous floor slabs usually allows to neglect out-of-plane and torsional deformations of the joint.

That is why the attempt to describe the connections response can be reduced to a description of the in-plane behaviour. In contrast to the idealising assumptions of beam-to-column connections as hinges or full restraints, with the main attention aimed on their resistance, the interest of actual research activities lies in the assessment of the non-linear joint response of the whole M-f curve with the following three main characteristics:

- **Initial rotational stiffness**
- **Moment resistance**
- **Rotation capacity**

It is true that the conventional simplifications of continuous or simple framing bring the advantage of a very simple calculation, but reality again lies in between the extreme limiting cases, what especially affects the serviceability limit state and the stability of the whole structure.

The joint representation covers all necessary actions to come from a specific joint configuration to its reproduction within the frame analysis. These actions are the

- **<u>Joint characterisation</u>**: determination of the joint response in terms of M-f curves reflecting the joint behaviour in view of bending moments and shear (in case of moment imbalance)
- **<u>Joint classification</u>**: if aiming at conventional modelling the actual joint behaviour may be compared with classification limits for stiffness, strength and ductility; if the respective requirements are fulfilled a joint then may be modelled as a hinge or as a rigid restraint.
- **<u>Joint idealisation</u>**: for semi-continuous joint modelling the M-f behaviour has to be taken into consideration; depending on the desired accuracy and the type of global frame analysis the non-linear curves may be simplified as bi- or tri linear approximations.
- **<u>Joint modelling</u>**: reproduction (computational model) of the joint´s M-f behaviour within the frame modelling for global analysis.

Figure 8 shows how the moment-rotation behaviour of joints can be represented in global structural analysis in a practical way.

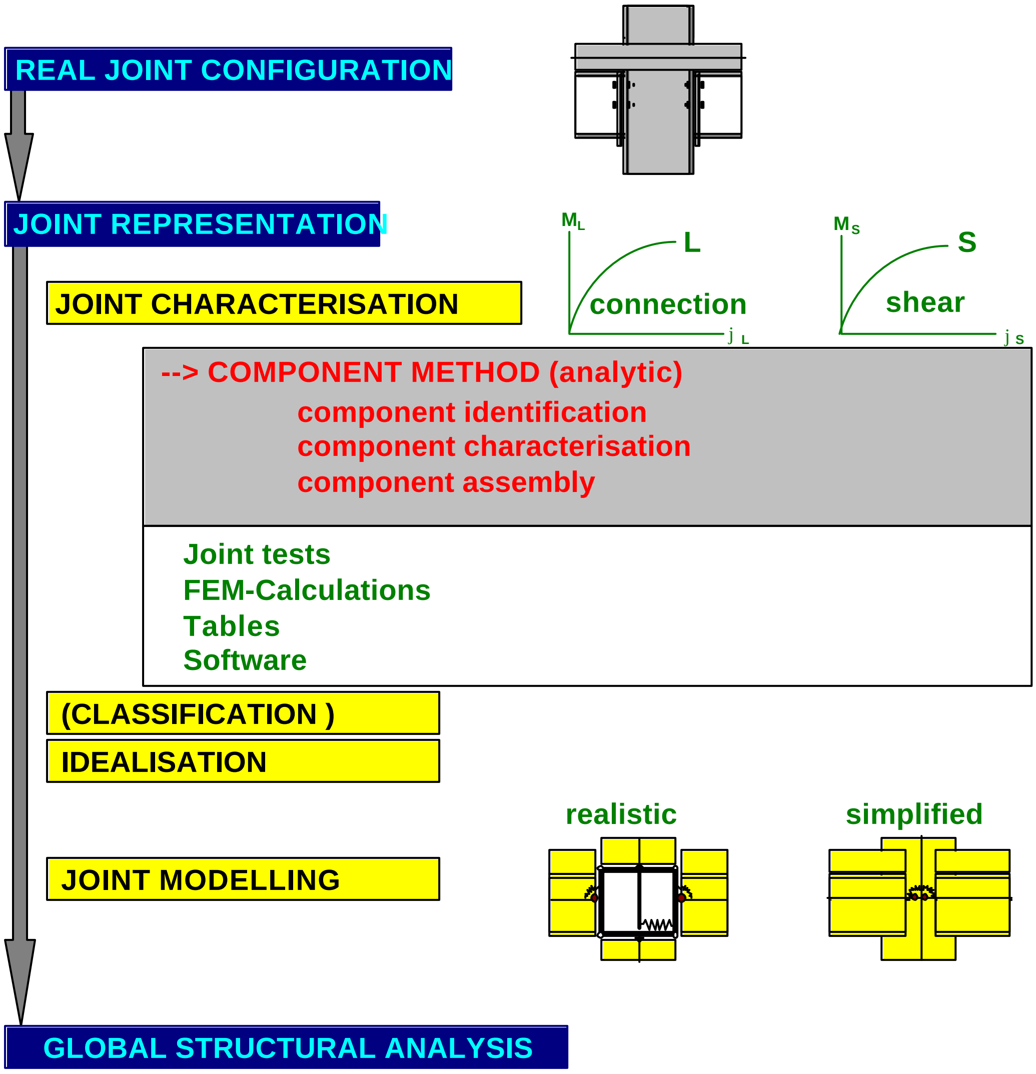


Figure 8 Practical procedure of joint representation

2.1 Joint characterisation

This chapter describes how to derive the M-f curves, representing the necessary input data for the joint models.
To determine these, several possibilities exist:

- Joint tests
- Finite element calculations
- **An analytical approach**

The general background for any joint model comprises three separate curves:

- One for the left hand connection
- One for the right hand connection
- One for the column web panel in shear

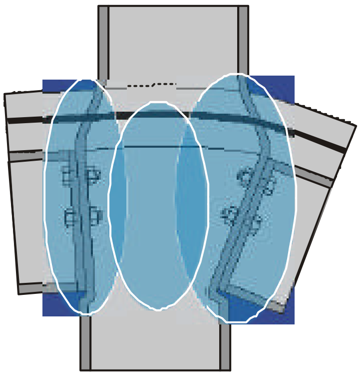


Figure 9 Left connection, column web panel in shear, right connection

2.1.1 Component method

A description of the rotational behaviour of a joint has to take into account all sources of deformations within the joint area. Furthermore all possibilities of local plastic deformations and instabilities have to be covered by such an analytical model. The multitude of influencing parameters was the reason, why the first attempts to develop a component method directly based on full-scale joint tests have been doomed to failure. Also a second research tendency with the aim of finite element modelling did not really succeed because of local detailing problems. So for the analytical determination of the non-linear joint response a macroscopic inspection by subdividing the complex finite joint into so-called simple "**components**" proved to be successful. In contrast to the finite element method (FEM) the components - as logical subsystems of the joint – are exposed to internal forces and moments and not to stresses. As it will be described later in detail these components can be understood as translational springs with a non-linear force-deformation behaviour. All considered components can be tested in clear, relatively cheap so-called "component tests", on the basis of which theoretical models can be developed. Finally the total joint response (reproduced in the "joint model for global frame analysis") can be derived by assembling all influencing components in the correct manner based on the so-called "**component model**". This method offers the advantages of a minimum of testing costs, clear physical calculation models for the components and a maximum of flexibility for the designer, who is able to combine a multitude of available components to a most economic joint configuration. Numerous full-scale joint tests confirm this approach.

In Figure 10 the development of joint modelling is shown. Conventionally rigid or hinged joints of infinite small size have been assumed in the global analysis. But as any real joint has a finite size, deformations occur under relevant member forces. So in the traditional view the beam and column stubs within the realistic joint area (b_j, h_j according to Figure 10) already can be understood as an attempt to model the deformation of the real joint. The first step of improvement was to regard the joint as a separate element of finite size. The second step was to describe the force-deformation behaviour of all individual components by non-linear translational springs. Setting together a joint out of these components, taking into consideration their location and compatibility conditions, results in the component model.

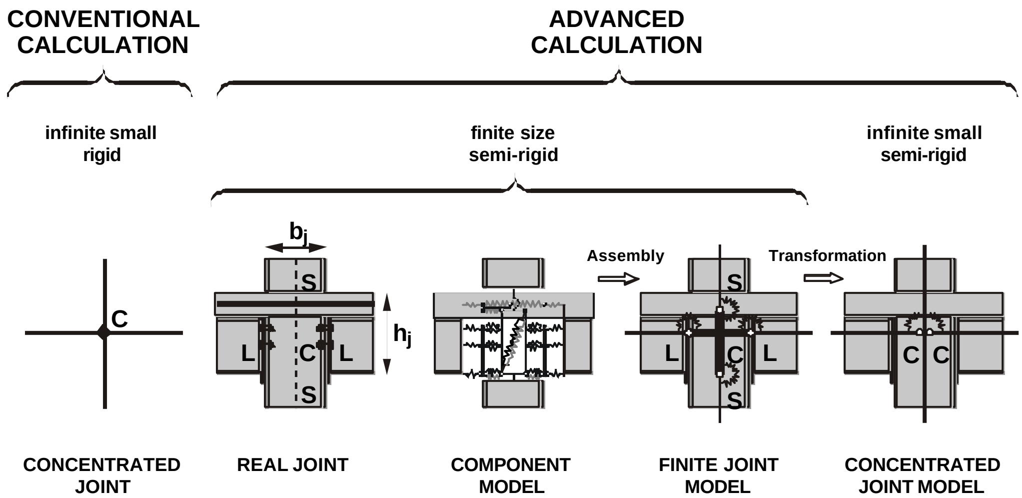


Figure 10 Modelling of joints – conventional and advanced

For a further treatment it is important to define three different locations within the finite joint area:

- **C** represents the Centre point of the joint located at the intersection between beam and column axes.
- **L** is the Loading point located at the centre of rotation in view of tension and compression at the front of the column flange (simplifying L can be assumed at the level of the beam axis).
- **S** is the Shear point located at the top and bottom of the joint's lever arm z along the column axis.

In the third step the components are assembled to rotational springs in L and S forming the finite joint model, which then can be used in the advanced global analysis. There the joint region (b_j, h_j) is set infinite stiff and all deformations are assigned to the flexural springs at the joint edges (L and S). A further possibility in advanced global analysis is to concentrate the full joint's flexibility within flexural springs at the axes intersection points C (separately for the left and right hand side). Doing so the joint again is reduced to an infinite small point within the global analysis, however this requires a transformation from the finite joint model to this concentrated joint model.

An analytical description of the behaviour of a joint has to cover all sources of deformabilites, local plastifications, plastic redistribution of forces within the joint itself and local instabilities. Due to the multitude of influencing parameters, a macroscopic inspection of the complex joint, by

subdividing it into "components", has proved to be most appropriate. In comparison with the finite element method, these components, which can be modelled by translational spring with non-linear force-deformation response, are exposed to internal forces and not to stresses.
The procedure can be expressed in three steps:

- Component **identification**
  determination of contributing components in compression, tension and shear in view of connecting elements and load introduction into the column web panel.

- Component **characterisation**
  determination of the component´s individual force-deformation response with the help of analytical mechanical models, component tests or FEM-simulations.

- Component **assembly**
  assembly of all contributing translational component springs to overall rotational joint springs according to the chosen component model.

2.1.2 Component models (spring models)

The component model for steel joints and later that for composite joints, has been developed on the basis of considerations of how to divide the complex finite joint into logical parts exposed to internal forces and moments and therefore being the sources of deformations. So in horizontal direction one has to distinguish between the connecting and the panel zones, in vertical direction between the tension, the compression and the shear region, all together forming six groups as illustrated in Figure 11.

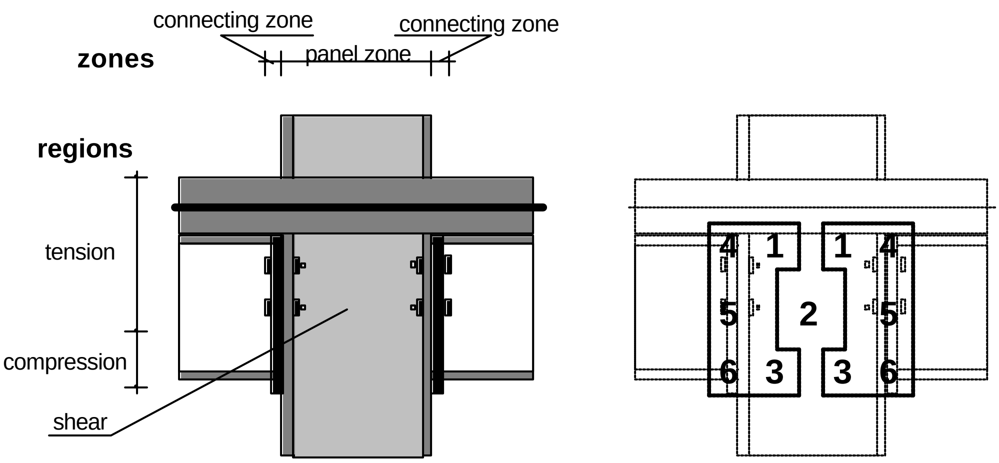


Figure 11 Groups of component model [4]

Group	
1	panel in tension
2	panel in shear
3	panel in compression
4	connection in tension
5	vertical shear connection
6	connection in compression

Table 1 Groups of component model

According to prEN 1993-1-8 [2] a basic component of a joint is a specific part of a joint that makes-dependent on the type of loading-an identified contribution to one or more of its structural properties. When identifying the contributing components within a joint one can distinguish between components loaded in tension (or bending), compression and shear. Beside the type of loading one can distinguish between components linked to the connecting elements and those linked to load-introduction into the column web panel (both are included in the connection) and the component column web panel in shear. The nodal subdivision for the most general case of a composite beam-to-column joint leads to a sophisticated component model like that developed in Innsbruck (Figure 12). There, the interplay of the several components is modelled in a very realistic way. Any composite joints (beam–to-column, beam-to-beam or beam splice), even steel joints can be derived as being only a special case of this general model. The sophisticated component model leads to a complex interplay of components and therefore to iterations within the joint characterisation itself. For simplifications concerning the component interplay a simplified component model has been developed (Figure 12).

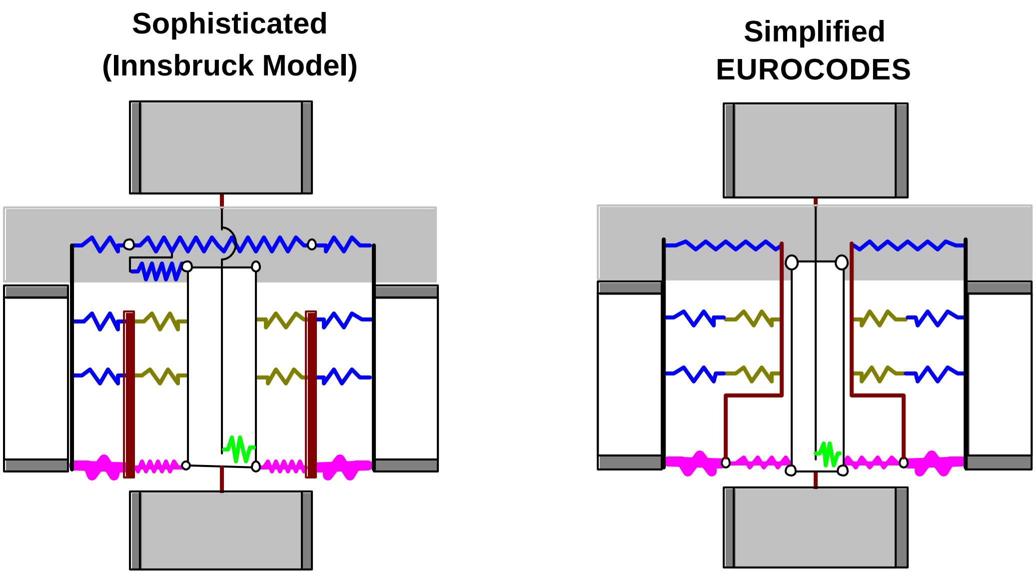
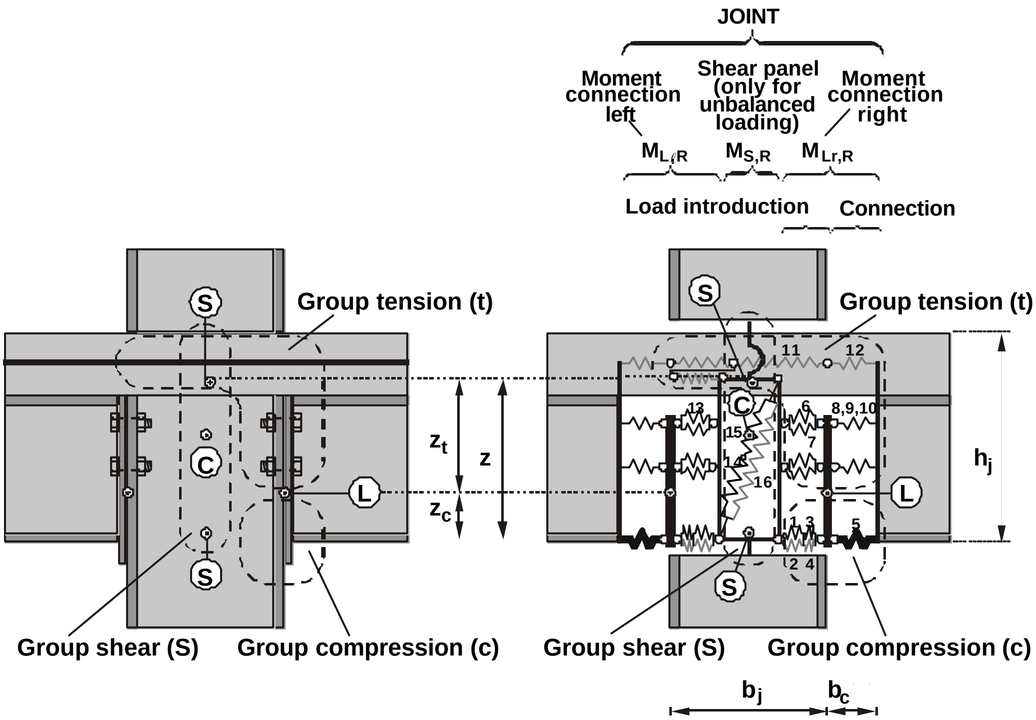


Figure 12 Sophisticated and simplified spring models



i	COMPONENT	GROUP
1	interior steel web panel	com-presssion
2	concrete encasement	
3	exterior steel web panel (column flange and local effects)	
4	effect of concrete encasement on exterior spring	
5	beam flange (local effects), contact plate, end plate	
6	steel web panel incl. part of flange, fillet radius	tension
7	stiffener in tension	
8	column flange in bending (stiffened)	
9	end plate in bending, beam web in tension	
10	bolts in tension	
11	reinforcement (within panel) in tension	
12	slip of composite beam (due to incomplete interaction)	
13	redirection of unbalanced forces	
14	steel web panel in shear	shear
15	steel web panel in bending	
16	concrete encasement in shear	

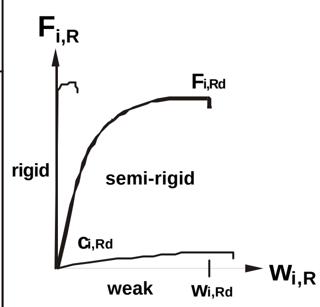


Figure 13 Refined component model

Through familiarity with the spring model (component model), which represents the interplay of all deformation influences, the principles of economic joint construction with respect to stiffness, resistance failure modes and ductility can be logically derived. Assuming a specific component with a given resistance and deformation ability, it is self-evident that by increasing its lever arm the joint´s moment resistance increases whilst the rotation capacity decreases.

Providing resistance in the compression region far beyond the resistance of the tension region is uneconomic. Similarly strengthening the connecting elements will not make the joint capable of sustaining more load, if failure is already dominated by the column web in tension, compression or shear. It also can be recognised immediately that it makes no sense to combine a relatively weak and ductile slab reinforcement with a stiff and brittle steelwork connection.
These examples give ideas on how to use the component model directly for a qualitative plastic redistribution within the joint itself, all components are used up to their plastic resistance and provide a further yield plateau for redistribution of moments within the whole frame.

Figure 13 shows the real joint situation on the left side and the corresponding component model on the right. The method of component modelling started with beam-to-column joints in steel. The bolt-rows in tension are modelled by corresponding spring rows, separating the influences of *load-introduction* within the column web panel and the *connection* to the column flange. The same philosophy has been followed for the compression. For an unbalanced loaded joint, the deformations due to the shear have to be modelled by an additional shear spring.

It has been proved by tests on full scale joints that the measured moment-rotation behaviour is in good agreement with the calculated curves using the component method. Out of this excellent experience the component method has been extended to composite joints, where only the additional concrete components had to be analysed. Thus the composite component model can be seen as the most general tool covering all considerable joint types. In this sense steel joints or special types of joints like splices, beam-to-beam joints or weak axis joints can be regarded as special cases of this general model (see Figure 14).

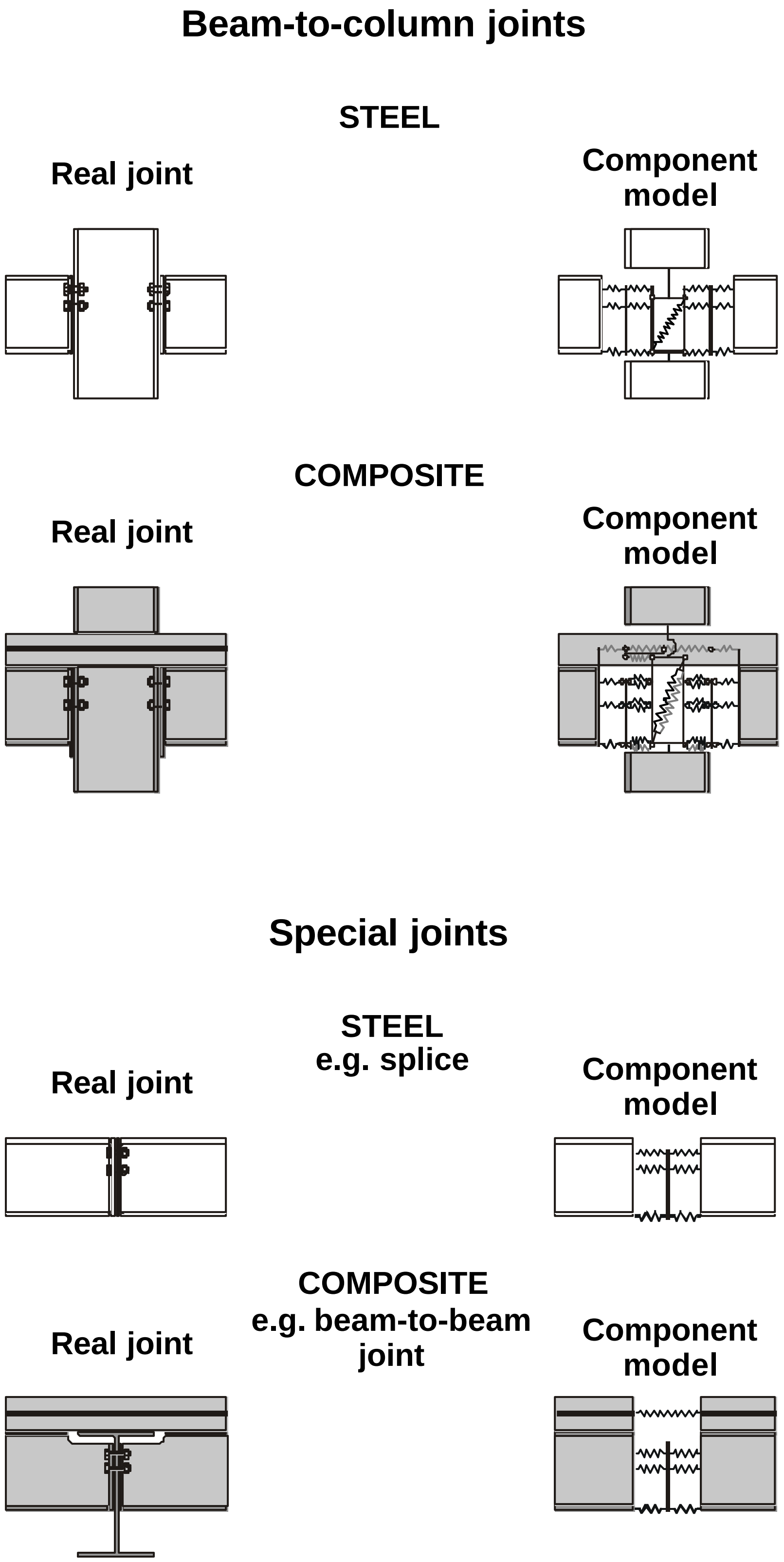


Figure 14 Modelling of joints – examples

2.1.3 Component assembly

The transfer from the force-deformation curves of the individual basic joint components to the moment-rotation curves representing the connection or the shear panel has to be done based on the component modes fulfilling the requirements of compatibility and equilibrium. Doing so it is assured that the joint model behaves exactly in the same way than the complex component model with respect to applied moments. Depending on the intended level of accuracy the assembly can be done for the main rotational key values only (initial rotational stiffness, moment resistance, rotation capacity) or for the full shape of the resulting M-f curves.

As already mentioned the assembly of the sophisticated component model leads to iteration loops due to the complex interplay of components.
For simplification, iterations can be avoided by using the simplified component model used in the Eurocodes, where the sum of all basic component springs can be derived by adding them step by step acting parallel or in series. (See Figure 15and Figure 16)

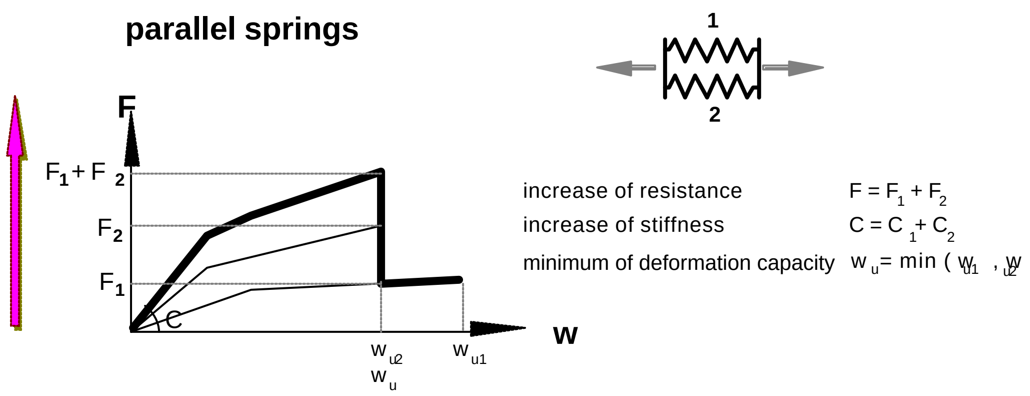


Figure 15 Assembly of parallel spring groups

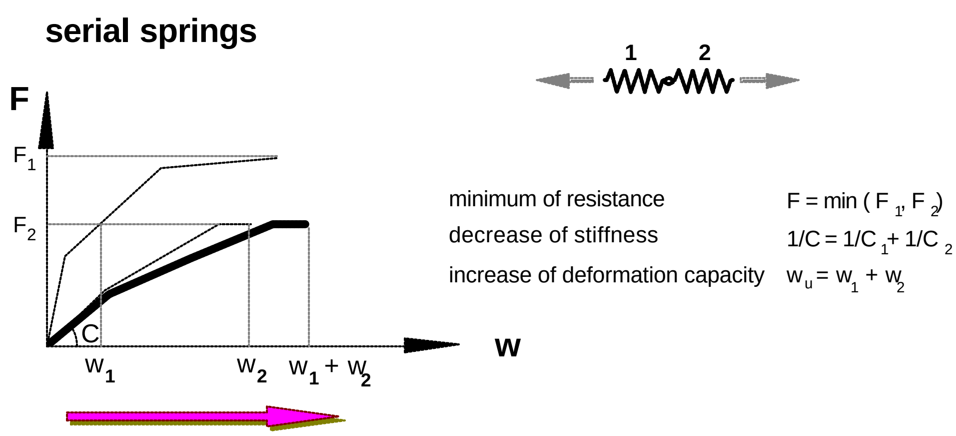


Figure 16 Assembly of serial spring groups

A component model for a composite beam-to-column joint configuration is shown in Figure 13.

Sixteen different components have been identified and these may be grouped as compression, tension and shear.

2.1.4 Rotational stiffness

It will be noted that these rotational stiffnesses represent the deformability of the column web panel in shear separately from the other sources of deformation. For a single-sided joint configuration, the total rotational stiffness can be expressed directly in terms of the effective translational stiffness in shear and compression and the equivalent translational stiffness in tension:

$$S_{j,ini} = z^2 \sum \frac{1}{c_i} \tag{1}$$

where c_i represents the effective or equivalent stiffness of the region i.

For a double-sided joint configuration, the extent of the shear in the column web panel is influenced by the ratio of unbalance between the moments at the two connections. In prEN 1993-1-8 [2] this influence is included through a parameter ß.

2.1.5 Design moment resistance

For resistance calculation has shown the tension region as limited by the deformation capacity of the second row. Recommendations to ensure ductile behaviour in steel joints are given in prEN 1993-1-8 [2]. These are applicable to steelwork parts of composite joints.
Detailing rules given in this document avoid brittle failure modes associated with composite action, provided that high-ductility reinforcement is allocated for the composite joint.
Provided a plastic distribution of bolt forces is allowed by prEN 1993-1-8 [2], the design moment of resistance M_{Rd} may be expressed as:

$$M_{Rd} = \sum F_{Lt,i,Rd} \; h_i \tag{2}$$

where the summation is over all rows of longitudinal slab reinforcement and bolt-rows in the tension region.
In practice though the resistance of the connection in compression or of the web panel in shear may be lower than that of the group of components in tension. For equilibrium the total tensile force $\sum F_{Lt,i,Rd}$ must not exceed the design resistance of the compression group $F_{Lc,Rd}$ and the shear resistance $V_{S,Rd}$ / ß. If this condition is reached at a tension row i, the contribution to the moment resistance of all other tension rows closer to the centre of compression is neglected.

2.1.6 Moment resistance

It has to be demonstrated that the internal moments due to the design loads, obtained by global analysis, do not exceed the corresponding resistance values. So strictly only the internal moment at the face of the column, not that at the column centre-line, has to be compared with the joint's resistance. The difference in internal moment might be important for short-span beams with deep column profiles.

2.1.7 Stiffness

- <u>Transformation of rotational stiffness</u>

By assembling appropriate components, rotational stiffnesses are obtained to represent load-introduction and the connection at L and shear deformation at S. These are then transformed separately from L to C and S to C, taking account of the beam and columns stubs introduced by the simplified model. The transformed stiffness are then combined to obtain the total rotational stiffness at C.

The transformation from S to C:

$$S_{column} = 2\,\frac{f}{z}\,E\,I_c \tag{3}$$

where:

f = 1 for a joint at the top of the column;
f = 2 for a joint within a continuous length of the column;
I_c is the second moment of area of the adjacent column section;
z is the lever arm of the joint.

For transformation from L to C:

$$S_{beam} = \frac{E\,I_b}{L_j} \tag{4}$$

where:

I_b is the second moment of area of the adjacent beam section in hogging bending;
L_j is half of the depth of the column section.

- <u>Transformation of component stiffness</u>

As a further simplification, stiffness of individual components can be calibrated to include the flexibility of the beam and column stubs. Thus quasi-transformed stiffness values can be given in design codes, leaving the designer to assemble the components according to the joint configuration. The result is a total rotational stiffness for the concentrated joint model at C.

2.1.8 Basic components of a joint

The design moment-rotation characteristic of a joint depends on the properties of its basic components. [2] 5.1.4

The following basic joint components are identified in this lecture:

- Column web panel in shear
- Column web in compression
- Column web in tension
- Column flange in bending
- End-plate in bending
- Beam flange and web in compression
- Beam web in tension
- Bolts in tension
- Longitudinal slab reinforcement in tension
- Contact plate in compression

A major-axis beam-to-column composite connection consists of a combination of some of the basic components listed above, excluding the column web panel in shear. In particular, it always includes the following:

- Column web in compression
- Longitudinal slab reinforcement in tension

Methods for determining the properties of the basic components of a joint are given: [1] 8.1.2

– For resistance in 3.3 [1] 8.3.3
– For elastic stiffness in 3.4 [1] 8.4.2

Relationships between the properties of the basic components of a joint and the structural properties of the joint are given:

- For moment resistance in 3.2 [1] 8.3.4
- For rotational stiffness in 3.4 [1] 8.4.1
- For rotation capacity in 3.5 [1] 8.5

2.2 Classification of beam-to-column joints

In conventional design of building structures, the framing is treated as simple or continuous, even though practical joints always possess some moment resistance and show some flexibility. Traditionally, engineering judgement has been used to ensure that joint behaviour approximates to that required for these forms of construction.

However, the concept of semi-continuous construction requires a more precise statement of the joint behaviour. In prEN 1993-1-8 [2] this is provided by a classification system based on joint resistance and stiffness. Table 2 shows how the types of joint model (representing behaviour), the form of construction and the method of global analysis are all related. [2] 7.2.2 [2] 7.2.3

Method of global analysis	Classification of joint		
Elastic	Nominally pinned	Rigid	Semi-rigid
Rigid-Plastic	Nominally pinned	Full-strength	Partial-strength
Elastic-Plastic	Nominally pinned	Rigid and full-strength	Semi-rigid and partial -strength Semi-rigid and full -strength Rigid and partial-strength
Type of joint model	Simple	Continuous	Semi-Continuous

Table 2 Type of joint model

2.2.1 Classification by strength

A beam-to-column joint may be classified as full-strength, nominally pinned or partial strength by comparing its moment resistance with the moment resistances of the members that it joins. The purpose is to indicate whether the joint or the adjacent member cross-sections will limit resistance. In the first case, the joint is either "partial strength" or "nominally pinned". In the second case the joint is "full-strength". Thus a beam-to-column joint will be classified as full-strength if the following criteria are satisfied.

<u>For a joint at the top of a column:</u>

$$M_{j,Rd} = M_{b,pl,Rd} \quad (5)$$

or:

$$M_{j,Rd} = M_{c,pl,Rd} \quad (6)$$

where:

$M_{b,pl,Rd}$ design plastic moment resistance of the composite beam in hogging bending immediately adjacent to the joint;

$M_{c,pl,Rd}$ design plastic moment resistance of the column.

<u>For a joint within a continuous length of column:</u>

$$M_{j,Rd} = M_{b,pl,Rd} \qquad (7$$

or:

$$M_{j,Rd} = 2\,M_{c,pl,Rd} \qquad (8$$

A joint may be classified as nominally pinned if its moment resistance $M_{j,Rd}$ is not greater than 0,25 times the moment resistance required for a full-strength joint, provided that it also has sufficient rotation capacity.

With composite beams the moment resistance depends on whether the member is in sagging or hogging bending. With braced construction, beam end connections are subject to hogging bending and the moment resistances of joints and beams should therefore be those applicable to this situation. Under such moment, EN 1994-1-1 [1] requires full shear connection and therefore the beam´s resistance should be calculated on this basis. Plastic analysis should be used for sections Class 1 and Class 2, including beam sections with Class 3 webs upgraded to Class 2 using the "hole-in-the web" approach.

[1] 5.3

The classification of the joint has implications for its design criteria. With a partial-strength or nominally pinned joint, it is the joint rather than the member section which requires rotation capacity. Tests have shown that rotation capacity may be limited.

2.2.2 *Classification by rotational stiffness*

A beam-to-column joint may be classified as rigid, nominally pinned or semi-rigid according to its stiffness, by determining its initial rotational stiffness $S_{j,ini}$ and comparing this with classification boundaries.

As shown in Figure 17, the classification given by prEN 1993-1-8 [2] compares stiffness of the joint with that of the connected member. The purpose is to indicate whether account has to be taken of the influence of joint flexibility on the frame response. The classification boundaries were determined from consideration of ultimate limit states.

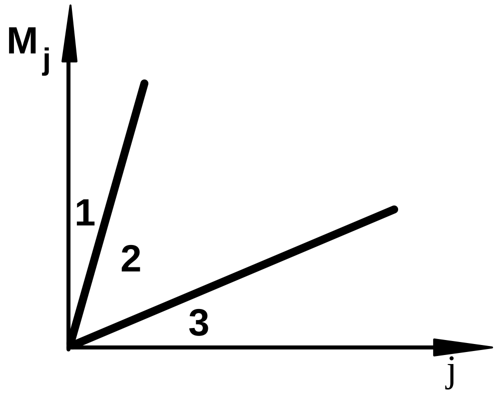


Figure 17 Boundaries for stiffness classification of beam-to-column joints

Zone 1: rigid, if $S_{j,ini} = 8\dfrac{E\,I_b}{L_b}$ (9)

Zone 2: semi-rigid, all joints in zone 2 should be classified as semi-rigid. More accurately, joints in zones 1 or 3 may also be treated as semi-rigid.

Zone 3: nominally pinned, if $S_{j,ini} = 0{,}5\ \frac{E\,I_b}{L_b}$ (10)

where: $E\,I_b$ is the uncracked flexural stiffness for a cross-section of a composite beam;
L_b is the span of a beam (centre-to-centre of columns).

For composite joints, it is necessary to decide whether the classification should be related to the cracked or uncracked flexural rigidity of the beam´s cross-section. The boundary for rigid joints has been determined by providing restraint against column collapse. In a braced frame the hogging beam end moments then decrease. As the unloading stiffness is taken as equal to the initial stiffness, the uncracked properties of the equivalent steel section should be used. To be strictly consistent with elastic global analysis, the modular ratio should be determined in accordance with EN 1994-1-1 [1]. For simplification, it is recommended that the short-term [1] 5.1.4
modular ratio should be taken as 7,0 irrespective of the grade of concrete.

2.3 Idealisation

In prEN 1993-1-8 [2], it is considered that the full non-linear M-f curve consists of three parts, as shown in Figure 18. Up to a level of 2/3 of the design moment resistance ($M_{j,Rd}$), the curve is assumed to be linear elastic. The corresponding stiffness is the so called initial stiffness $S_{j,ini}$. Between 2/3 ($M_{j,Rd}$) and $M_{j,Rd}$ the curve is non-linear. After the moment in the joint reaches $M_{j,Rd}$, a yield plateau could appear. The end of this M-f curve indicates the rotation capacity (f $_{Cd}$) of the joint.

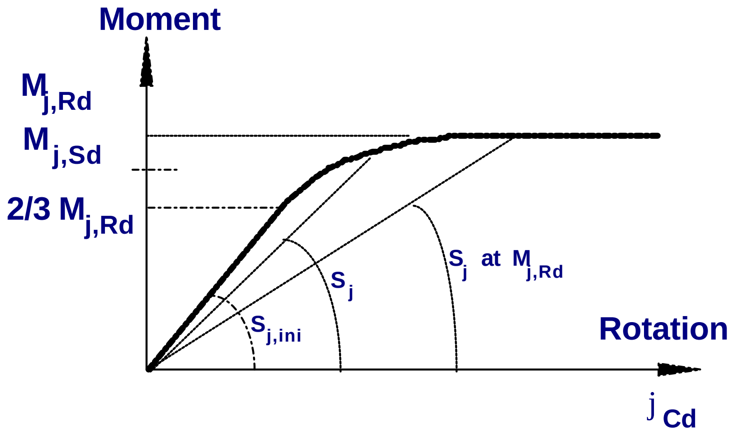


Figure 18 Non-linear M- f curve

The characterisation adopted by Eurocode 3 assumes a fixed ratio between the initial stiffness $S_{j,ini}$ and the secant stiffness at the intersection between the non-linear part and the yield plateau (S_j at the level $M_{j,Rd}$), see Figure 18. For composite joints with bolted end-plates, this ratio is taken equal to 3,0. Contact plate joints have a less gradual decrease in stiffness and the ratio is taken as 2,0.
The shape of the non-linear part for a bending moment $M_{j,Sd}$ between 2/3 ($M_{j,Rd}$) and $M_{j,Rd}$ can be [1] 8.4.1(2)
found with the following interpolation formula:

$$S_j = \frac{S_{j,ini}}{\left(\frac{1{,}5\ M_{j,Sd}}{M_{j,Rd}}\right)^{\psi}} \qquad (11)$$

where ? = 2,7 for joints with bolted flush end-plates and 1,7 for joints with contact plates. In this interpolation formula, the value of S_j is therefore dependent on $M_{j,Sd}$.

Depending on the available software either the full non-linear shape of all joint-curves discussed until now or multi -linear simplifications of them can be assigned to the respective flexural springs. Figure 19 shows curve idealisations proposed in prEN 1993-1-8 [2]. It is evident that the required input as well as the capability required from the software both increase if high accuracy is intended.

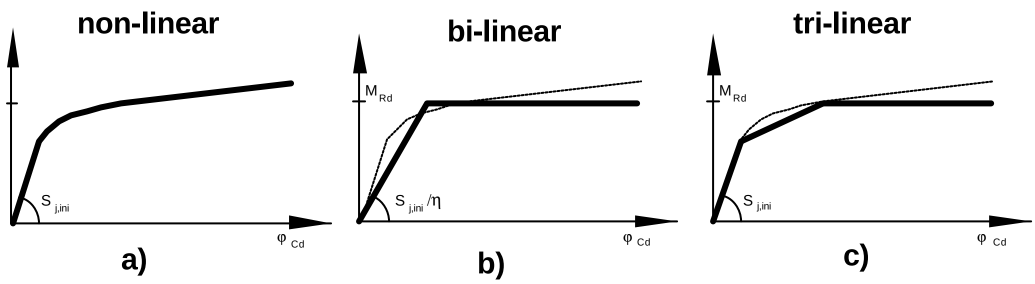


Figure 19 Possibilities for curve idealisation

The actual moment-rotation response of joints usually is described by means of a non-linear curve, see (Figure 19a). However, the use of such non–linear curves requires sophisticated frame analysis programs. In order to enable a (more simple) linear calculation, e.g. an **elastic global frame analysis** (this is still the current practice in most European countries), the non-linear curve may be simplified by straight lines. As a conservative assumption each curve lying below the non-linear curve in general may be used in the frame analysis. For example a tri-linear curve is shown in Figure 19c. The most simple curve is a bi-linear curve. In order to use a bi-linear curve which provides the most efficient solutions (i.e. the highest stiffness), comparative studies were performed to calibrate an idealised joint stiffness S_j*. The idealised joint stiffness S_j* is constant for all values of applied moments smaller than the design moment resistance of the joint. prEN 1993-1-8 [2] gives guidelines on how to derive such a simplified bi-linear curve as shown in Figure 19b. The idealised joint stiffness S_j* can easily be calculated by dividing the initial joint stiffness $S_{j,ini}$ with a stiffness modification factor ?:

$$S_j{*}= S_{j,ini} /? \qquad (12)$$

The stiffness modification factor ? depends on the type of connection (contact plate, bolted end-plate) (See Table 3). [1] 8.2.1.2 Table 8.1

When elastic global analysis is used, the joints should be classified according to their stiffness. In the case of a semi-rigid joint, its rotational stiffness S_j for use in the global analysis should generally be taken as equal to the value of S_j , corresponding to the bending moment $M_{j,Sd}$.

As a simplification, the procedure illustrated in Figure 17 may be adopted, as follows:
Provided that the moment $M_{j,Sd}$ does not exceed 2/3 $M_{j,Rd}$ the initial rotational stiffness of the joint $S_{j,ini}$ may be used in the global analysis.

Where the moment $M_{j,Sd}$ exceeds 2/3 $M_{j,Rd}$ the rotational stiffness should be taken as $S_{j,ini}$/?, where ? is the stiffness modification coefficient from Table 8.1 of EN 1994-1-1 [1] or Table 3 below [1] Table 8.1

Type of connection	Value of ?
Contact plate	1,5
Bolted end-plate	2

Table 3 Stiffness modification coefficient ?

As a further simplification, the rotational stiffness may be taken as $S_{j,ini}$/? in the global analysis, for all values of the moment $M_{j,Sd,}$ as shown in Figure 20.

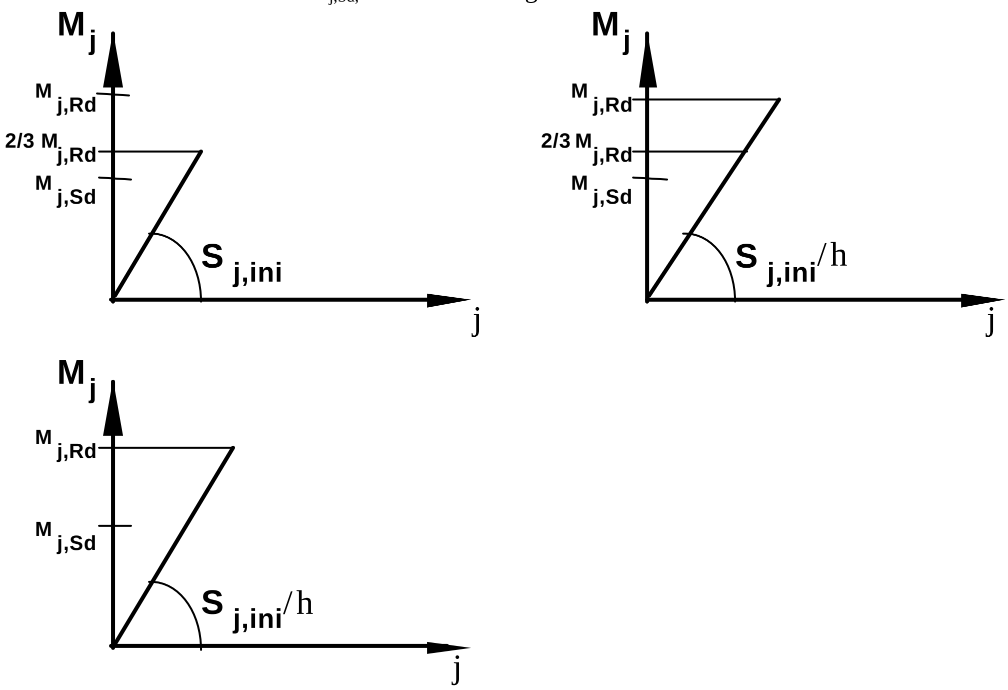


Figure 20 Rotational stiffness to be used in elastic global analysis

When **rigid-plastic global analysis** is used, joints should be classified according to their strength. [1] 8.2.2.2

When **elastic-plastic global analysis** is used, the joints should be classified according to both stiffness and strength.

3 Joint Representation of Composite Beam-to-Column Joints, Design Provisions

3.1 Design provisions

3.1.1 Basis of design

Design provisions in this lecture are based on the component method for steel joints described in prEN 1993-1-8 [2]. The simplified model for a composite joint is shown in Figure 21.

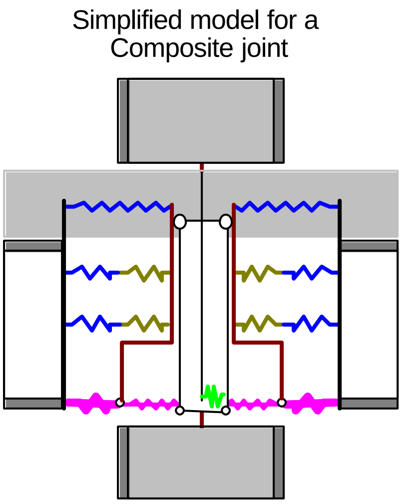


Figure 21 Simplified model according to EN 1994-1-1 [1]

The prEN 1993-1-8 [2] already provides expressions for the design resistance and initial stiffness of the following components:

Compression region:

- Column web in compression
- Beam flange and web in compression

Tension region:

- Column flange in bending
- Column web in tension
- End-plate in bending
- Beam web in tension
- Bolts in tension

Shear region:

- Column web panel in shear

For composite joints, the following additional basic components are relevant:

- **Longitudinal slab reinforcement in tension;**
- **Contact plate in compression.**

Although not regarded as a separate basic component, account may also need to be taken of concrete encasement to the column. This is treated as a form of stiffening.
Expressions for design resistance and initial stiffness are given in 3.3. The stiffness coefficients k_i for components affected by concrete encasement are transformed into equivalent all-steel values, using a modular ratio. This enables a single value for modulus of elasticity E to be used to determine the rotational stiffness of the joint, in the same way as for steel joints in prEN 1993-1-8 [2]. A similar transformation enables the modulus of elasticity of reinforcement to have a different value to that for structural steel.

Unlike the sophisticated model, the simplified Eurocode model shown in Figure 21 does not make explicit allowance for the following:

- Slab concrete in bearing against the column;
- Transverse slab reinforcement;
- Slip of the beam´s shear connection.

Account is taken of these actions either through detailing rules to exclude their influence or (for slip) by reduction factor on the stiffness.

3.2 Design moment resistance

To simplify calculation, plastic theory is used to determine the design moment resistance. This moment is therefore taken as the maximum evaluated on the basis of the following criteria:

- The internal forces are in equilibrium with the forces applied to the joint
- The design resistance of each component is not exceeded
- The deformation capacity of each component is not exceeded
- Compatibility is neglected

<u>Contact plate joint:</u>
In such joints, the steelwork connection provides no resistance to tension arising from bending. The distribution of internal forces is therefore easy to obtain. As can be seen from Figure 22, the compression force is assumed to be transferred at the centroid of the lower beam flange and the tension force at the centroid of the reinforcement.

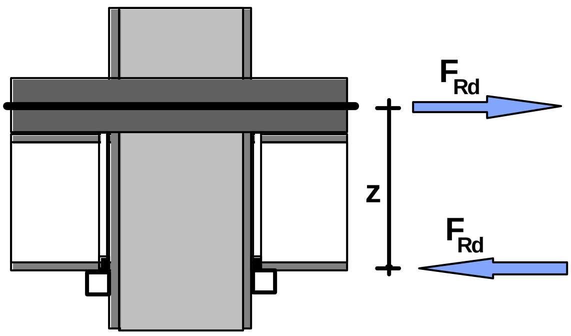


Figure 22 Contact plate joint with one row of reinforcement

The design moment resistance of the joint $M_{j,Rd}$ is dependent on the design resistance F_{Rd} of the weakest joint component; for this joint, the relevant components are assumed to be the reinforcement in tension, the column web in compression, the beam flange and web in compression or the column web panel in shear. So:

$$M_{j,Rd} = F_{Rd} \cdot z \tag{13}$$

where z is the lever arm of the internal forces. [1] 8.2.4.1

For connections with contact plates, the centre of compression should be assumed to be in line with the mid-thickness of compression flange.

For connections with contact plates and only one row of reinforcement active in tension, the lever arm z should be taken as the distance from the centre of compression to the row of reinforcement in tension.

For connections with contact plates and two rows of reinforcement active in tension the lever arm z should be taken as the distance from the centre of compression to a point midway between these two rows, provided that the two rows have the same cross-sectional area.(See Figure 23)

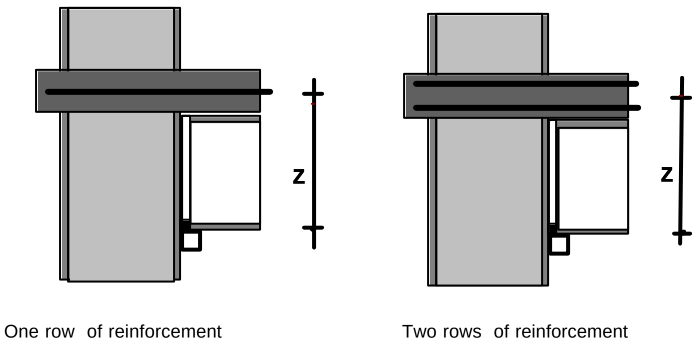


Figure 23 Determination of the lever arm z

For connections with other types of steelwork connections the lever arm z should be taken as equal to z_{eq} obtained using the method given in 5.3.3.1 of prEN 1993-1-8 [2]. [2]5.3.3.1

For the assumption to be correct, detailing rules need to ensure that, under unbalanced loading, failure does not occur by crushing of concrete against the column section.

<u>Joints with steelwork connection effective in tension</u>:
A composite flush end-plate connection, such as that shown in Figure 24, is an example of such a joint. Tension arising from bending is resisted by the combined action of the reinforcement and the upper part of the steelwork connection. As there is more than one row of components in the tension region, the distribution is now more complex. It is assumed that as the moment increases the reinforcement bars reach their design resistance before the top row of bolts. High-ductility reinforcement, as defined in EN 1992-1 [10], is to be used and therefore redistribution of internal forces can take place. Thus each bolt-row in turn may reach its resistance, commencing with the top row. [10]

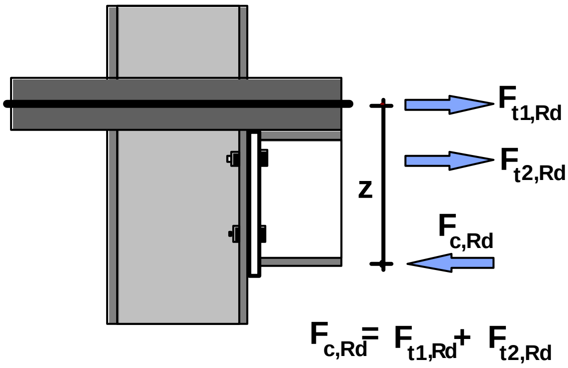


Figure 24 Composite flush end-plate joint

[1] 8.3.3

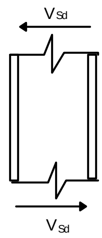
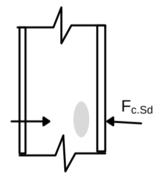
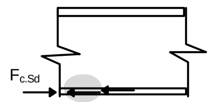
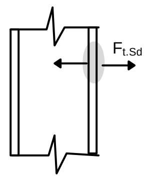
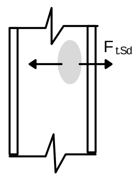
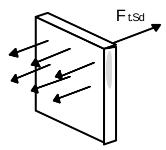
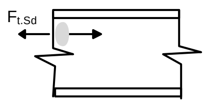
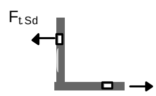
3.3 Resistance of basic components

3.3.1 General

In Table 4 a list of components covered by prEN 1993-1-8 [2] is shown.

For composite joints, the following additional basic components are relevant:

- **Longitudinal slab reinforcement in tension**
- **Contact plate in compression**

N°	Component	
1	Column web panel in shear	
2	Column web in compression	
3	Beam flange and web in compression	
4	Column flange in bending	
5	Column web in tension	
6	End-plate in bending	
7	Beam web in tension	
8	Flange cleat in bending	

[3] Module 15

[2] 5.2.6

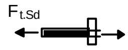
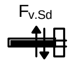
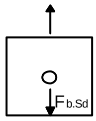
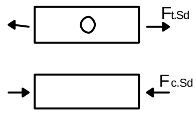
9	Bolts in tension	
10 11	Bolts in shear Bolts in bearing (on beam flange, column flange, end-plate or cleat)	 
12	Plate in tension or compression	

Table 4 List of components covered by prEN 1993-1-8 [2]

3.4 Rotational stiffness

3.4.1 Basic model [2] 5.3

The rotational stiffness of a joint should be determined from the flexibilities of its basic components, each represented by its elastic stiffness coefficient k_i. These elastic stiffness coefficients are of general application. The numbering of stiffness coefficients is consistent with that in prEN 1993-1-8 [2] . The elastic translational stiffness of a component i is obtained by multiplying k_i with E_a.

For connections with more than one layer of components in tension, the stiffness coefficients k_i for the related basic components should be combined.

In a bolted connection with more than one bolt-row in tension, as a simplification, the contribution of any bolt-row may be neglected, provided that the contributions of all other bolt-rows closer to the centre of compression are also neglected. The number of bolt–rows retained need not necessarily be the same for the determination of the moment resistance.

Provided that the axial force N_{Sd} in the connected member does not exceed 10% of the resistance $N_{pl,Rd}$ of its cross-section, the rotational stiffness S_j of a joint, for a moment $M_{j,Sd}$ less than the moment resistance $M_{j,Rd}$ of the joint, may be obtained with sufficient accuracy from:

$$S_j = \frac{E_a z^2}{\mu \sum_i \frac{1}{k_i}} \qquad (14)$$

where:

E_a modulus of elasticity of steel;
k_i stiffness coefficient for basic joint component i;
z lever arm, see Figure 23;
μ stiffness ratio $S_{j,ini} / S_j$, see below;
$S_{j,ini}$ initial rotational stiffness of the joint, given by the expression above with μ=1,0.

The stiffness ratio μ should be determined from the following:

- if $M_{j,Sd} = 2/3\, M_{j,Rd}$ $\mu = 1$ (15)

- if $2/3\, M_{j,Rd} < M_{j,Sd} = M_{j,Rd}$ $\mu = (1{,}5\ M_{j,Sd}\ /\ M_{j,Rd})^{\psi}$ (16)

in which the coefficient ? is obtained from Table 8.2 of EN 1994-1-1 [1] or Table 5 below.

Type of connection	Value of ?
Contact plate	1,7
Bolted end-plate	2,7

Table 5 Value of the coefficient ?

3.4.2 Initial stiffness $S_{j,ini}$

The initial rotational stiffness $S_{j,ini}$ is derived from the elastic translational stiffness of the joint components. The elastic behaviour of each component is represented by a spring. The force-deformation relationship of this spring is given by:

$$F_i = E\, k_i\, w_i \tag{17}$$

F_i the force in the spring i;
E the modulus of elasticity of structural steel;
k_i the translational stiffness coefficient of the spring i;
w_i the deformation of spring i.

Separate rotational stiffness can be derived for the connection and the web panel in shear. In simplified joint modelling a value for the overall joints is all that is required. The derivation of this is now explained.

3.4.2.1 Connections with one layer of components in tension

Contact plate joint:

Figure 25 shows the spring model for a contact plate joint in which tensile forces arising from bending are carried only by one layer of reinforcement.

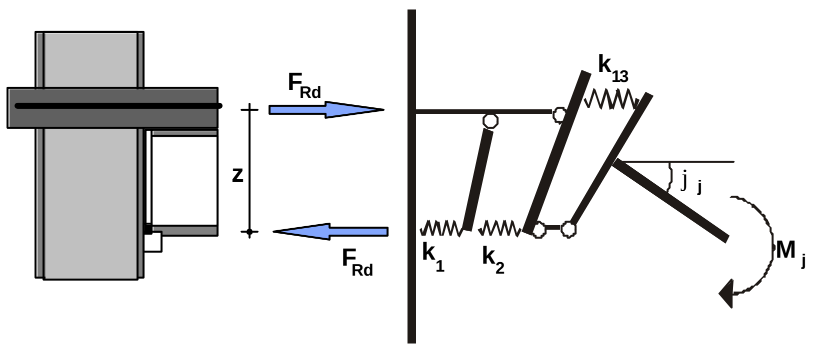


Figure 25 Spring model for a contact plate composite joint

Here, k_1 represents the column web panel in shear, k_2 the unstiffened column web subject to compression from the contact plate and k_{13} the longitudinal reinforcement bars in tension. The contact plate itself is assumed to have infinite stiffness.

The force in each spring is equal to F. The moment M_j acting in the spring model is equal to F·z, where z is the distance between the centroid of the reinforcement in tension and the centre of compression (assumed located in the centre of the lower beam flange). The rotation f $_j$ in the joint is equal to $(w_1+w_2+w_{13})/z$. In other words:

$$S_{j,ini} = \frac{M}{\varphi_j} = \frac{Fz}{\frac{\Sigma w_i}{z}} = \frac{Fz^2}{\frac{F}{E}\Sigma\frac{1}{k_i}} = \frac{Ez^2}{\Sigma\frac{1}{k_i}} \tag{18}$$

3.5 Rotation capacity

It is not usual for designers to calculate either the required or available rotation capacity of structural elements. Rotation capacity is essential though if redistribution of bending moments is assumed in the global analysis. For members, well-known classification systems are used to ensure that adequate rotation capacity is available.
With semi-continuous construction, rotation capacity may be required of the joints rather than the members. As a result, prEN 1993-1-8 [2] gives guidance on joint ductility.

Component models can be used to calculate the rotation capacity of a joint, provided the limiting deformation capacity of each active component is known. For steel joints though it is often sufficient to rely on the observed behaviour of critical joint components. Thus for bolted joints the prEN 1993-1-8 [2] permits the designer to assume sufficient rotation capacity for plastic global analysis provided that the moment resistance of the joint is governed by the resistance of one of the following:

- The column web panel in shear;
- The column flange in bending;
- The beam end-plate in bending.

In the latter two cases, the thickness of the flange or the end-plate must also be limited to avoid fracture of the bolts.
For composite joints, yielding of the slab reinforcement in tension is the main source of predictable deformation capacity. The rotation capacity corresponding to this failure mode can be calculated from a simplified component model.

When plastic global analysis is used, the partial-strength joints should have sufficient rotation capacity. Where necessary, see EN 1994-1-1 [1] 8.2.3.3, full-strength joints should also have sufficient rotation capacity. [1] 8.5 (1) [1] 8.2.3.3

When elastic global analysis is used, joints should have sufficient rotation capacity if the conditions given in EN 1994-1-1 [1] 8.2.3.2 are not satisfied. [1] 8.5 (2) [1] 8.2.3.2

A joint with a bolted connection, in which the moment resistance $M_{j,Rd}$ is governed by the resistance of bolts in shear, should not be assumed to have sufficient rotation capacity for plastic global analysis.

In the case of members of steel grades S235, S275 and S355, the provisions given below may be used for joints in which the axial force N_{Sd} in the connected member does not exceed 10% of the resistance $N_{pl.Rd}$ of its cross-section. However, these provisions should not be applied in the case of members of steel grades S420 and S460.

A beam-to-column joint in which the moment resistance of the joint $M_{j,Rd}$ is governed by the resistance of the column web panel in shear, may be assumed to have sufficient rotation capacity for plastic global analysis. [1] 8.5 (3)

The steelwork parts of a composite joint with a bolted connection with end-plates may be assumed to have sufficient rotation capacity for plastic analysis, provided that both of the following conditions are satisfied: [1] 8.5 (4)

- The moment resistance of the steelwork connection is governed by the resistance of either:
  - The column flange in bending;
  - The beam end-plate in bending.
- The thickness t of either the column flange or beam end-plate satisfies:

$$t \le 0{,}36\ d\sqrt{\frac{f_{ub}}{f_y}} \qquad (19)$$

where:

d is the nominal diameter of the bolts;

f_{ub} is the ultimate tensile strength of the bolts;

f_y is the yield strength of the relevant basic component.

The rotation capacity of a composite joint may be determined by testing. Alternatively, appropriate calculation models may be used. [1] 8.5 (5)

The design rotation capacity determined from a tested structure or element should be adjusted to take account of possible variations of the properties of materials from specified characteristic values. [1] 8.5 (6)

4 Summary

4.1 General

Following configurations and connection types are described

- Double-sided beam-to-column joint configurations
- Single-sided beam-to-column joint configurations

Types of steelwork connections:

- Connections with contact plates
- Connections with partial-depth end-plates
- Connections with flush end-plates

4.2 Field of application of the calculation procedures

- Static loading
- Small normal force N in the beam
  ($N/N_{pl} < 0{,}1$ where N_{pl} is the squash load of the beam)
- Strong axis beam-to-column joints
- In the case of beam-to-beam joints and two-sided beam-to-column joints, only a slight difference in beam depth on each side is possible
- Steel grade: S235 to S355
- H and I hot-rolled cross-sections
- Flush end-plates: one bolt-row in tension zone

- All bolt property classes as in prEN 1993-1-8 [2], it is recommended to use 8.8 or 10.9 grades of bolts
- One layer of longitudinal reinforcement in the slab
- Encased or bare column sections

4.3 Summary of key steps

The calculation procedures are based on the component method which requires three steps:

- Definition of the active components for the studied joint;
- Evaluation of the stiffness coefficients (k_i) and/or strength ($F_{Rd,i}$) characteristics of each individual basic component;
- Assembly of the components to evaluate the stiffness (S_j) and/or resistance (M_e, M_{Rd}) characteristics of the whole joint.

The stiffness (k_i) and the design resistance ($F_{Rd,i}$) of each component are evaluated from analytical models. The assembly is achieved as follows:

Initial stiffness:

$$S_{j,ini} = \frac{E_a \, z^2}{\sum\limits_{i=1,n} \frac{1}{k_i}} \qquad (20)$$

where:

z relevant lever arm;
n number of relevant components;
E_a steel elastic modulus.

Nominal stiffness:

$$S_j = \frac{S_{j,ini}}{1,5}$$ for beam-to-column joints with contact plates

$$S_j = \frac{S_{j,ini}}{2,0}$$ for beam-to-column joints with flush end – plates

Plastic design moment resistance:

$$F_{Rd} = \min\,[F_{Rd,i}] \qquad (21)$$

$$M_{Rd} = F_{Rd} \cdot z \qquad (22)$$

Elastic design moment resistance:

$$M_e = 2/3\, M_{Rd} \qquad (23)$$

4.4 Concluding summary

Conventionally joints have been treated either as pinned or as fully rigid due to a lack of more realistic guidance in view of modelling. In reality both assumptions may be inaccurate and uneconomic and do only represent the boundaries of the real moment-rotation behaviour. They may lead to a wrong interpretation of the structural behaviour in terms of load resistance and deflections. So whereas up to now the joint construction expensively has been adopted to the possibilities of calculation, the new approach is to develop efficient joint types first and to take their realistic behaviour into consideration within the global frame analysis afterwards.

In contrast to the idealising assumptions of beam-to-column connections as hinges or full restraints with the main attention to their resistance, the interest of recent research activities lies in the assessment of the whole non-linear joint response with the three main characteristics.

- **Initial rotational stiffness**
- **Moment resistance**
- **Rotation capacity**

The moment-rotation behaviour of joints can be represented in structural analysis in a practical way:
The joint representation covers all necessary actions to come from a specific joint configuration to its reproduction within the frame analysis. These actions are the:

- **<u>Joint characterisation:</u>** determination of the joint response in terms of M-f curves reflecting the joint behaviour in view of bending moments and shear (in case of moment imbalance).
- **<u>Joint classification</u>:** if aiming at conventional modelling the actual joint behaviour may be compared with classification limits for stiffness, strength and ductility; if the respective requirements are fulfilled a joint then may be modelled as a hinge or as a rigid restraint.
- **<u>Joint idealisation:</u>** for semi-continuous joint modelling the M- f behaviours have to be taken into consideration; depending on the desired accuracy and the type of global frame analysis the non-linear curves may be simplified as bi- or tri linear approximations.
- **<u>Joint modelling</u>:** reproduction (computational model) of the joint´s M- f behaviour within the frame modelling for global analysis.

An analytical description of the behaviour of a joint has to cover all sources of deformabilities, local plastifications, plastic redistribution of forces within the joint itself and local instabilities. Due to the multitude of influencing parameters, a macroscopic inspection of the complex joint by subdividing it into "components" has proved to be most appropriate. In comparison with the finite element method, these components, which can be modelled by translational spring with non-linear force-deformation response, are exposed to internal forces and not to stresses.
The procedure of the COMPONENT METHOD can be expressed in three steps:

- Component **identification**
  determination of contributing components in compression, tension and shear in view of connecting elements and load introduction into the column web panel.
- Component **characterisation**
  determination of the component´s individual force-deformation response with the help of analytical mechanical models, component tests of FE-simulations.
- Component **assembly**
  assembly of all contributing translational component springs to overall rotational joint springs according to the chosen component mode.

Basic components of a joint are described and detailed design provisions for resistance and rotational stiffness are given.

Structural Steelwork Eurocodes

Development of a Trans-National Approach

Course: Eurocode 4

Lecture 9 : Composite joints Annex A

Summary:

- Traditionally structural joints are considered as rigid or pinned
- The lecture is intended to introduce the concept of advanced, semi-rigid joints
- The requirements relating to stiffness, strength and rotation capacity are explained
- The steps of joint representation are shown
- The principles of the component method (including component identification, component characterisation and component assembly) for joint characterisation are presented
- Design provisions for composite joints

Pre-requisites:

- Basic knowledge about frame analysis and design
- Basic definitions and concepts on semi-rigid joints
- Plastic response of frames
- Knowledge of SSEDTA-1 Module 5

Notes for Tutors:

- This is the extended, detailed version with furthermore additional information and background knowledge for designing composite joints.
- In Annex B there are calculation procedures and worked examples for composite joints

References:

- [1] EN 1994-1-1: Design of composite steel and concrete structures Part 1.1
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- [2] prEN 1993-1-8: Eurocode 3: Design of steel structures
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- [3] SSEDTA (1): Structural Steelwork Eurocodes Development of A Trans-national Approach
  Module 5: Structural Joints
- [4] Gerald Huber: Non linear calculations of composite sections and semi-continuous joints
  Doctoral Thesis, Verlag Ernst & Sohn, December1999
- [5] COST C1: Composite steel-concrete joints in frames for buildings: Design provisions
  Brussels, Luxembourg 1999
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  Proceedings of the international conference Liège, 17 to 19 September 1998
- [7] COST C1: Composite steel-concrete joints in braced frames for buildings
  Brussels, Luxembourg 1996
- [8] Teaching Modules of A-MBT,1999/2000
  Application Centre Mixed Building Technology , Innsbruck -Austria,
- [9] ESDEP: European Steel Design Education Programme; CD-ROM 1996
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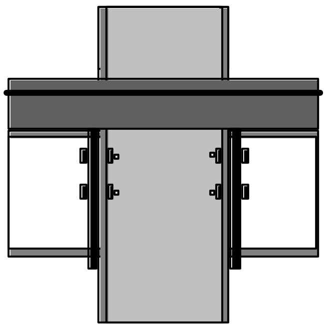
Contents:

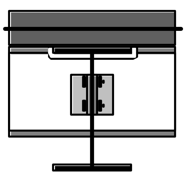
1 Introduction

This lecture concerns building frames with composite steel-concrete beams. The primary aim is to explain how to design joints under hogging bending moment as composite elements.

1.1 Definition and terminology

- **Composite joint** [1] 1.4.2
  A joint between composite members, in which reinforcement is intended to contribute to the resistance and the stiffness of the joint
- **Basic component** (of a joint)
  specific part of a joint that makes an identified contribution to one or more of its structural properties
- **Connected member**
  member that is supported by the member to which it is connected
- **Connection**
  location at which two members are interconnected, and the means of interconnection
- **Joint**
  assembly of basic components that enables members to be connected together in such a way that the relevant internal forces and moments can be transferred between them
- **Joint configuration**
  type or layout of the joint or joints in a zone within which the axes of two or more inter-connected members intersect
- **Structural properties** (of a joint)
  its resistance to internal forces and moments in the connected members, its rotational stiffness and its rotation capacity


Beam-to-column joint


beam-to-beam joint

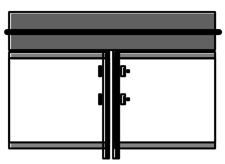

splice

Figure 1 Types of joints

The key feature is the provision of continuous slab reinforcement to act in tension across the joint. This increases substantially both resistance and stiffness for little increase in work on site.

Generally there is a lack of full continuity between the floor system and the column. Frames with composite joints are therefore semi-continuous in nature. This type of framing enables advantage to be taken of the stiffness and moment resistance inherent in many forms of connection whilst avoiding the expense of rigid and full-strength steelwork connections. The recognition of this approach is widely regarded as one of the advances in prEN 1993-1-8 [2], compared to earlier standards.

Conventionally, joints have been treated as nominally pinned without any strength or stiffness (simple joints) or as rigid with full strength (continuous joints). The differences between conventional design and semi-continuous construction can be seen from Figure 2 to Figure 4. The types of framing depend on the joint characteristics, particularly on the initial stiffness and the moment resistance relative to the connected members. The semi-continuous approach provides greater freedom, enabling the designer to choose connections to meet the particular requirements of the structure.

1.1.1 Types of joints

The construction of composite joints used to be based on the already well-known conventional steel joints (see Figure 1 and Figure 5). The composite floor system was connected to the columns by hinged or semi-rigid steel connections, as e.g. welded, angle cleats and flush endplate joints. However a gap separated the concrete res. composite slab from the column to prevent an interaction with the joint. This constructional separation was necessary due to a lack of knowledge in view of modelling the interaction. As the gap had to be provided by expensive constructional means and furthermore the joint’s stiffness and strength were relatively low in comparison to the adjacent composite members this solution was not economic.

Nowadays in advanced composite joints the floor system is integrated into the joint (Figure 5), leading to much higher values of stiffness and strength. This very efficient solution became possible, because the interaction problem between the slab and the column has been solved in comprehensive studies.

Generally one has to distinguish between beams located below the slab (conventional floors) and beams which are already integrated into the slab (slim floors). A further distinction concerns the type of connection before concreting of the slab. In that cases where already a semi-rigid steel connection is provided (e.g. welded, angle cleats, flush or partial depth endplates) the slab supplies additional stiffness and strength after hardening of the concrete. In the very economic case of hinged steel joints during erection, by adding contact pieces and due to the integration of the slab into the joint a high bearing capacity and stiffness can be gained without any further bolting or welding on site (joints with brackets and additional contact plates for the compression transfer or hinged steel joints with fins or angle cleats, see Figure 5). For slim floor decks the authors in any case recommend to design hinged steel joints, which then by concreting are automatically converted to semi-rigid composite joints with considerable resistance and rotation ability.

A further increase of joint stiffness and strength can be obtained by concreting the column sections (encased composite columns). Figure 5 gives an overview of all mentioned joint types using H-shaped column sections. Regarding the fire resistance and also the appearance, hollow column sections, which can be filled with concrete, turned out to be forward-looking.

1.1.1.1 Conventional joints:

- Simple joints

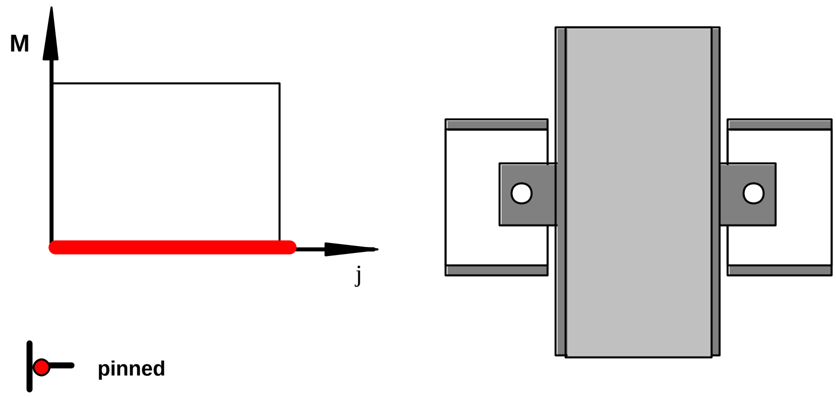


Figure 2 Simple joint (pinned)

- Continuous joints

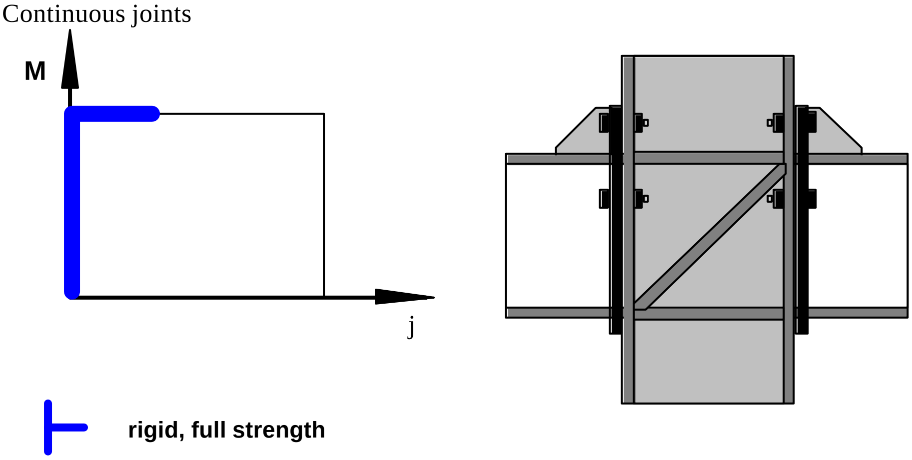


Figure 3 Rigid joint (continuous)

1.1.1.2 Advanced joints

- Semi continuous joints

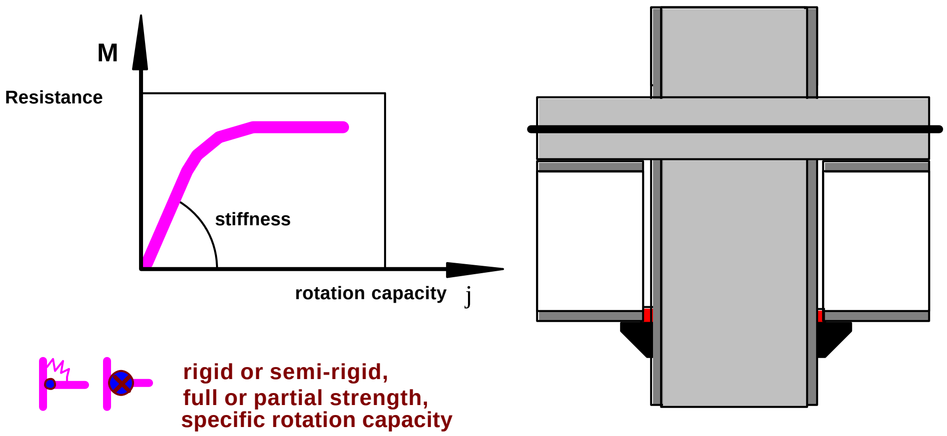


Figure 4 Advanced composite joint (semi continuous)

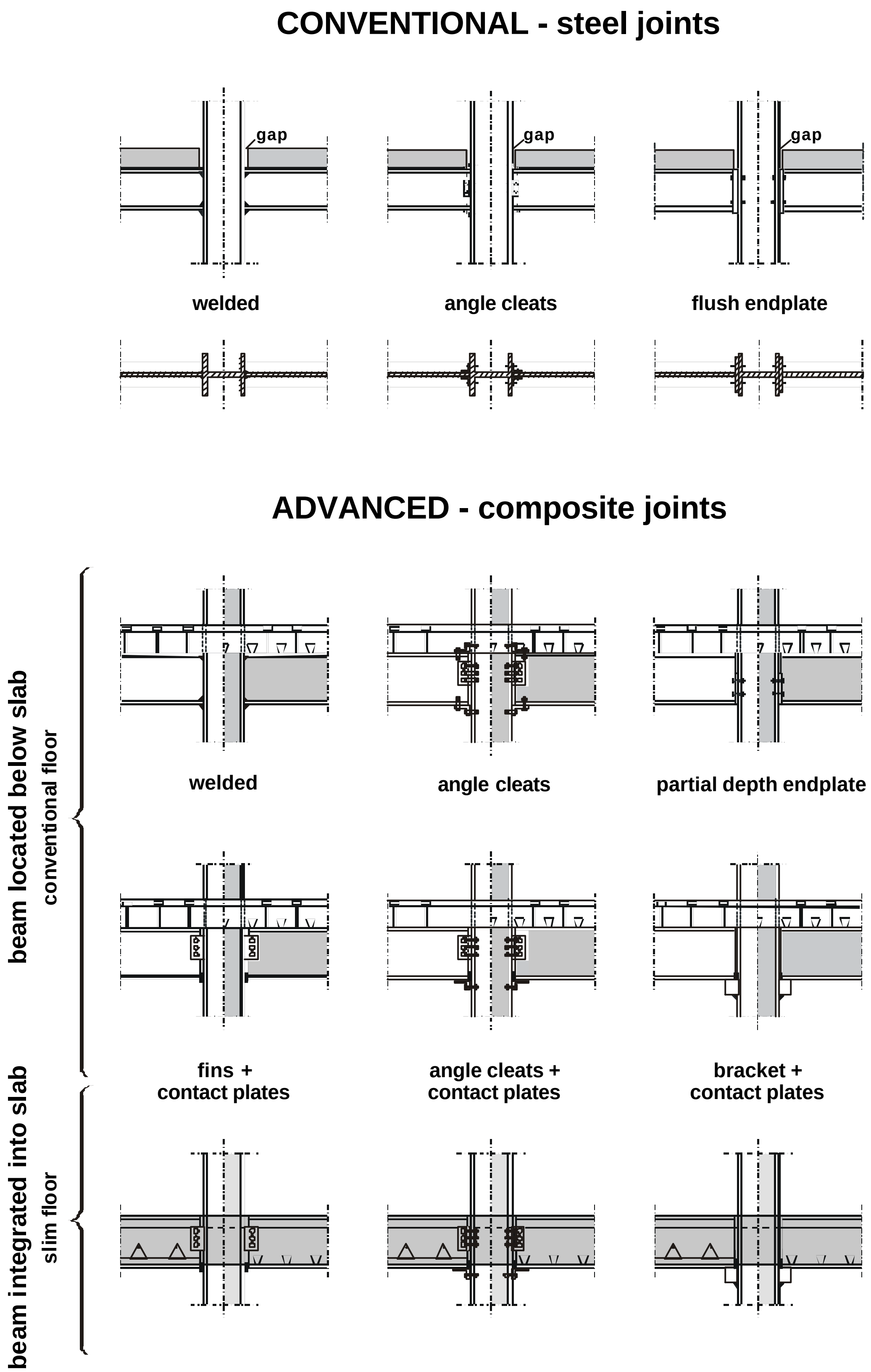


Figure 5 Types of joints- H- shaped column

As semi continuous joints influence the response of the frame to load, they should be modelled in the global frame analysis. Chapter 3 describes suitable methods for doing this.
Traditionally the joints simply have not been considered in the global calculation as a separate element. However as a joint strictly consists of parts of the column, parts of the beams and parts of the slabs, connecting elements and sometimes also includes stiffening elements, so the real behaviour can only be taken into consideration by defining the joint as a separate element (Figure 6) within the structure, additional to the beams and column elements.

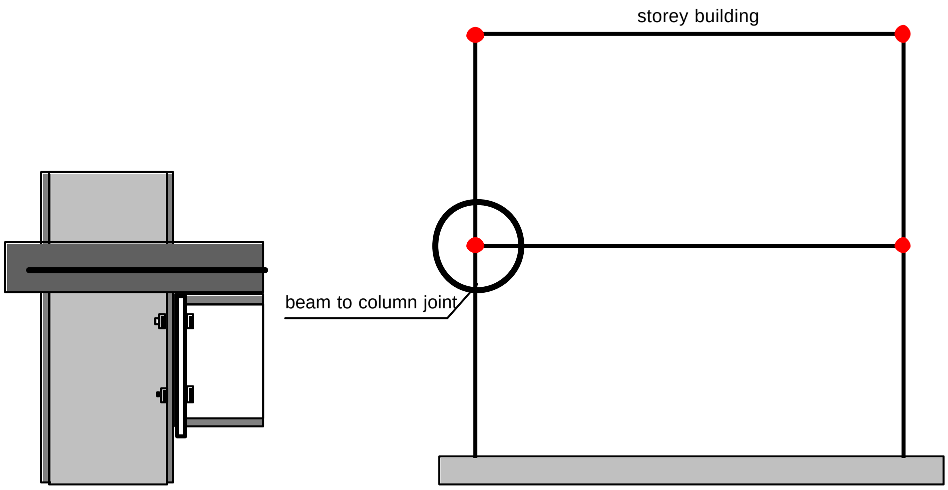


Figure 6 Joint as a separate element within the structure

This enables more efficient constructions but the influence of the joints on the global behaviour is so important that the old-fashioned philosophy of perfect hinges or fully continuous restraints does not describe the real behaviour of a semi continuous joint. (See Figure 7)

It is conventional to simplify the framing as simple or continuous framing, which has the advantage of straight-forward calculation. However with modern design approaches it is appropriate to replace the conventional calculation methods by more advanced ones which treat the joints in a realistic manner. As already noted, semi-continuous construction enables advantage to be taken of the stiffness and moment resistance inherent in many forms of connection, without the expense of forming the rigid and/or full-strength joints necessary for continuous construction.

Although structural behaviour is three-dimensional, the usual presence of stiff floor slabs normally allows the designer to neglect out-of-plane and torsional deformations of the joint. The joint characterisation is therefore usually in the form of a moment rotation relationship (Figure 7).

According to this new approach, joints may be assessed with regard to the following three main characteristics.

- **The initial rotational stiffness $S_{j,ini}$:**

A joint with a very small rotational stiffness and which therefore carries no bending moment is called a hinge. A rigid joint is one whose rigidity under flexure is more or less infinite and which thus ensures a perfect continuity of rotations. In between these two extreme boundaries we speak about semi-rigid joints.

- **The design moment resistance $M_{j,Rd}$:**

In contrast to a hinge, a joint whose ultimate strength is greater than the ultimate resistance (ultimate strength) of the parts whose linkage it ensures is called a full strength joint. Again a partial strength joint represents a middle course between these extremes. (For simplicity from now on "resistance" will mostly be used for the ultimate resistance value; the terms "resistance" and "strength" are used in the Eurocodes with an identical meaning.)

- **The design rotation capacity f_{Cd}:**

Brittle behaviour is characterised by fracture under slight rotation, usually without plastic deformations. Ductile behaviour is characterised by a clear non-linearity of the moment-rotation curve with a large plateau before fracture. It usually indicates the appearance of plastic deformations. The ductility coefficient is the ratio between the ultimate rotation and the elastic rotation limit. Semi-ductility falls in between brittle and ductile behaviour.

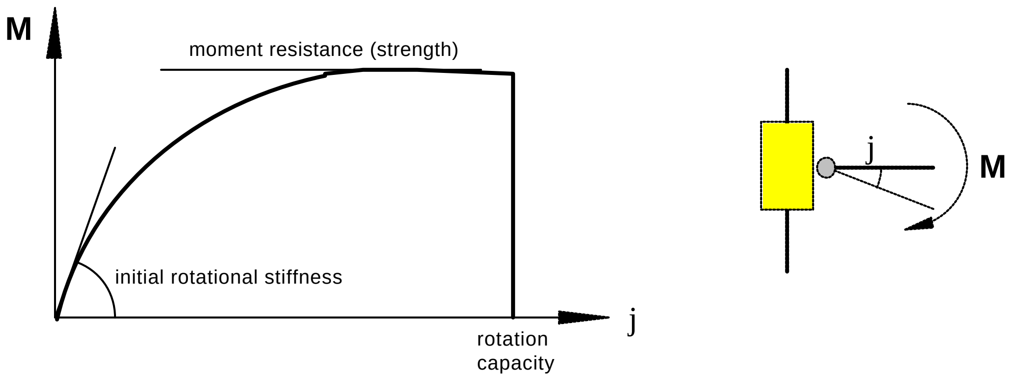


Figure 7 Joint response [4]

Chapter 3 describes how these can be calculated. As is usual in the Eurocodes, partial safety factors are applied to loads and strengths during ultimate limit state verifications, but not to the modulus of elasticity nor to limiting strains.

By analysing full-scale joints it could be seen very quickly that the number of influencing parameters is too large. So world-wide the so-called component method is accepted universally as the best method to describe the joint behaviour analytically. In contrast to the common finite element method (FEM), which often fails to consider local load introduction problems, the joint here is divided into logical parts exposed to internal forces. So while the FEM works on the level of strains and stresses, the component method concentrates on internal forces and deformations of the component springs.

In recent years all over the world extensive testing programs have been performed worldwide for studying the non-linear behaviour of individual components and their assembly to gain the non-linear moment-rotation characteristic of the whole joint formed by these components.

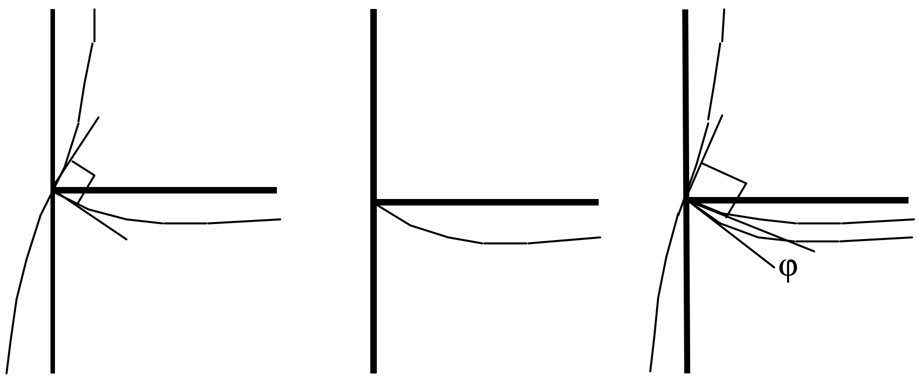
Traditionally, engineering judgement has been used to ensure that joint behaviour approximates to that required for simple and continuous construction. However, the concept of semi-continuous construction requires a more precise statement of what constitutes each joint. In prEN 1993-1-8 [2] this is provided by a classification system based on joint resistance and stiffness. This is extended to composite constructions. Advice is also given on types of connections suitable for each type of construction and on detailing rules.

A consistent approach for structural joints

The rotational behaviour of actual joints is well recognised as being often intermediate between the two extreme situations, i.e. rigid or pinned.

Later in this lecture, the difference between *joints* and *connections* will be introduced. For the time being, examples of joints between one beam and one column only will be used.

Consider now the bending moments and the related rotations at a joint (See Figure 8):



(a) Rigid joint (b) Pinned joint (c) Semi-rigid joint

Figure 8 Relationship of bending moment and rotations at a joint

When all the different parts in the joint are sufficiently stiff (i.e. ideally infinitely stiff), the joint is *rigid*, and there is no difference between the respective rotations at the ends of the members connected at this joint (Figure 8a). The joint experiences a single global rigid-body rotation which is the nodal rotation in the commonly used analysis methods for framed structures.

Should the joint be without any stiffness, then the beam will behave just as simply supported whatever the behaviour of the other connected member (Figure 8b). This is a *pinned* joint.

For intermediate cases (non zero and non infinite stiffness), the transmitted moment will result in a difference f between the absolute rotations of the two connected members (Figure 8c). The joint is *semi-rigid* in this case.

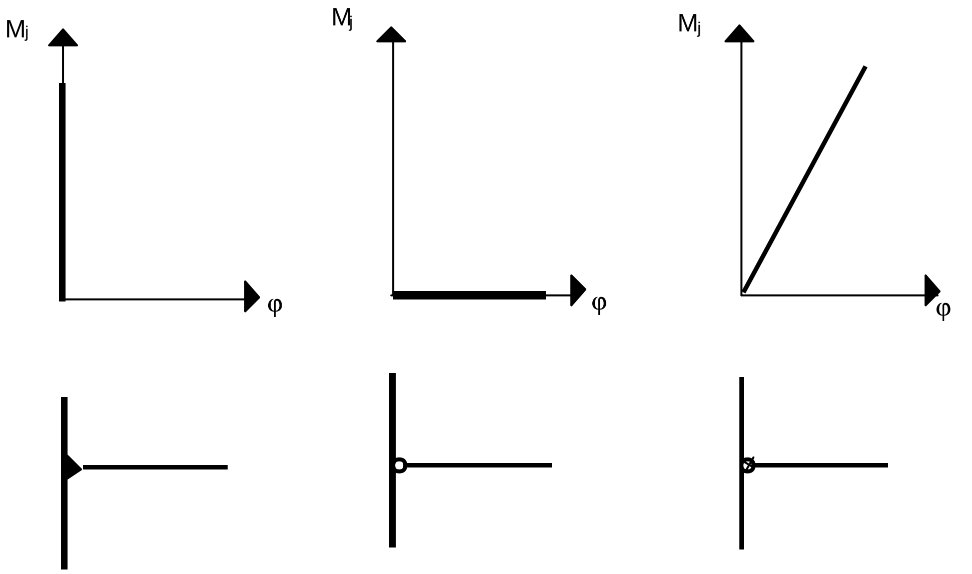
The simplest means for representing the concept is a rotational (spiral) spring between the ends of the two connected members. The rotational stiffness *S* of this spring is the parameter that links the transmitted moment M_j to the relative rotation f , which is the difference between the absolute rotations of the two connected members.

When this rotational stiffness *S* is zero, or when it is relatively small, the joint falls back into the pinned joint class. In contrast, when the rotational stiffness *S* is infinite, or when it is relatively high, the joint falls into the rigid joint class. In all intermediate cases, the joint belongs to the semi-rigid joint class.

For semi-rigid joints the loads will result in both a bending moment M_j and a relative rotation f between the connected members. The moment and the relative rotation are related through a constitutive law which depends on the joint properties. This is illustrated in Figure 9, where, for the sake of simplicity, the global analysis is assumed to be performed with linear elastic assumptions.

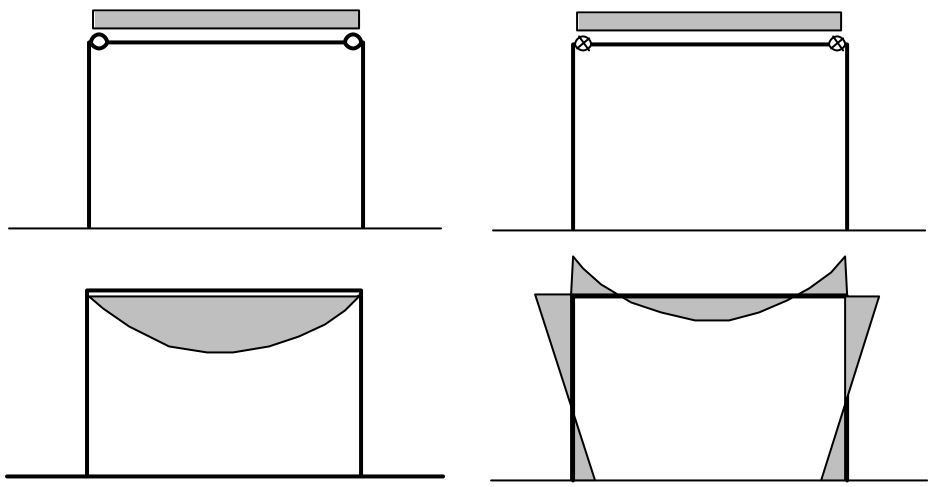
At the global analysis stage, the effect of having semi-rigid joints instead of rigid or pinned joints is to modify not only the displacements, but also the distribution and magnitude of the internal forces throughout the structure.

As an example, the bending moment diagrams in a fixed-base simple portal frame subjected to a uniformly distributed load are given in Figure 10 for two situations, where the beam-to-column joints are respectively either pinned or semi-rigid. The same kind of consideration holds for deflections.



(a) Rigid joint (f = 0) (b) Pinned joint ($M_j = 0$) (c) Semi-rigid joint (M_j and f ≠ 0)

Figure 9 Modelling of joints (case of elastic global analysis)



(a) Pinned joints (b) Semi-rigid joints

Figure 10 Elastic distribution of bending moments in a simple portal frame

1.2 Composite joints for simple framing

Most of the joints in composite frames are currently treated as nominally pinned and the frames are therefore of "simple" construction. This provides the advantage that the global analysis is straightforward because the structure is easier to calculate.

Nominally pinned joints are cheap to fabricate and easy to erect. They may be the most appropriate solution when significant ground settlement is expected. However, deeper beam sections will be required compared to other forms of construction, leading to an increased height of the structure and greater cladding costs. Due to the beam end rotations, substantial cracking can occur in the joint area if the slab is continuous over internal supports.

The joint has to be designed to transfer safely the vertical shear and any axial forces. The resistance of the slab to vertical shear is small and is neglected. As any continuity in the slab is also neglected, a nominally pinned joint for a composite beam is therefore designed in the same way as a simple steelwork connection.

If the slab is constructed as continuous, uncontrolled cracking is permitted by Eurocode 2 if the exposure conditions are Class 1. According to EN 1992-1 [10], most interiors of buildings for offices or normal habitation are in this class, which implies that there is no risk of corrosion of reinforcement. Appearance requires a floor finish with ductile behaviour or provision of a covering. Even so, minimum areas of reinforcement are specified to prevent fracture of the bars or the formation of very wide cracks under service loading. Where cracks are avoided by measures such as the provision of joints in the slab, these should not impair the proper functioning of the structure or cause its appearance to be unacceptable.

In industrial buildings, no additional finish or covering is included and the slab itself provides the floor surface. In such cases continuous or semi-continuous construction is preferable, so that crack widths can be adequately controlled.

Although joint ductility is essential in simple construction, it is not usual for designers to calculate either the required or available rotation capacity. For steel connections it is sufficient to rely on the observed behaviour of joint components.

Rotation capacity can be provided by bolt slip and by designing the components in such a way that they behave in a ductile manner. Local yielding of thin steel end-plates and the use of a wide transverse bolt spacing are examples of this approach. Fracture of bolts and welds should not be the failure mode. However, as the slab reinforcement is assumed to have no influence on the behaviour at the ultimate limit state, fracture of this does not limit the rotation capacity of the joint.

1.3 Joints in semi-continuous construction

Composite moment-resisting joints provide the opportunity to improve further the economy of composite construction. Several possible arrangements for composite joints are shown in Figure 5, demonstrating the wide variety of steelwork connection that may be used. In the interest of economy though, it is desirable that the steelwork connection is not significantly more complicated than that used for simple construction of steel frames. With conventional unpropped construction, the joints are nominally pinned at the construction phase. Later the steelwork connection combines with the slab reinforcement to form a composite joint of substantial resistance and stiffness.

A particularly straightforward arrangement arises with "boltless" steelwork connections. At the composite stage all of the tensile resistance is provided by the slab reinforcement, and no bolts act in tension. The balancing compression acts in bearing through plates or shims, inserted between the end of the lower flange of the beam and the face of the column to make up for construction tolerances. Additional means to resist vertical shear must also be provided, for example by a seating for the beam.

The benefits which result from composite joints include reduced beam depths and weight, improved service performance, including control of cracking, and greater robustness. Besides the need for a more advanced calculation method and the placement of reinforcement, the principal disadvantage is the possible need for transverse stiffeners to the column web, placed opposite the lower beam flange. These are required if the compression arising from the action of the joint exceeds the resistance of the unstiffened column web. Alternatively, stiffeners can be replaced by concrete encasement.

1.4 Scope

In the following, detailed provisions are given for:

- Joints with flush or partial-depth end-plates;
- Joints with boltless connections and contact plates.

The joints are intended for conventional composite beams in which the structural steel section is beneath the slab. It is assumed that the connections are subject to hogging bending moment due to static or quasi-static loading. Profiled steel decking may be used both to form the in-situ concrete and to act as tensile reinforcement, creating a composite slab; alternatively, the slab may be of in-situ reinforced-concrete construction or may use precast units.
Steel sections for columns may be H- or I-shaped and may act compositely with concrete encasement.

2 Joint Representation (General)

Conventionally beam-to-column joints have been treated either as pinned without any strength or stiffness or as fully rigid with full strength due to lack of more realistic guidance in view of joint representation. In reality both assumptions may be inaccurate and uneconomic and do only represent the limiting cases of the real moment-rotation behaviour. They may lead to a wrong interpretation of the structural behaviour in terms of load resistance and deflections. So whereas up to now the joint construction expensively has been adopted to the possibilities of calculation, the new approach is to develop efficient joint types first and to take their realistic behaviour into consideration within the frame analysis afterwards. Due to the interaction between joints and members an overall cost-optimisation is only possible if both design tasks are taken over by the same party. So the designer at least should be able to bring in a first good guess of the joint characteristics depending on the chosen joint configuration.

First discoveries on the importance of joint representation released a real innovation boom, which can be seen by a tremendous number of research activities all over the world.
So e.g. comprehensive research projects on representation and design of structural steel and composite connections and their effect on the frame response have been co-ordinated by the "European Co-operation in the Field of Scientific and Technical Research (COST) [5],[6],[7].

Moment resisting joints have to transfer moments and forces between members with an adequate margin of safety. Their behaviour obviously influences the distribution of moments and forces within the structure. Therefore the list of construction elements as beams, slabs and columns has to be extended by the joints. An overall account of the behaviour would need to recognise its three-dimensional nature. However the presence of rather stiff continuous floor slabs usually allows to neglect out-of-plane and torsional deformations of the joint.
That is why the attempt to describe the connections response can be reduced to a description of the in-plane behaviour. In contrast to the idealising assumptions of beam-to-column connections as hinges or full restraints, with the main attention aimed on their resistance, the interest of actual research activities lies in the assessment of the non-linear joint response of the whole M-

f curve with the following three main characteristics:

- **Initial rotational stiffness**
- **Moment resistance**
- **Rotation capacity**

It is true that the conventional simplifications of continuous or simple framing bring the advantage of a very simple calculation, but reality again lies in between the extreme limiting cases, what especially affects the serviceability limit state and the stability of the whole structure.

The joint representation covers all necessary actions to come from a specific joint configuration to its reproduction within the frame analysis. These actions are the

- **Joint characterisation**: determination of the joint response in terms of M-f curves reflecting the joint behaviour in view of bending moments and shear (in case of moment imbalance)
- **Joint classification**: if aiming at conventional modelling the actual joint behaviour may be compared with classification limits for stiffness, strength and ductility; if the respective requirements are fulfilled a joint then may be modelled as a hinge or as a rigid restraint.
- **Joint idealisation**: for semi-continuous joint modelling the M-f behaviour has to be taken into consideration; depending on the desired accuracy and the type of global frame analysis the non-linear curves may be simplified as bi- or tri linear approximations.
- **Joint modelling**: reproduction (computational model) of the joint´s M-f behaviour within the frame modelling for global analysis.

Figure 11 shows how the moment-rotation behaviour of joints can be represented in global structural analysis in a practical way.

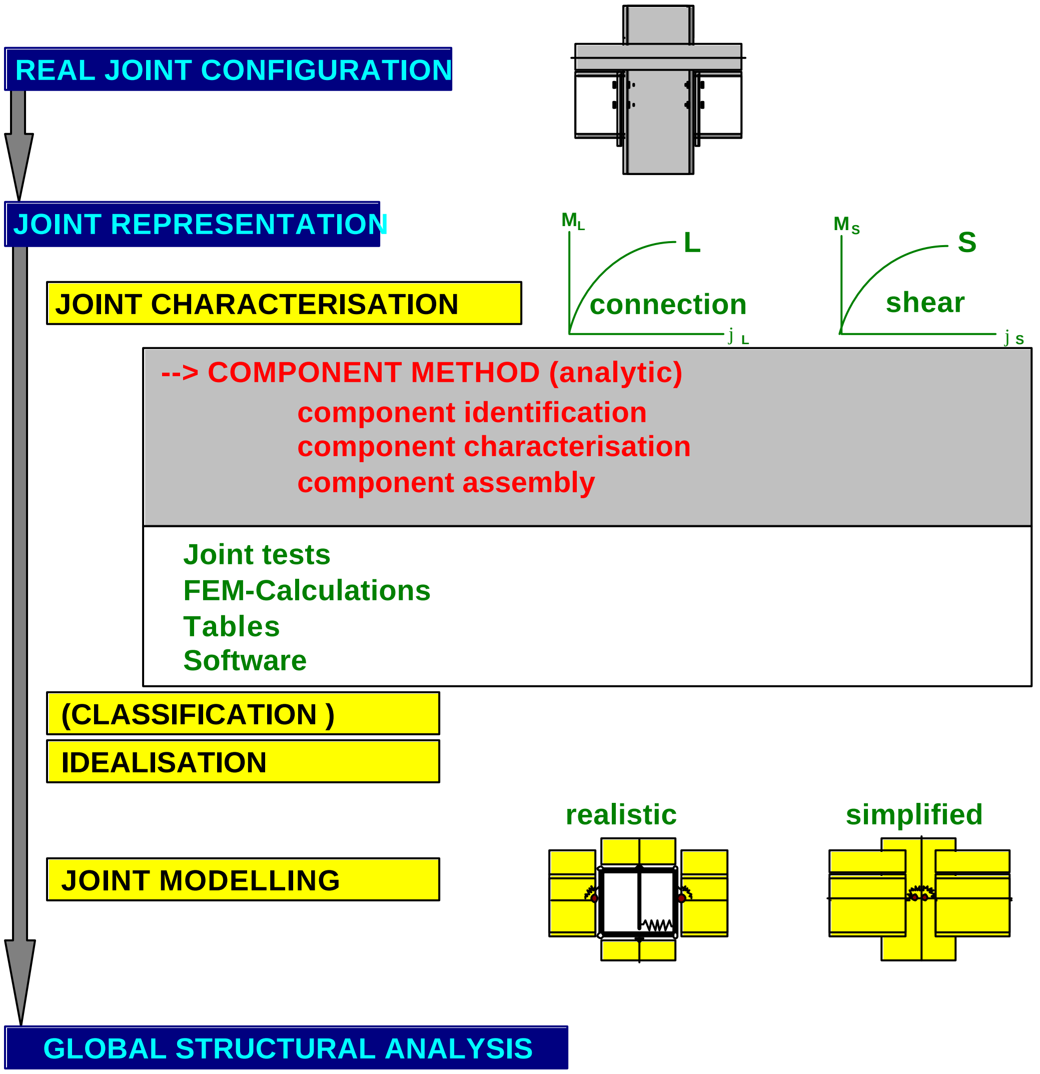


Figure 11 Practical procedure of joint representation

2.1 Joint characterisation

This chapter describes how to derive the M-f curves, representing the necessary input data for the joint models.
To determine these, several possibilities exist:

- Joint tests
- Finite element calculations
- **An analytical approach**

The general background for any joint model comprises three separate curves:

- One for the left hand connection
- One for the right hand connection
- One for the column web panel in shear

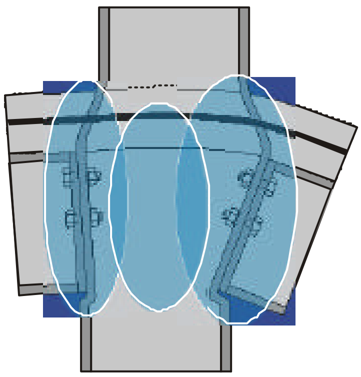


Figure 12 Left connection, column web panel in shear, right connection

2.1.1 Component method

A description of the rotational behaviour of a joint has to take into account all sources of deformations within the joint area. Furthermore all possibilities of local plastic deformations and instabilities have to be covered by such an analytical model. The multitude of influencing parameters was the reason, why the first attempts to develop a component method directly based on full-scale joint tests have been doomed to failure. Also a second research tendency with the aim of finite element modelling did not really succeed because of local detailing problems. So for the analytical determination of the non-linear joint response a macroscopic inspection by subdividing the complex finite joint into so-called simple "**components**" proved to be successful. In contrast to the finite element method (FEM) the components - as logical subsystems of the joint – are exposed to internal forces and moments and not to stresses. As it will be described later in detail these components can be understood as translational springs with a non-linear force-deformation behaviour. All considered components can be tested in clear, relatively cheap so-called "component tests", on the basis of which theoretical models can be developed. Finally the total joint response (reproduced in the "joint model for global frame analysis") can be derived by assembling all influencing components in the correct manner based on the so-called "**component model**". This method offers the advantages of a minimum of testing costs, clear physical calculation models for the components and a maximum of flexibility for the designer, who is able to combine a multitude of available components to a most economic joint configuration. Numerous full-scale joint tests confirm this approach.

In Figure 13 the development of joint modelling is shown. Conventionally rigid or hinged joints of infinite small size have been assumed in the global analysis. But as any real joint has a finite size, deformations occur under relevant member forces. So in the traditional view the beam and column stubs within the realistic joint area (b_j, h_j according to Figure 13) already can be understood as an attempt to model the deformation of the real joint. The first step of improvement was to regard the joint as a separate element of finite size. The second step was to describe the force-deformation behaviour of all individual components by non-linear translational springs. Setting together a joint out of these components, taking into consideration their location and compatibility conditions, results in the component model.

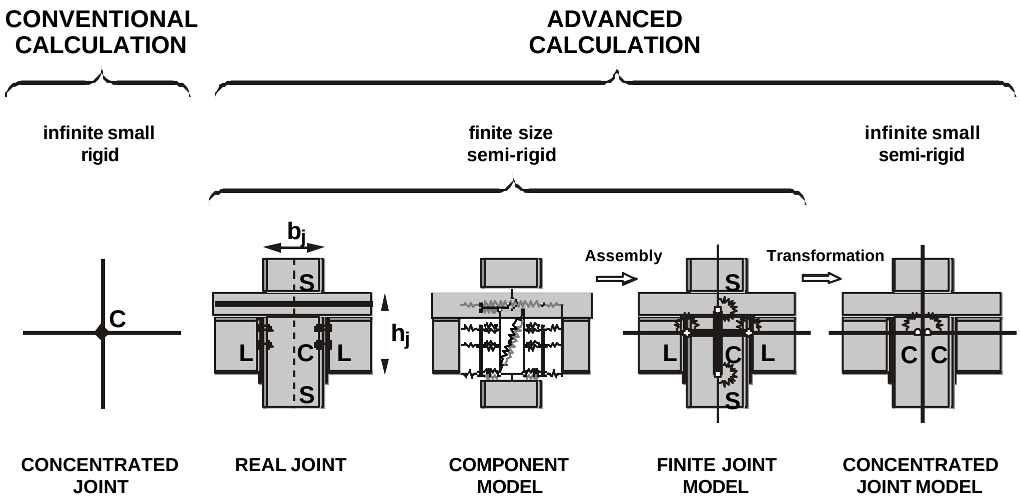


Figure 13 Modelling of joints – conventional and advanced

For a further treatment it is important to define three different locations within the finite joint area:

- **C** represents the Centre point of the joint located at the intersection between beam and column axes.
- **L** is the Loading point located at the centre of rotation in view of tension and compression at the front of the column flange (simplifying L can be assumed at the level of the beam axis).
- **S** is the Shear point located at the top and bottom of the joint's lever arm z along the column axis.

In the third step the components are assembled to rotational springs in L and S forming the finite joint model, which then can be used in the advanced global analysis. There the joint region (b_j, h_j) is set infinite stiff and all deformations are assigned to the flexural springs at the joint edges (L and S). A further possibility in advanced global analysis is to concentrate the full joint's flexibility within flexural springs at the axes intersection points C (separately for the left and right hand side). Doing so the joint again is reduced to an infinite small point within the global analysis, however this requires a transformation from the finite joint model to this concentrated joint model.

An analytical description of the behaviour of a joint has to cover all sources of deformabilites, local plastifications, plastic redistribution of forces within the joint itself and local instabilities.

Due to the multitude of influencing parameters, a macroscopic inspection of the complex joint, by subdividing it into “components”, has proved to be most appropriate. In comparison with the finite element method, these components, which can be modelled by translational spring with non-linear force-deformation response, are exposed to internal forces and not to stresses.
The procedure can be expressed in three steps:

- Component **identification**
  determination of contributing components in compression, tension and shear in view of connecting elements and load introduction into the column web panel.

- Component **characterisation**
  determination of the component´s individual force-deformation response with the help of analytical mechanical models, component tests or FEM-simulations.

- Component **assembly**
  assembly of all contributing translational component springs to overall rotational joint springs according to the chosen component model.

2.1.2 Component models (spring models)

The component model for steel joints and later that for composite joints, has been developed on the basis of considerations of how to divide the complex finite joint into logical parts exposed to internal forces and moments and therefore being the sources of deformations. So in horizontal direction one has to distinguish between the connecting and the panel zones, in vertical direction between the tension, the compression and the shear region, all together forming six groups as illustrated in Figure 14.

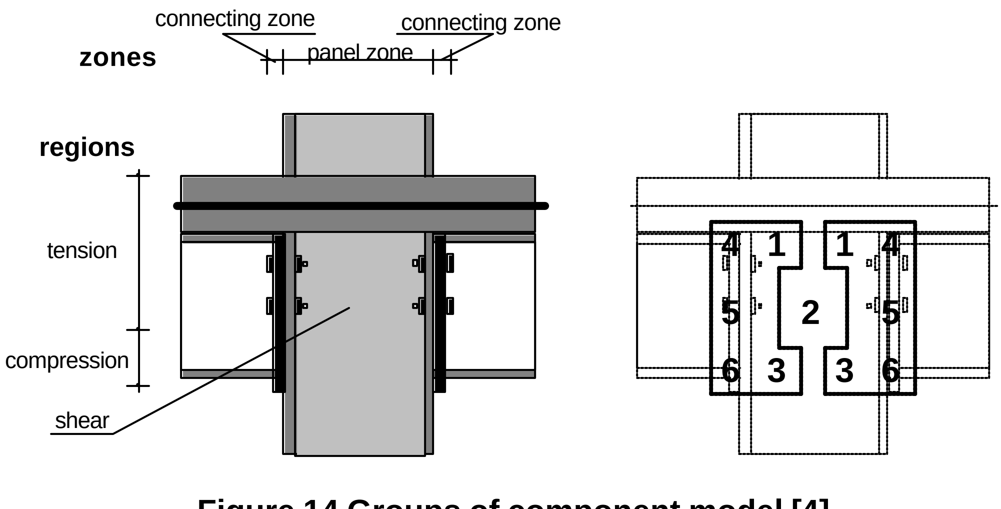


Figure 14 Groups of component model [4]

Group	
1	panel in tension
2	panel in shear
3	panel in compression
4	connection in tension
5	vertical shear connection
6	connection in compression

Table 1 Groups of component model

According to prEN 1993-1-8 [2] a basic component of a joint is a specific part of a joint that makes-dependent on the type of loading-an identified contribution to one or more of its structural properties. When identifying the contributing components within a joint one can distinguish between components loaded in tension (or bending), compression and shear. Beside the type of loading one can distinguish between components linked to the connecting elements and those linked to load-introduction into the column web panel (both are included in the connection) and the component column web panel in shear. The nodal subdivision for the most general case of a composite beam-to-column joint leads to a sophisticated component model like that developed in Innsbruck (Figure 15). There, the interplay of the several components is modelled in a very realistic way. Any composite joints (beam–to-column, beam-to-beam or beam splice), even steel joints can be derived as being only a special case of this general model. The sophisticated component model leads to a complex interplay of components and therefore to iterations within the joint characterisation itself. For simplifications concerning the component interplay a simplified component model has been developed (Figure 15).

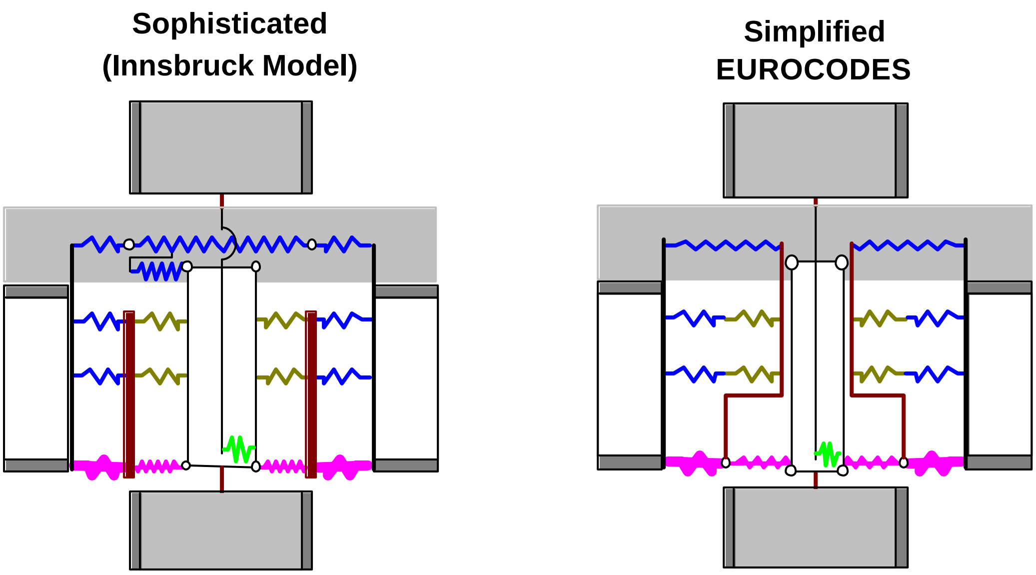
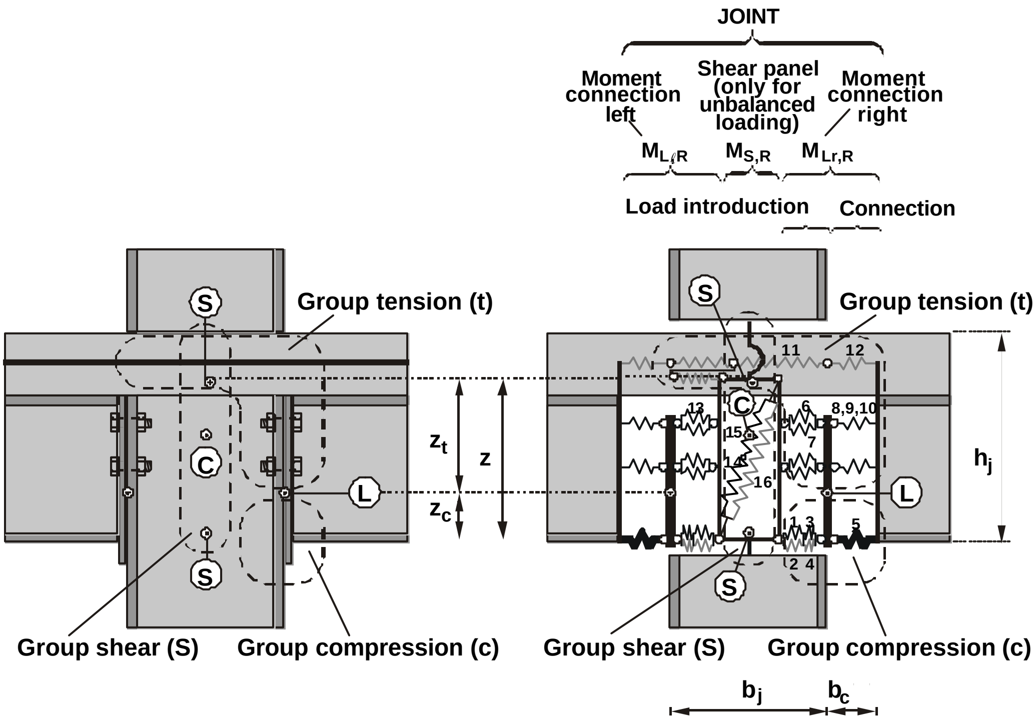


Figure 15 Sophisticated and simplified spring models



i	COMPONENT	GROUP
1	interior steel web panel	com-presssion
2	concrete encasement	
3	exterior steel web panel (column flange and local effects)	
4	effect of concrete encasement on exterior spring	
5	beam flange (local effects), contact plate, end plate	
6	steel web panel incl. part of flange, fillet radius	tension
7	stiffener in tension	
8	column flange in bending (stiffened)	
9	end plate in bending, beam web in tension	
10	bolts in tension	
11	reinforcement (within panel) in tension	
12	slip of composite beam (due to incomplete interaction)	
13	redirection of unbalanced forces	
14	steel web panel in shear	shear
15	steel web panel in bending	
16	concrete encasement in shear	

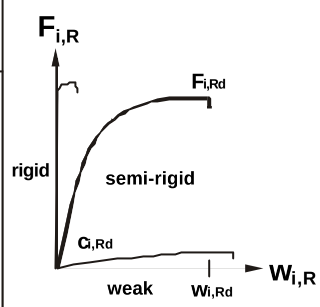


Figure 16 Refined component model

Through familiarity with the spring model (component model), which represents the interplay of all deformation influences, the principles of economic joint construction with respect to stiffness, resistance failure modes and ductility can be logically derived. Assuming a specific component with a given resistance and deformation ability, it is self-evident that by increasing its lever arm

the joint´s moment resistance increases whilst the rotation capacity decreases.
Providing resistance in the compression region far beyond the resistance of the tension region is uneconomic. Similarly strengthening the connecting elements will not make the joint capable of sustaining more load, if failure is already dominated by the column web in tension, compression or shear. It also can be recognised immediately that it makes no sense to combine a relatively weak and ductile slab reinforcement with a stiff and brittle steelwork connection.
These examples give ideas on how to use the component model directly for a qualitative plastic redistribution within the joint itself, all components are used up to their plastic resistance and provide a further yield plateau for redistribution of moments within the whole frame.

Figure 16 shows the real joint situation on the left side and the corresponding component model on the right. The method of component modelling started with beam-to-column joints in steel. The bolt-rows in tension are modelled by corresponding spring rows, separating the influences of *load-introduction* within the column web panel and the *connection* to the column flange. The same philosophy has been followed for the compression. For an unbalanced loaded joint, the deformations due to the shear have to be modelled by an additional shear spring.

It has been proved by tests on full scale joints that the measured moment-rotation behaviour is in good agreement with the calculated curves using the component method. Out of this excellent experience the component method has been extended to composite joints, where only the additional concrete components had to be analysed. Thus the composite component model can be seen as the most general tool covering all considerable joint types. In this sense steel joints or special types of joints like splices, beam-to-beam joints or weak axis joints can be regarded as special cases of this general model (see Figure 17).

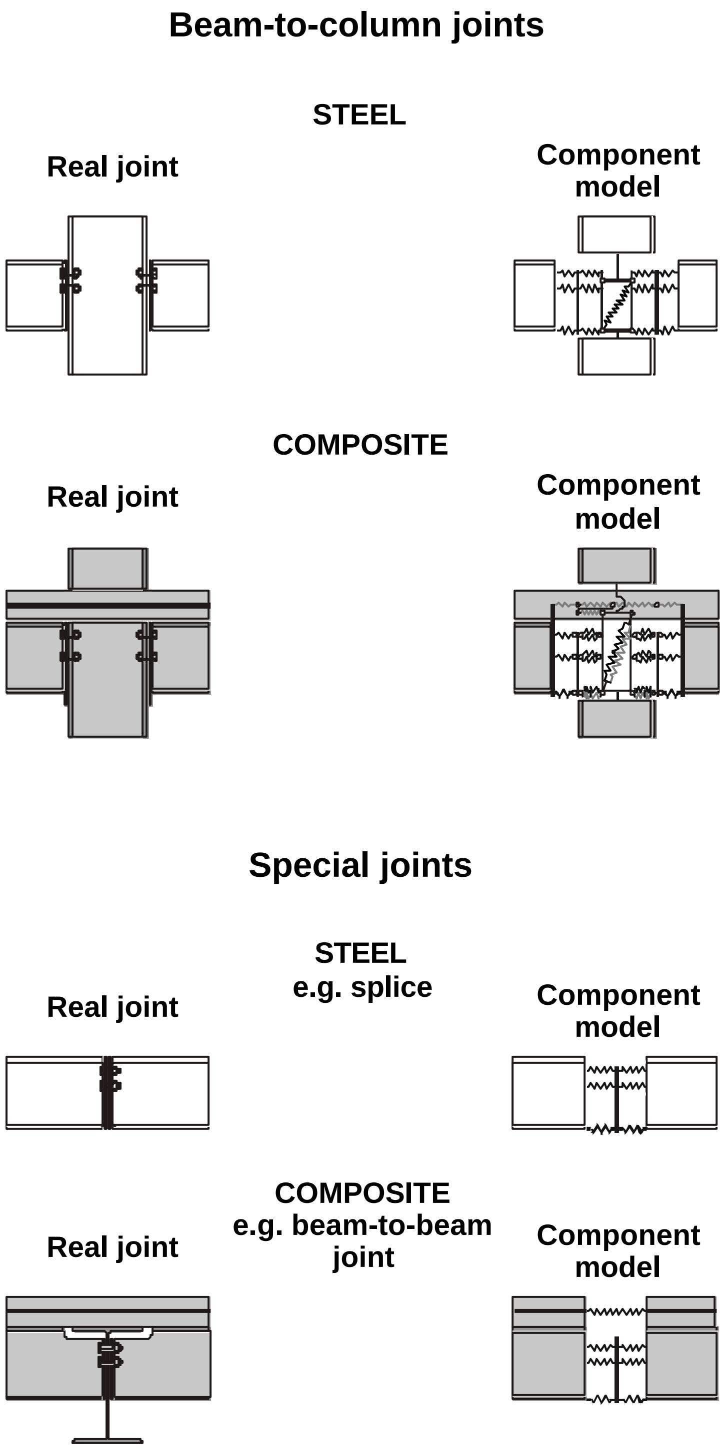


Figure 17 Modelling of joints – examples

2.1.3 Component assembly

The transfer from the force-deformation curves of the individual basic joint components to the moment-rotation curves representing the connection or the shear panel has to be done based on the component modes fulfilling the requirements of compatibility and equilibrium. Doing so it is assured that the joint model behaves exactly in the same way than the complex component model with respect to applied moments. Depending on the intended level of accuracy the assembly can be done for the main rotational key values only (initial rotational stiffness, moment resistance, rotation capacity) or for the full shape of the resulting M-f curves.

As already mentioned the assembly of the sophisticated component model leads to iteration loops due to the complex interplay of components.
For simplification, iterations can be avoided by using the simplified component model used in the Eurocodes, where the sum of all basic component springs can be derived by adding them step by step acting parallel or in series. (See Figure 18and Figure 19)

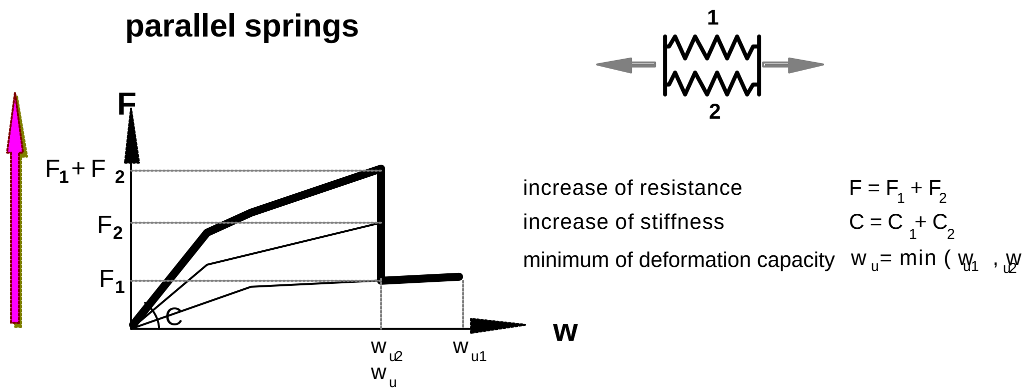


Figure 18 Assembly of parallel spring groups

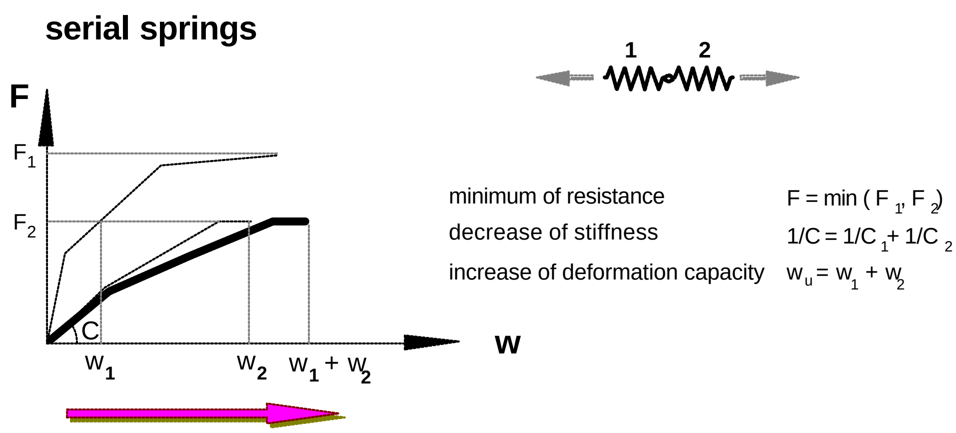


Figure 19 Assembly of serial spring groups

A component model for a composite beam-to-column joint configuration is shown in Figure 16. Sixteen different components have been identified and these may be grouped as compression, tension and shear. The assembly of components to determine the structural properties is now explained.

2.1.3.1 Linear grouping

Individual components are represented by translational springs. The first step is therefore to group components which act together directly in parallel or in series. Translational stiffness, resistance and deformation capacity are considered separately. Each of the regions in compression, tension or shear is also considered separately. The assembly is illustrated in Figure 20, in which the translational stiffness is denoted by c_i, defined by:

$$c_i = F_i / w_i \tag{1}$$

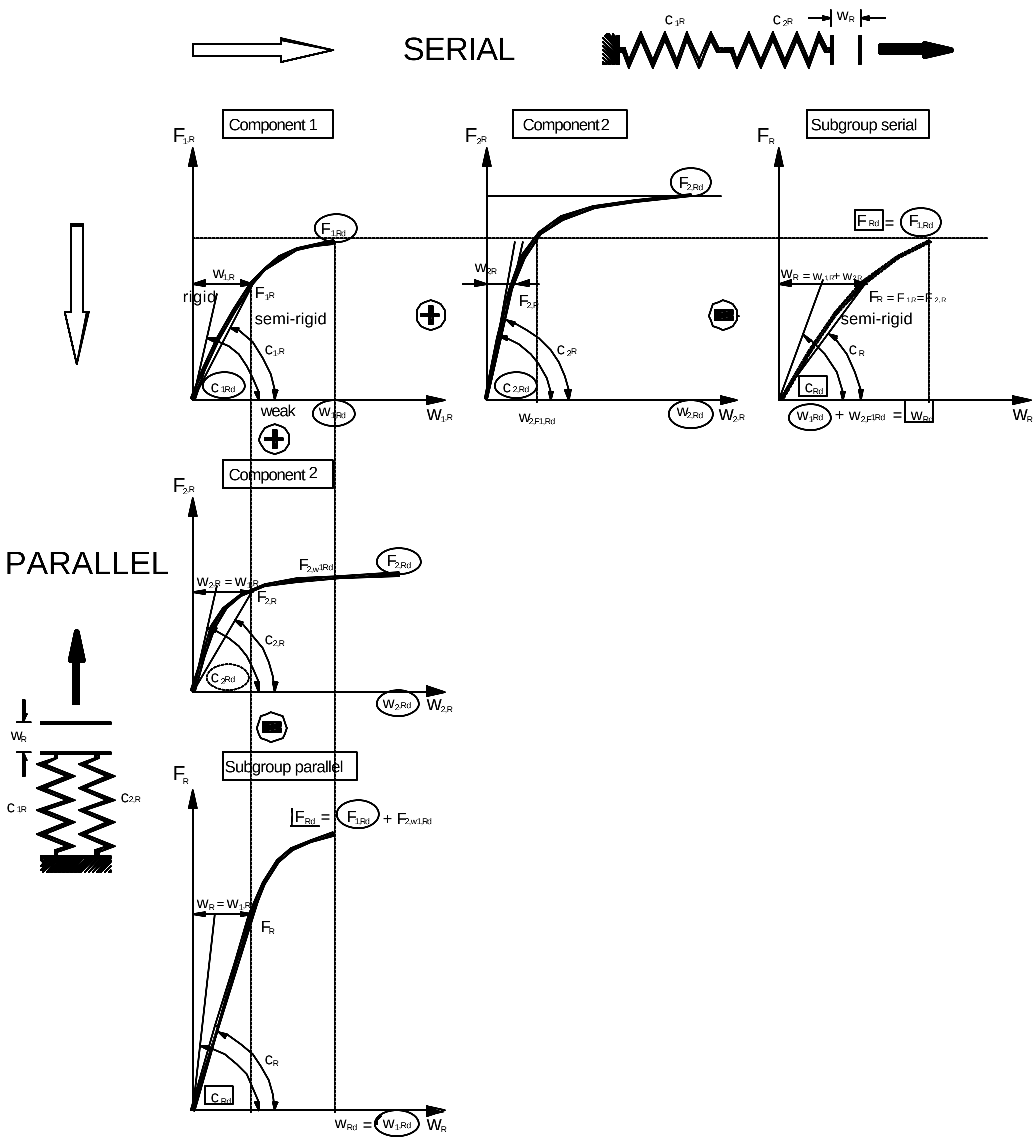
The stiffness coefficients k_i (see 3.5.3) and used elsewhere in this document, are related to c_i by:

$$k_i = c_i / E_i \tag{2}$$

For composite joints it would be more convenient to use a stiffness c_i rather than a coefficient k_i. This is because a composite joint includes more than one material. Despite this, stiffness coefficients are used elsewhere in this lecture, to avoid differing from prEN 1993-1-8 [2].
For components acting in parallel - for example, the column web of a steel column section and concrete encasement- the initial stiffness and resistances can be added. However, the smallest of the deformation capacities governs that of the group. Such behaviour is represented by the diagrams in the lower part of Figure 20.

For components effectively acting in series - for example, a single bolt-row comprising the end-plate in bending, the bolts themselves in tension and the column flange in bending- the initial stiffness is obtained by a reciprocal relationship, but the resistance is that of the weakest component. The deformation capacity is the sum of the weakest component´s capacity and the corresponding deformations of the other components at that load level. Such behaviour is represented by the diagrams in the upper part of Figure 20.

For the regions in compression and shear, the linear grouping results in one effective translational spring per group, each with its own translational stiffness, resistance and deformation capacity. However, the tension region may consist of several rows of springs, each representing a layer of longitudinal slab reinforcement or a bolt-row. This may also be reduced to one equivalent translational spring, as shown in Figure 21 by considering the rotational behaviour of the joint.



CHARACTERISTICS	SUBGROUPS (resp. GROUPS)	
	PARALLEL	SERIAL
INITIAL STIFFNESS c_{Rd}	$c_{1,Rd} + c_{2Rd}$	$1/(1/c_{1,Rd} + 1/c_{2,Rd})$
DESIGN RESISTANCE F_{Rd}	$F_{1,Rd} + F_{2,w1,Rd}$	$F_{1,Rd}$
DEFORMATION CAPACITY w_{Rd}	w_{1Rd}	$w_{1Rd} + w_{2,F1,Rd}$

Figure 20 Linear grouping

2.1.3.2 Rotational grouping

It is assumed in the simplified Eurocode component model that the centre of rotation for all tension rows is located at the centre of the flange of the beam. This is valid for connections with relatively stiff load-introduction in compression. It can be readily shown that the rotational stiffness $S_{eff,i}$ is related to the translational stiffness $c_{eff,i}$ by:

$$S_{eff,i} = c_{eff,i}\, z_i^2 \tag{3}$$

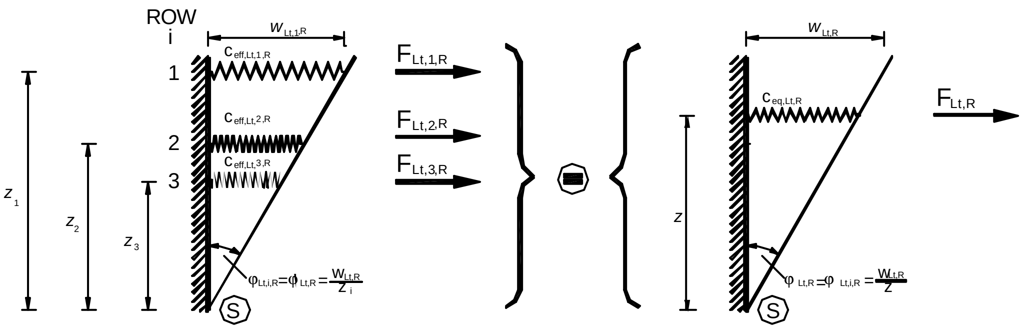
where:

z_i is the distance from the centre of rotation to the effective spring i.

It is also required that the moment-rotation behaviour of each of the systems shown in Figure 21 is equal. An additional condition is that equilibrium of forces is maintained. As a result the effective stiffness of the tension rows can be replaced by a single equivalent translational stiffness c_{eq} at an equivalent lever arm z. The formulae are given in Figure 21, where the derivation is also summarised.

This figure also gives the resulting expression for resistance and deformation capacity of the tension region, assuming as a particular case that the second tension row limits the deformation capacity.

The second (and final) stage in the rotational grouping converts the translational properties into moment-rotation relationships at S and L (the edges of the finite joint model defined in Figure 13). The conversion is illustrated in Figure 21 in which the resulting formulae are also given at the bottom left. The effective translational stiffness for the shear region is converted into a flexural spring by multiplication with z^2. The effective translational stiffness for the compression region and the equivalent translational stiffness for the compression region are added in series; the resulting translational stiffness is then converted to a rotational spring by multiplication with z^2.



CHARACTERISTICS	ROW i=1,2,3
INITIAL STIFFNESS	$S_{eff,Lt,i,R} = c_{eff,Lt,i,R} * z_i^2$
DESIGN RESISTANCE	$M_{Lt,i,R} = F_{Lt,i,R} * z_i$
DEFORMATION CAPACITY	$\varphi_{Lt,i,R} = w_{Lt,i,R} * \frac{1}{z_i}$

1.CONDITION: $\sum_i M = 0$ $\sum_i M_{Lt,i,R} = M_{Lt,i,R}$	2.CONDITION: $\sum_i H = 0$ $\sum_i F_{Lt,i,R} = F_{Lt,i,R}$
$\sum_i \overbrace{\overbrace{\overbrace{\varphi_{Lt,i,R} * z_i}^{w_{Lt,i,R}} * c_{eff,Lt,i,R}}^{F_{Lt,i,R}} * z_i}^{M_{Lt,i,R}} = \overbrace{\overbrace{\overbrace{\varphi_{Lt,R} * z}^{w_{Lt,R}} * c_{eff,Lt,R}}^{F_{Lt,R}} * z}^{M_{Lt,R}}$ (1)	$\sum_i \overbrace{\overbrace{\varphi_{Lt,i,R} * z_i}^{w_{Lt,i,R}} * c_{eff,Lt,i,R}}^{F_{Lt,i,R}} = \overbrace{\overbrace{\varphi_{Lt,R} * z}^{w_{Lt,R}} * c_{eff,Lt,R}}^{F_{Lt,R}}$ (2)

For (1) and (2):

$$z = \frac{\sum_i c_{eff,Lt,i,R} * z_i^2}{\sum_i c_{eff,Lt,i,R} * z_i} \quad \text{and} \quad c_{eq,Lt,R} = \frac{\sum_i c_{eff,Lt,i,R} * z_i^2}{z^2}$$

CHARACTERISTICS	Assumption: Row 2 has the smallest rotation ability		
INITIAL STIFFNESS	$c_{eq,Lt,Rd}$ =	$\sum_i c_{eff,Lt,i,R} * z_i^2$	$* \frac{1}{z_i^2}$
DESIGN RESISTANCE	$F_{1,Rd}$ =	$(F_{Lt,2,Rd} * z_2 + \sum_{i=1,3} F_{Lt,i,w_{Lt,2,Rd},R} * z_i)$	$* \frac{1}{z_i}$
ROTATION CAPACITY	$W_{Lt,Rd}$ =	$w_{Lt,2,Rd} * \frac{1}{z_2}$	$* z$

Figure 21 Grouping of tension springs

2.1.3.3 Rotational stiffness

It will be noted that these rotational stiffnesses represent the deformability of the column web panel in shear separately from the other sources of deformation. For a single-sided joint configuration, the total rotational stiffness can be expressed directly in terms of the effective translational stiffness in shear and compression and the equivalent translational stiffness in tension:

$$S_{j,ini} = z^2 \sum \frac{1}{c_i} \tag{4}$$

where c_i represents the effective or equivalent stiffness of the region i.

For a double-sided joint configuration, the extent of the shear in the column web panel is influenced by the ratio of unbalance between the moments at the two connections. In prEN 1993-1-8 [2] this influence is included through a parameter ß (see Table 4).

2.1.3.4 Design moment resistance

For resistance calculation, Figure 21, has shown the tension region as limited by the deformation capacity of the second row. Recommendations to ensure ductile behaviour in steel joints are given in prEN 1993-1-8 [2]. These are applicable to steelwork parts of composite joints.
Detailing rules given in this document avoid brittle failure modes associated with composite action, provided that high-ductility reinforcement is allocated for the composite joint.
Provided a plastic distribution of bolt forces is allowed by prEN 1993-1-8 [2], the design moment of resistance M_{Rd} may be expressed as:

$$M_{Rd} = \sum F_{Lt,i,Rd} \; h_i \tag{5}$$

where the summation is over all rows of longitudinal slab reinforcement and bolt-rows in the tension region.
In practice though the resistance of the connection in compression or of the web panel in shear may be lower than that of the group of components in tension. For equilibrium the total tensile force $\sum F_{Lt,i,Rd}$ must not exceed the design resistance of the compression group $F_{Lc,Rd}$ and the shear resistance $V_{S,Rd}$ / ß. If this condition is reached at a tension row i, the contribution to the moment resistance of all other tension rows closer to the centre of compression is neglected.

2.1.3.5 Transformation of joint characteristics

The procedures described above provide moment-rotation relationships at S and L (the edges of the finite joint model defined in Figure 13). In such models the connection springs are arranged at the edges of the finite joint area, as shown in Figure 29. In the simplified joint model, the combined joint springs for shear and connections are located at the intersection C of the beam and column axes. It is obvious that the acting bending moment usually increases from the joint edge to the axes intersection point. The consideration of the higher value for the applied moment therefore leads to a more conservative design. On the other hand, the weakness of the "extended" beam and column stubs within the finite joint area adds an extra rotation to the joints, leading to an overestimation of the global frame deformation.
These effects can be compensated by transforming the properties which apply at L and S to properties at C.

2.1.3.6 Moment resistance

It has to be demonstrated that the internal moments due to the design loads, obtained by global analysis, do not exceed the corresponding resistance values. However, the moment resistance of a joint relates to the point L at the periphery of the finite joint area (see Figure 29). So strictly only the internal moment at the face of the column, not that at the column centre-line, has to be compared with the joint's resistance. The difference in internal moment might be important for short-span beams with deep column profiles.

2.1.3.7 Stiffness

Two approaches have been proposed to allow for the additional flexibility which results from extending the beam and column into the joint area:

- <u>Transformation of rotational stiffness</u>

By assembling appropriate components, rotational stiffnesses are obtained to represent load-introduction and the connection at L and shear deformation at S. These are then transformed separately from L to C and S to C, taking account of the beam and columns stubs introduced by the simplified model. The transformed stiffness are then combined to obtain the total rotational stiffness at C.

The transformation from S to C:

$$S_{column} = 2\,\frac{f}{z}\,E\,I_c \qquad (6)$$

where:
$f = 1$ for a joint at the top of the column;
$f = 2$ for a joint within a continuous length of the column;
I_c is the second moment of area of the adjacent column section;
z is the lever arm of the joint.

For transformation from L to C:

$$S_{beam} = \frac{E\,I_b}{L_j} \qquad (7)$$

where:
I_b is the second moment of area of the adjacent beam section in hogging bending;
L_j is half of the depth of the column section (see Figure 30).

- <u>Transformation of component stiffness</u>

As a further simplification, stiffness of individual components can be calibrated to include the flexibility of the beam and column stubs. Thus quasi-transformed stiffness values can be given in design codes, leaving the designer to assemble the components according to the joint configuration. The result is a total rotational stiffness for the concentrated joint model at C.

2.1.4 Basic components of a joint

The design moment-rotation characteristic of a joint depends on the properties of its basic components. [2] 5.1.4

The following basic joint components are identified in this lecture:
- Column web panel in shear
- Column web in compression

- Column web in tension
- Column flange in bending
- End-plate in bending
- Beam flange and web in compression
- Beam web in tension
- Bolts in tension
- Longitudinal slab reinforcement in tension
- Contact plate in compression

A major-axis beam-to-column composite connection consists of a combination of some of the basic components listed above, excluding the column web panel in shear. In particular, it always includes the following:

- Column web in compression
- Longitudinal slab reinforcement in tension

Methods for determining the properties of the basic components of a joint are given: [1] 8.1.2

- For resistance in 3.3 [1] 8.3.3
- For elastic stiffness in 3.5.3 [1] 8.4.2

Relationships between the properties of the basic components of a joint and the structural properties of the joint are given:

- For moment resistance in 3.2.2 [1] 8.3.4
- For rotational stiffness in 3.5 [1] 8.4.1
- For rotation capacity in 3.6 [1] 8.5

2.2 Classification of beam-to-column joints

In conventional design of building structures, the framing is treated as simple or continuous, even though practical joints always possess some moment resistance and show some flexibility. Traditionally, engineering judgement has been used to ensure that joint behaviour approximates to that required for these forms of construction.

However, the concept of semi-continuous construction requires a more precise statement of the joint behaviour. In prEN 1993-1-8 [2] this is provided by a classification system based on joint resistance and stiffness. Table 2 shows how the types of joint model (representing behaviour), the form of construction and the method of global analysis are all related. [2] 7.2.2 [2] 7.2.3

Method of global analysis	Classification of joint		
Elastic	Nominally pinned	Rigid	Semi-rigid
Rigid-Plastic	Nominally pinned	Full-strength	Partial-strength
Elastic-Plastic	Nominally pinned	Rigid and full-strength	Semi-rigid and partial -strength Semi-rigid and full -strength Rigid and partial-strength
Type of joint model	Simple	Continuous	Semi-Continuous

Table 2 Type of joint model

2.2.1 Classification by strength

A beam-to-column joint may be classified as full-strength, nominally pinned or partial strength by comparing its moment resistance with the moment resistances of the members that it joins. For this purpose, the moment resistance of the joint is compared with the moment resistance of the connected members. The purpose is to indicate whether the joint or the adjacent member cross-sections will limit resistance. In the first case, the joint is either "partial strength" or "nominally pinned". In the second case the joint is "full-strength". Thus a beam-to-column joint will be classified as full-strength if the following criteria are satisfied.

<u>For a joint at the top of a column:</u>

$$M_{j,Rd} = M_{b,pl,Rd} \quad (8)$$

or:

$$M_{j,Rd} = M_{c,pl,Rd} \quad (9)$$

where:

$M_{b,pl,Rd}$ design plastic moment resistance of the composite beam in hogging bending immediately adjacent to the joint;

$M_{c,pl,Rd}$ design plastic moment resistance of the column.

<u>For a joint within a continuous length of column:</u>

$$M_{j,Rd} = M_{b,pl,Rd} \quad (10)$$

or:

$$M_{j,Rd} = 2\,M_{c,pl,Rd} \quad (11)$$

A joint may be classified as nominally pinned if its moment resistance $M_{j,Rd}$ is not greater than 0,25 times the moment resistance required for a full-strength joint, provided that it also has sufficient rotation capacity.
With composite beams the moment resistance depends on whether the member is in sagging or hogging bending. With braced construction, beam end connections are subject to hogging bending and the moment resistances of joints and beams should therefore be those applicable to this situation. Under such moment, EN 1994-1-1 [1] requires full shear connection and therefore the beam´s resistance should be calculated on this basis. Plastic analysis should be used for sections Class 1 and Class 2, including beam sections with Class 3 webs upgraded to Class 2 using the "hole-in-the web" approach. [1] 5.3

The classification of the joint has implications for its design criteria. With a partial-strength or nominally pinned joint, it is the joint rather than the member section which requires rotation capacity. Tests have shown that rotation capacity may be limited.

2.2.2 Classification by rotational stiffness

A beam-to-column joint may be classified as rigid, nominally pinned or semi-rigid according to its stiffness, by determining its initial rotational stiffness $S_{j,ini}$ and comparing this with classification boundaries.

As shown in Figure 22, the classification given by prEN 1993-1-8 [2] compares stiffness of the joint with that of the connected member. The purpose is to indicate whether account has to be taken of the influence of joint flexibility on the frame response. The classification boundaries were determined from consideration of ultimate limit states.

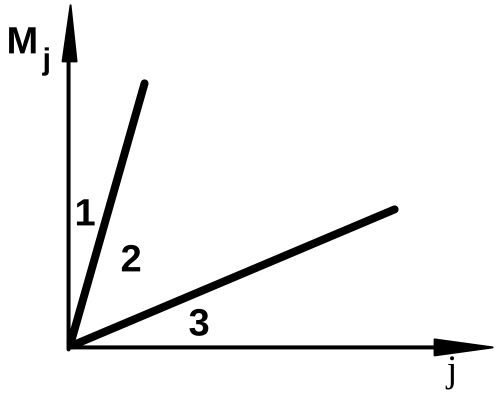


Figure 22 Boundaries for stiffness classification of beam-to-column joints

Zone 1: rigid, if $S_{j,ini} = 8 \frac{E\,I_b}{L_b}$ (12)

Zone 2: semi-rigid, all joints in zone 2 should be classified as semi-rigid. More accurately, joints in zones 1 or 3 may also be treated as semi-rigid.

Zone 3: nominally pinned, if $S_{j,ini} = 0{,}5 \frac{E\,I_b}{L_b}$ (13)

where: $E\,I_b$ is the uncracked flexural stiffness for a cross-section of a composite beam;
L_b is the span of a beam (centre-to-centre of columns).

For composite joints, it is necessary to decide whether the classification should be related to the cracked or uncracked flexural rigidity of the beam´s cross-section. The boundary for rigid joints has been determined by providing restraint against column collapse. In a braced frame the hogging beam end moments then decrease. As the unloading stiffness is taken as equal to the initial stiffness, the uncracked properties of the equivalent steel section should be used. To be strictly consistent with elastic global analysis, the modular ratio should be determined in accordance with EN 1994-1-1 [1]. For simplification, it is recommended that the short-term [1] 5.1.4
modular ratio should be taken as 7,0 irrespective of the grade of concrete.

2.3 Idealisation

In prEN 1993-1-8 [2], it is considered that the full non-linear M-f curve consists of three parts, as shown in Figure 23. Up to a level of 2/3 of the design moment resistance ($M_{j,Rd}$), the curve is assumed to be linear elastic. The corresponding stiffness is the so called initial stiffness $S_{j,ini}$. Between 2/3 ($M_{j,Rd}$) and $M_{j,Rd}$ the curve is non-linear. After the moment in the joint reaches $M_{j,Rd}$, a yield plateau could appear. The end of this M-f curve indicates the rotation capacity (f $_{Cd}$) of the joint.

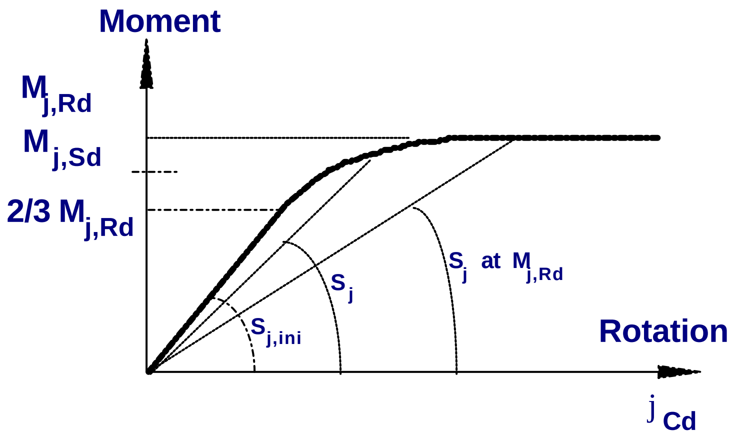


Figure 23 Non-linear M- f curve

The characterisation adopted by Eurocode 3 assumes a fixed ratio between the initial stiffness $S_{j,ini}$ and the secant stiffness at the intersection between the non-linear part and the yield plateau (S_j at the level $M_{j,Rd}$), see Figure 23. For composite joints with bolted end-plates, this ratio is taken equal to 3,0. Contact plate joints have a less gradual decrease in stiffness and the ratio is taken as 2,0.

The shape of the non-linear part for a bending moment $M_{j,Sd}$ between 2/3 ($M_{j,Rd}$) and $M_{j,Rd}$ can be found with the following interpolation formula: [1] 8.4.1(2)

$$S_j = \frac{S_{j,ini}}{\left(\frac{1{,}5\ M_{j,Sd}}{M_{j,Rd}}\right)^{\psi}} \qquad (14)$$

where ?= 2,7 for joints with bolted flush end-plates and 1,7 for joints with contact plates. In this interpolation formula, the value of S_j is therefore dependent on $M_{j,Sd}$.

Depending on the available software either the full non-linear shape of all joint-curves discussed until now or multi -linear simplifications of them can be assigned to the respective flexural springs. Figure 24 shows curve idealisations proposed in prEN 1993-1-8 [2]. It is evident that the required input as well as the capability required from the software both increase if high accuracy is intended.

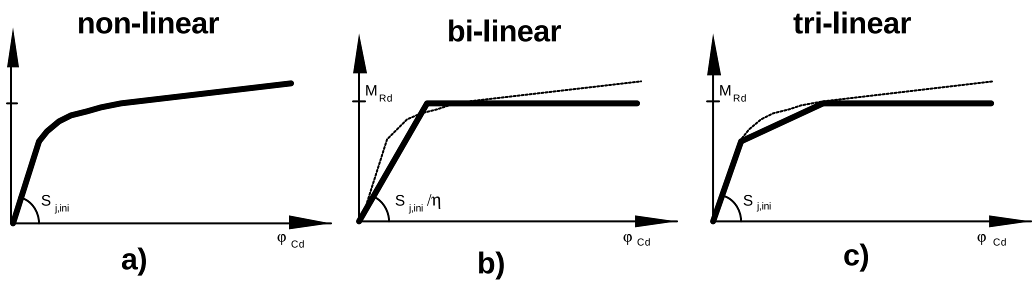


Figure 24 Possibilities for curve idealisation

The actual moment-rotation response of joints usually is described by means of a non-linear curve, see (Figure 24a). However, the use of such non–linear curves requires sophisticated frame analysis programs. In order to enable a (more simple) linear calculation, e.g. an **elastic global frame analysis** (this is still the current practice in most European countries), the non-linear curve may be simplified by straight lines. As a conservative assumption each curve lying below the non-linear curve in general may be used in the frame analysis. For example a tri-linear curve is shown in Figure 24c. The most simple curve is a bi-linear curve. In order to use a bi-linear curve which provides the most efficient solutions (i.e. the highest stiffness), comparative studies were performed to calibrate an idealised joint stiffness S_j*. The idealised joint stiffness S_j* is constant for all values of applied moments smaller than the design moment resistance of the joint. prEN 1993-1-8 [2] gives guidelines on how to derive such a simplified bi-linear curve as shown in Figure 24b. The idealised joint stiffness S_j* can easily be calculated by dividing the initial joint stiffness $S_{j,ini}$ with a stiffness modification factor ?:

$$S_j^* = S_{j,ini} /? \qquad (15)$$

The stiffness modification factor ? depends on the type of connection (contact plate, bolted end-plate) (See Table 3). [1] 8.2.1.2 Table 8.1

When elastic global analysis is used, the joints should be classified according to their stiffness. In the case of a semi-rigid joint, its rotational stiffness S_j for use in the global analysis should generally be taken as equal to the value of S_j , corresponding to the bending moment $M_{j,Sd}$.

As a simplification, the procedure illustrated in Figure 22 may be adopted, as follows:
Provided that the moment $M_{j,Sd}$ does not exceed 2/3 $M_{j,Rd}$ the initial rotational stiffness of the joint $S_{j,ini}$ may be used in the global analysis.

Where the moment $M_{j,Sd}$ exceeds 2/3 $M_{j,Rd}$ the rotational stiffness should be taken as $S_{j,ini}$/?, where ? is the stiffness modification coefficient from Table 8.1 of EN 1994-1-1 [1] or Table 3 below [1] Table 8.1

Type of connection	Value of ?
Contact plate	1,5
Bolted end-plate	2

Table 3 Stiffness modification coefficient ?

As a further simplification, the rotational stiffness may be taken as $S_{j,ini}$/? in the global analysis, for all values of the moment $M_{j,Sd,}$ as shown in Figure 25.

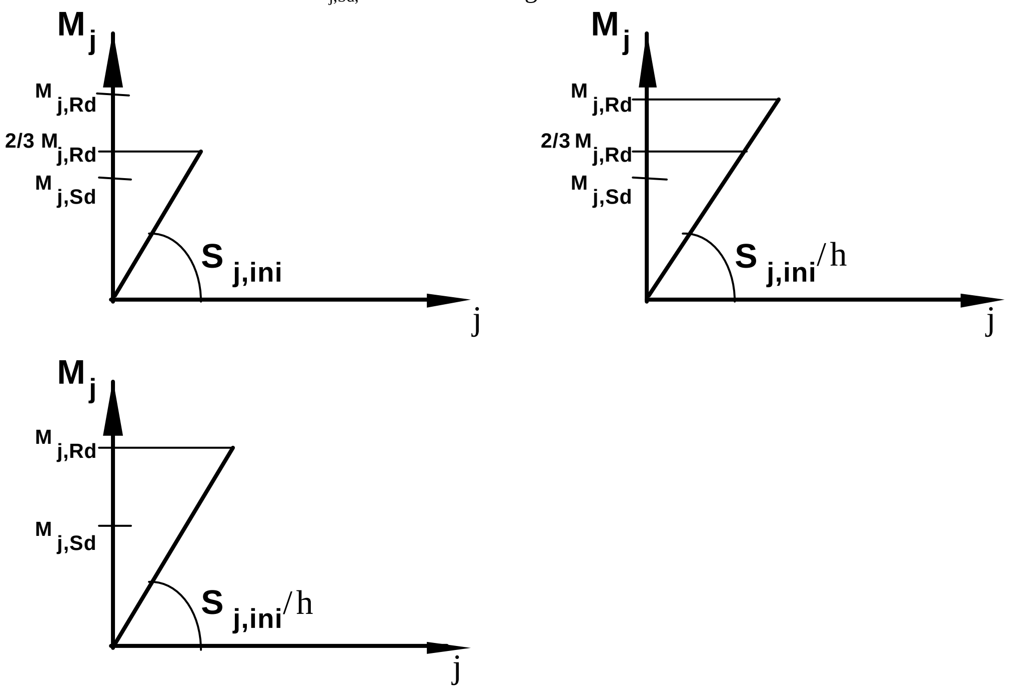


Figure 25 Rotational stiffness to be used in elastic global analysis

When **rigid-plastic global analysis** is used, joints should be classified according to their strength. [1] 8.2.2.2

When **elastic-plastic global analysis** is used, the joints should be classified according to both stiffness and strength.

2.4 Joint modelling of beam-to-column joints

As already mentioned there are three types of joint modelling which are closely linked to the method of global analysis and to the classification of the joint:
A continuous joint ensures full rotation continuity, a semi continuous joint ensures only partial rotational continuity and a simple joint prevents any rotational continuity between the connected members.

2.4.1 General

To model the deformation behaviour of a beam–to-column joint, account should be taken of the shear deformation of the web panel and the rotational deformation of the connection.
Beam-to-column joint configurations should be designed to resist the internal bending moments $M_{b1,Sd}$ and $M_{b2,Sd}$, normal forces $N_{b1,Sd}$ and $N_{b2,Sd}$ and shear forces $V_{b1,Sd}$ and $V_{b2,Sd}$ applied to the connections by the connected beams (see Figure 26).

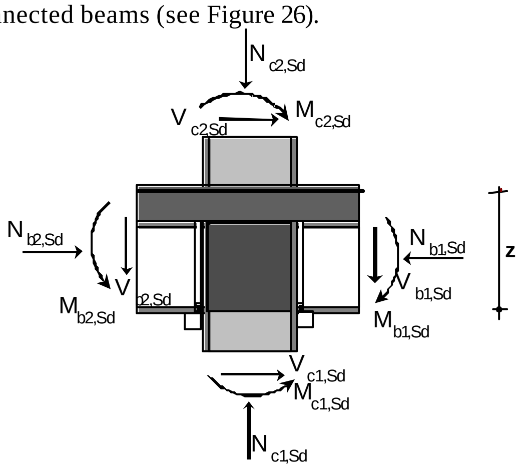


Figure 26 Forces and Moments acting on a joint

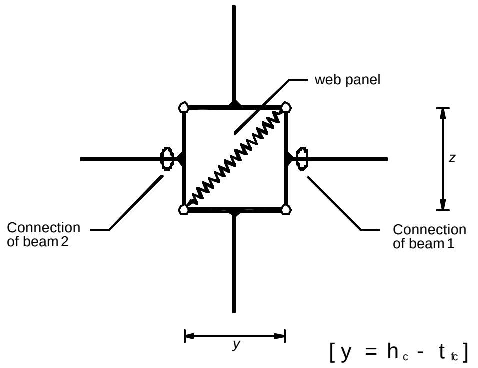
The resulting shear force $V_{wp,Sd}$ in the web panel should be obtained using:

$$V_{wp,Sd} = (M_{b1,Sd} - M_{b2,Sd}) / z - (V_{c1,Sd} - V_{c2,Sd}) / 2 \quad (16$$

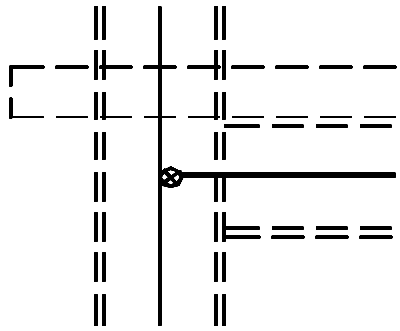
where:
z is the lever arm.
For global structural analysis, single-sided and double-sided beam to column joint configurations may be modelled either as indicated in 2.4.2 or using the simplified method given in 2.4.3.

2.4.2 Joint modelling reflecting the actual behaviour

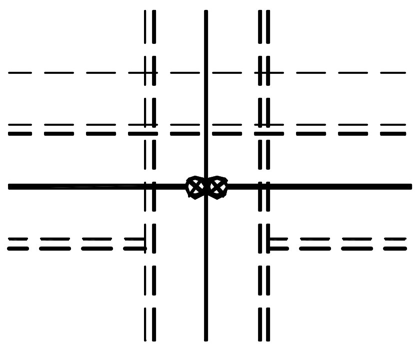
To model a joint in a way that closely reproduces the expected behaviour, the web panel and each of the connections should be modelled separately, taking account of the internal moments and forces in the members, acting at the periphery of the web panel (See Figure 26).
This may be achieved as shown in Figure 27a), in which the shear deformation of the web panel is reproduced by a separate translation spring and the rotational deformation of each connection is reproduced by flexural springs.



a) Modelling the web panel and the connections by separate springs



single-sided joint configuration



double-sided joint configuration

b) Modelling the connections as rotational springs

Figure 27 Modelling the joint deformability

Within a joint two separate sources of deformability have to be distinguished – those due to the connection (load-introduction into the column through the connecting elements) and those due to the shear panel (column web panel in shear due to the moment imbalance between the left and right hand sides). To reflect actual joint behaviour a cruciform joint configuration has to be modelled by two separate M_L-f $_L$ curves representing the left and the right connection and one additional M_s-f $_s$ curve for the shear panel, see Figure 28. It has to be noted that the panel is actually loaded in shear which results in a shear deformation, and therefore, the M_s-f $_s$ curve has to be derived from a V-? relationship for the shear panel. For accuracy a single-sided joint has to be modelled by two separate M-f curves, one for the connection and one for the shear. Figure 29 illustrates equally good possibilities of “joint modelling reflecting the actual behaviour by so-called finite–joint models”

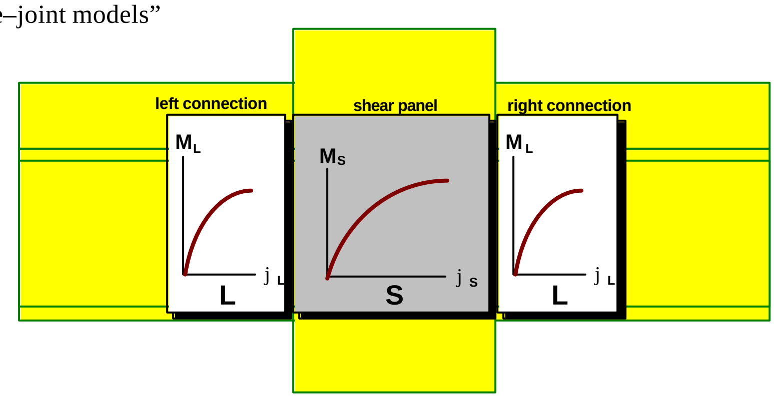


Figure 28 Joint modelling reflecting the actual behaviour

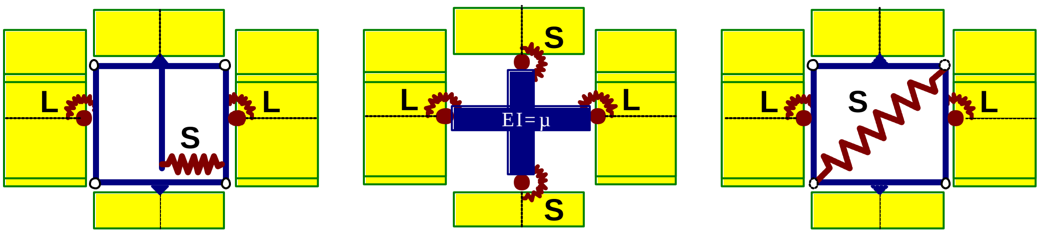


Figure 29 Application of joint modelling by finite joint models

If the software for frame analysis does not support flexural springs they may be substituted by equivalent beam stubs with appropriate bending stiffness. The derivation of the equivalent stub stiffness is shown in Figure 30 assuming a constant bending moment along the stub. Thus less deviation to the actual behaviour is achieved if the stub is short.

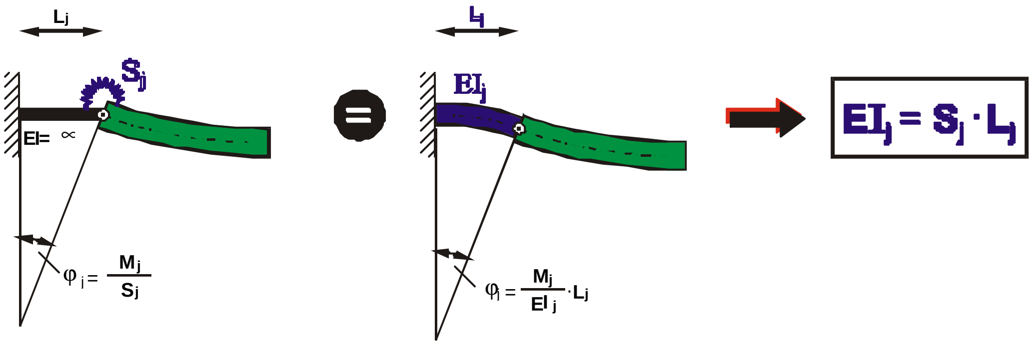


Figure 30 Equivalent stub instead of rotational spring

For sake of simplicity a simplified joint modelling has been introduced in prEN 1993-1-8 [2] for ever-day design, where the nodal zone is no longer taken into account with finite dimension. The aim is to represent the overall joint behaviour in one single rotational spring for each joint. Therefore simplifications are introduced as described below.

2.4.3 Simplified joint modelling

As a simplified alternative to the method of modelling, a single sided joint configuration may be modelled as a single joint, and a double-sided joint configuration may be modelled as two separate but inter-acting joints, one for each connection.

Each of these joints should be modelled as a separate rotational spring, as shown in Figure 31, in which each spring has a moment-rotation characteristic that takes into account the behaviour of the web panel as well as the influence of the relevant connection.

The finite size of the joint, neglected by this model, may be compensated by transforming the moment-rotation characteristic.

When determining the moment resistance and rotational stiffness for each of the joints, the possible influence of the web panel in shear should be taken into account by means of the transformation parameters $ß_1$ and $ß_2$ where:

$ß_1$ is the value of the transformation parameter ß for the right-hand side joint

$ß_2$ is the value of the transformation parameter ß for the left-hand side joint

Conservative values for $ß_1$ and $ß_2$ based on the values of the beam moments $M_{b1,Sd}$ and $M_{b2,Sd}$ at the periphery of the web panel may be obtained from Table 4:

[2] Table 7.3

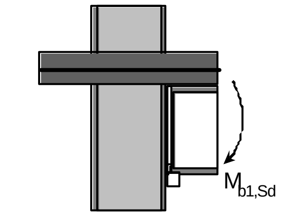
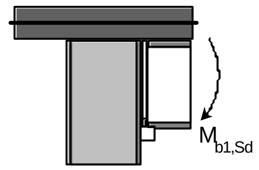
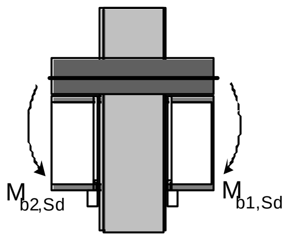
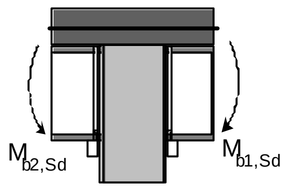
Type of Joint Configuration	Action	Value of b
 	$M_{b1,Sd}$	$\beta = 1$
	$M_{b1,Sd} = M_{b2,Sd}$	$\beta = 0$*
	$M_{b1,Sd} > 0$	$\beta = 1$
	$M_{b2,Sd} > 0$	
	$M_{b1,Sd} < 0$	$\beta = 1$
	$M_{b2,Sd} < 0$	
	$M_{b1,Sd} > 0$	$\beta = 2$
	$M_{b2,Sd} < 0$	
	$M_{b1,Sd} < 0$	$\beta = 2$
	$M_{b2,Sd} > 0$	
	$M_{b1,Sd} + M_{b2,Sd} = 0$	$\beta = 2$
* In this case, the value of β is the exact value rather than a approximation		

Table 4 Approximate values for the transformation parameter ß

Alternatively more accurate values of $ß_1$ and $ß_2$ based on the values of the beam moments $M_{j,b1,Sd}$ and $M_{j,b2,Sd}$ at the intersection of the member centrelines, may be determined from the simplified model as follows:

$$ß_1 = \left| 1 - (M_{j,b2,Sd} / M_{j,b1,Sd}) \right| \quad (17)$$

$$ß_2 = \left| 1 - (M_{j,b2,Sd} / M_{j,b2,Sd}) \right| \quad (18)$$

where:

$M_{j,b1,Sd}$ is the moment at the intersection from the right hand beam

$M_{j,b2,Sd}$ is the moment at the intersection from the left hand beam

For sake of simplicity a simplified joint modelling has been introduced in prEN 1993-1-8 [2] for "every – day design", where the nodal zone is no longer taken into account with finite dimensions. The aim is to represent the overall joint behaviour in one single rotational spring for each joint. Therefore simplifications are introduced as described below

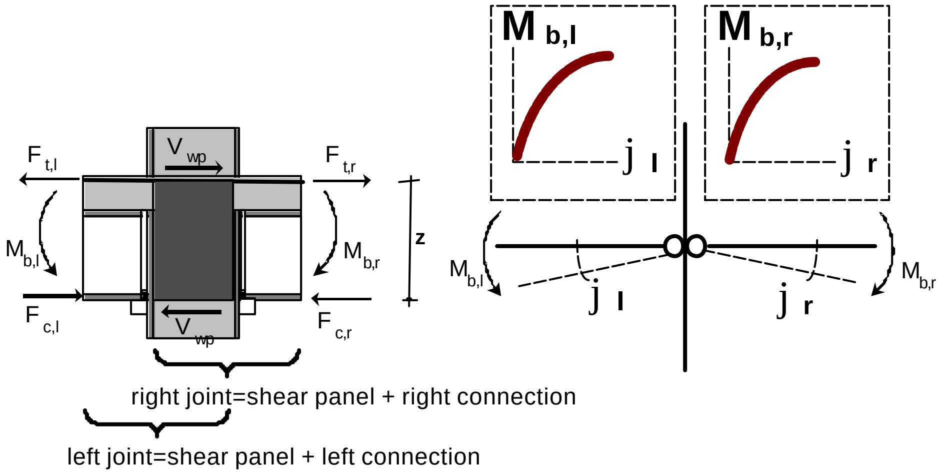


Figure 31 Modelling the web panel and the connections as combined rotational springs

The load–deformation (V-? curve of the column web panel in shear has to be converted into a moment-rotation (M_s-f $_s$) curve, using the lever arm z. However, the actual shear force V_{wp} in the column web panel is not equal to the "local" tension and compression forces within the connection (F_c=F_t=M / z), see Figure 31, due to the global shear forces acting in the column. For the simplified modelling according to prEN 1993-1-8 [2] this effect is neglected. In case of a single-sided beam-to-column joint configuration, both curves representing the web panel and the connection are added in series to one combined M-f curve for the joint, see Figure 32.
In case of a double-sided joint configuration, two separate but interacting joints may be modelled. Each of these joints takes into account the behaviour of the column web panel as well as the behaviour of the relevant connection. Thus, the web deformation would be taken into account twice but a so-called transformation factor ß (Table 4)can be introduced to consider the actual loading of the shear panel in relation to the loading of the individual joints. The validity and accuracy of this procedure in demonstrated in extensive background studies.

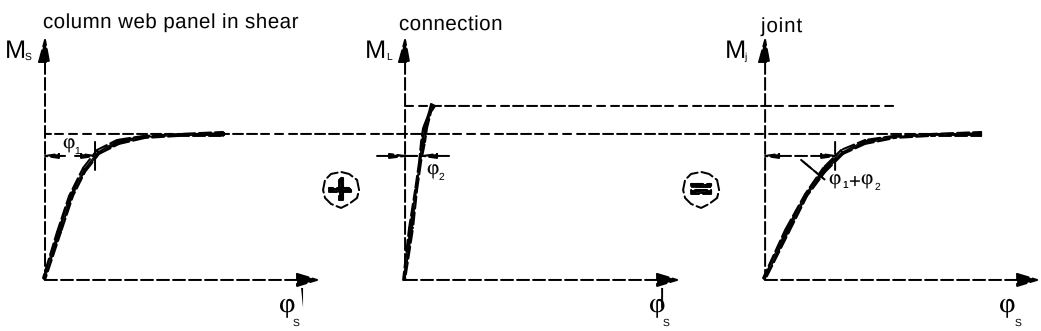


Figure 32 Derivation of a combined rotational spring for the simplified modelling

The second difference concerns the location of the rotational spring. In finite joint modes the connection springs are arranged at the edges of the finite joint area, as shown in Figure 29.
In the simplified joint modelling the combined joint springs for shear and connections are located

at the beam-to-column axes intersection point, see Figure 31. It is obvious that the acting consideration of the higher value for the applied moment therefore leads to a more conservative design. On the other hand the weakness of the "extended" beam and column overestimates the global frame deformation. This simplification is the same as that, made in the conventional design where simple or continuous modelling is used. The influence of both effects becomes more important with increasing joint dimensions in comparison to the beam span and column height. These effects can be compensated by a so-called "spring transformation", which leads to additional refining terms within the formulae characterising the joint properties.

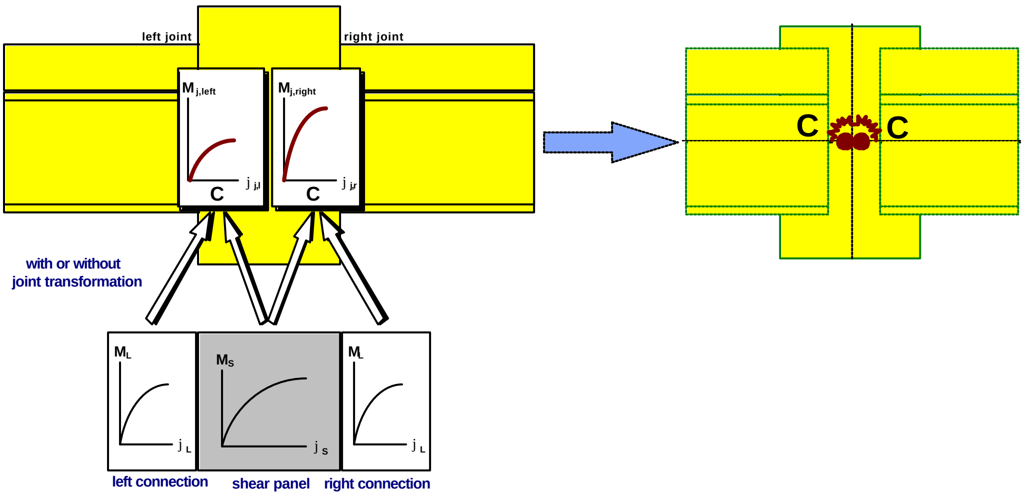


Figure 33 Simplified joint modelling

3 Joint Representation of Composite Beam-to-Column Joints, Design Provisions

3.1 Framing arrangements in composite construction

3.1.1 Introduction

In buildings, the structural frame provides the skeleton around which other elements are constructed. These include the external cladding, internal finishes and the services. Consequently, the frame should not be designed in isolation and the most efficient solution for the structure may not be the most appropriate for the total building. It is not possible therefore to prescribe standard framing arrangements for all buildings. However, guidance can be given on those factors which should be considered in developing a suitable layout.

3.1.2 General factors influencing framing arrangements

In determining the most appropriate arrangement, the structural engineer will need to consider the following factors.

- Site-dependent factors, including ground conditions and planning constraints
- Building use
- Design loading
- Building services and finishes
- External wall construction
- Lateral stiffness
- Structure costs, fabrication and construction.

Building use has a strong influence, which in recent years has led to wide clear spans. Studies have shown that composite construction (steel beams acting with the slab above or Slimfloor options) generally provides the most cost-effective solution for the floors. Greater stiffness and resistance means that beams can be shallower for the same span, leading to lower storey heights and reduced cladding costs. Alternatively, wider spans are possible within a given floor depth. The speed of construction is an important influence on the popularity of these structures.

3.1.3 Composite construction

Composite action within the slab itself is achieved by the well-known use of profiled steel sheeting. Although deep profiles are available, most floors use relatively shallow decking (typically in the range of 45-70 mm) spanning about 3m between beams. For these spans the decking does not normally require propping during concreting.

With slabs spanning this distance, secondary beams are needed. Alternative layouts for wide spans are shown in Figure 34. The need to consider together the steel framing and the particular services can be seen by considering these alternatives.

As the heavily-loaded primary beams in Figure 34a) have the shorter span, both these and the secondary beams will be of similar depth. The governing depth of floor construction is therefore minimised. Space to accommodate services exists between the beams. However if uninterrupted service space is required then that can only be provided by a void between the underside of the beams and the ceiling, or by making use of cellular beams. The overall depth for structure and services then increases.

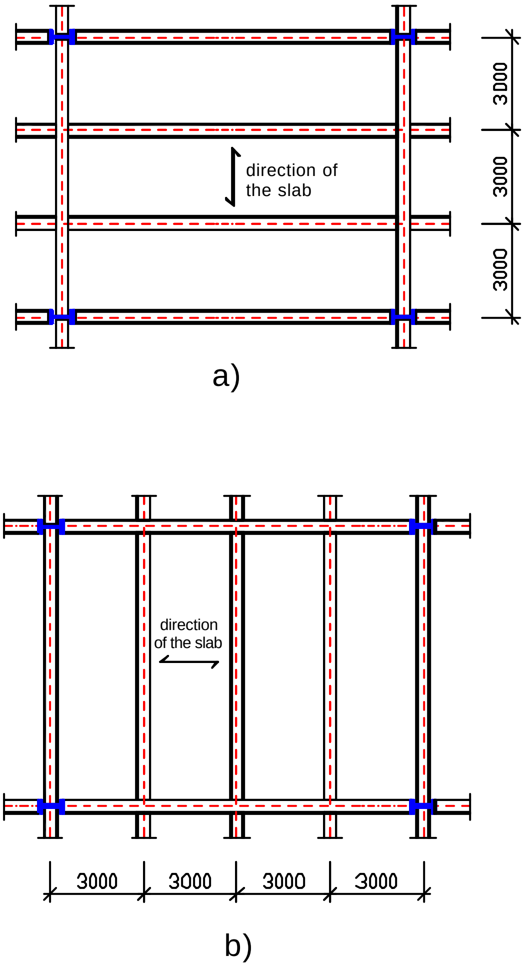


Figure 34 Alternative framing systems for floors

In Figure 34b) the secondary beams will be of less depth, but at the expanse of deep primary members. However, substantial zones now exist for services beneath the secondary members but within the overall floor depth governed by the size of the primary beams.

3.1.4 Sizing of composite beams

A simply supported composite beam is generally governed by one of the following conditions:

- Resistance to the ultimate design loads;
- Total deflection under serviceability loading;
- Vibration.

For a beam with composite end connections, the load capacity depends on the sagging moment resistance of the beam section in the mid-span region and also on the resistance of the connections to hogging moment. For the same load capacity, this leads to a smaller steel section than in simple construction.

It must also be remembered though that with unpropped beams the construction condition could govern. As the steelwork part of a composite joint resembles a nominally-pinned steel connection, the beam is simply-supported during construction. There is more likelihood therefore that this will govern. This does not mean that there is no benefit from the composite joints – by making the composite stage less critical a smaller steel section is more likely. Furthermore, over-design at the composite stage can be minimised by use of partial shear connection for sagging bending. For simply-supported composite beams, typically around 50% of the total deflection is due to sagging of the steel section at the construction stage. With composite connections, this proportion increases, because smaller loading may also affect the design. Detailed analysis of the dynamic response is beyond the scope of this document and reference should be made to specialised information.

When sections are governed by deflection, pre-cambering can be used to reduce the final sagging displacement. It should be noted though that the deflection at the end of construction may not be important. If deflection does not impair appearance, the reference level for displacement should be taken as the upper side of the composite beam. The deflection at the end of construction can therefore be neglected provided that:

- A false ceiling is provided;
- The top surface of the floor slab is level at the end of construction (as would normally be the case because of the flow of wet concrete);
- Consideration is given to the loading effect of ponding (increased depth of concrete due to deflection).

At the composite stage, composite joints are clearly beneficial because of the substantial increase in stiffness. It has been shown that even connections of modest stiffness cause significant reduction in beam deflection. A substantial increase in natural frequency is also obtained. In addition, composite joints permit cracking to be controlled, where it is required for appearance and/or durability.

3.1.5 Framing arrangements and composite joints

Considering the factors given above, the designer should determine:

- The layout of the beams and columns;
- The types of beams, for example rolled steel section acting compositely with the slab above or a Slimfloor option; non-composite steel beams may be included;
- The types of column, for example rolled H-sections or hollow sections; composite sections may be included;
- The orientation of the column sections (if appropriate);
- Types of joint, for example composite joints, moment-resisting steel joints, nominally-pinned joints;
- Types of connection, for example composite joints with contact plate, composite joints with full-depth steel end-plates, “simple” steel fin plates.

The choices given as examples show the freedom available to the designer to meet the particular requirements of the structure. This is characteristic of semi-continuous construction and is illustrated by the example shown in Figure 35. In the discussion which follows, it is assumed that the designer has sought to achieve beams of similar depth. It is assumed that for the building concerned composite connections are not required purely for crack control. The numbers refer to the joint types in the lower part of the figure and are also used to identify particular beams.

All internal beams are to be designed as composite members, but as the edge beams 1-2 and 3-3 carry less load, they may be designed as non-composite. In that case it follows that connections 1, 2 and 3 are also non-composite.

Connections to perimeter columns can be composite, provided that a region of slab exists beyond the column in which to anchor the tension reinforcement or reinforcement is looped around the column. Here it is assumed that connections to the perimeter columns will be bare steel, to avoid any difficulty in achieving this.

Each of connection types 1 and 2 may be either nominally-pinned or moment-resisting, as required to achieve the aim of beams of similar depth. In the interests of overall economy though, a stiffened connection should be avoided. For ease of fabrication and erection, connection 3 should be nominally-pinned. In any case, a column web has only very limited resistance under unbalanced moment. Joint 4 is also nominally pinned, because its one-sided nature.

The remaining connections, 5-7, may be designed compositely. By adjusting the reinforcement, the designer is able to vary at will the structural properties of the joint. The moment resistance and stiffness of the steel connection can also be varied by changing its details. Indeed, joint types 5-7 can be replaced by contact-plate joints, as shown in Figure 35. This freedom helps the designer to achieve economically a uniform large floor grid, the beams and their connections would be repeated many times. The resulting reductions in beam sections would therefore be widespread and there would be substantial repetition in fabrication and erection.
Finally, it will be noted that composite connections are used about both axes of the internal column. With a decking say 70 mm deep and a thin overall slab depth (say 120 - 130mm), problems could arise in accommodating the two layers of reinforcement in a limited thickness of slab. In such circumstances one connection could be bare steel. In the example (Figure 35) this should be connection 6; as it connects to the column flange, it could still be designed as moment-resistant. Alternatively, a greater thickness of concrete could be provided by using shallower decking.

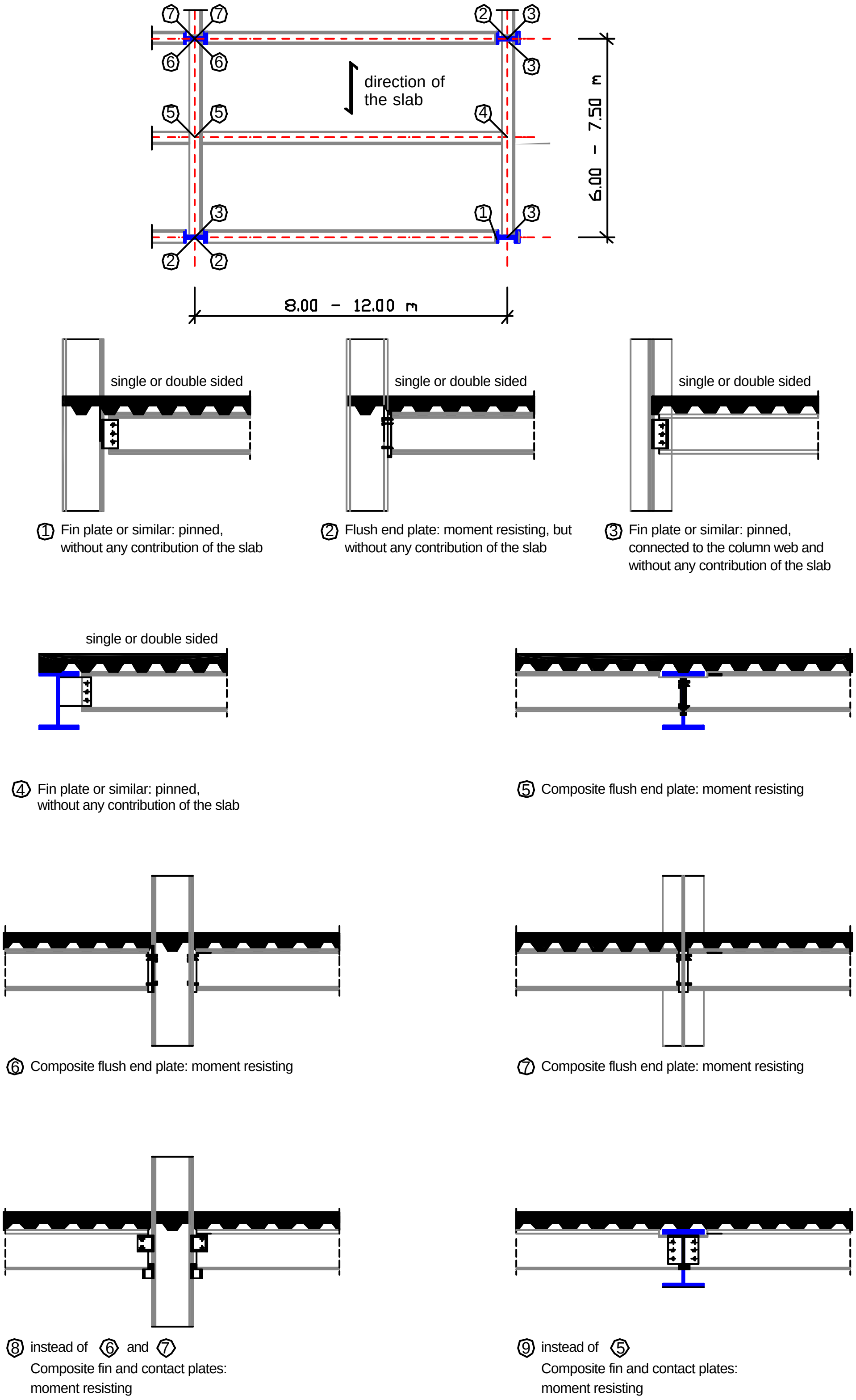


Figure 35 Choice of joint type

3.2 Design provisions

3.2.1 Basis of design

Design provisions in this lecture are based on the component method for steel joints described in prEN 1993-1-8 [2]. The simplified model for a composite joint is shown in Figure 36.

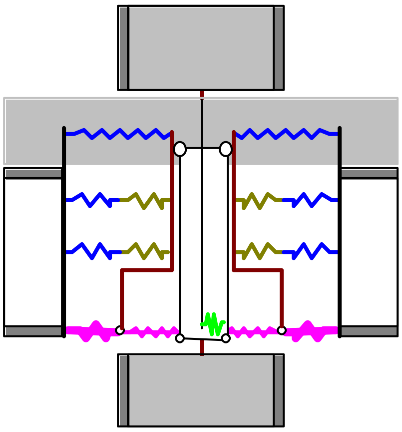


Figure 36 Simplified model according to EN 1994-1-1 [1]

The prEN 1993-1-8 [2] already provides expressions for the design resistance and initial stiffness of the following components:

Compression region:
- Column web in compression
- Beam flange and web in compression

Tension region:
- Column flange in bending
- Column web in tension
- End-plate in bending
- Beam web in tension
- Bolts in tension

Shear region:
- Column web panel in shear

For composite joints, the following additional basic components are relevant:
- **Longitudinal slab reinforcement in tension;**
- **Contact plate in compression.**

Although not regarded as a separate basic component, account may also need to be taken of concrete encasement to the column. This is treated as a form of stiffening.
Expressions for design resistance and initial stiffness are given in 3.3. The stiffness coefficients k_i for components affected by concrete encasement are transformed into equivalent all-steel

values, using a modular ratio. This enables a single value for modulus of elasticity E to be used to determine the rotational stiffness of the joint, in the same way as for steel joints in prEN 1993-1-8 [2]. A similar transformation enables the modulus of elasticity of reinforcement to have a different value to that for structural steel.

Unlike the sophisticated model, the simplified Eurocode model shown in Figure 36 does not make explicit allowance for the following:

- Slab concrete in bearing against the column;
- Transverse slab reinforcement;
- Slip of the beam´s shear connection.

Account is taken of these actions either through detailing rules to exclude their influence or (for slip) by reduction factor on the stiffness.

3.2.2 Design moment resistance

To simplify calculation, plastic theory is used to determine the design moment resistance. This moment is therefore taken as the maximum evaluated on the basis of the following criteria:

- The internal forces are in equilibrium with the forces applied to the joint
- The design resistance of each component is not exceeded
- The deformation capacity of each component is not exceeded
- Compatibility is neglected

<u>Contact plate joint:</u>
In such joints, the steelwork connection provides no resistance to tension arising from bending. The distribution of internal forces is therefore easy to obtain. As can be seen from Figure 37, the compression force is assumed to be transferred at the centroid of the lower beam flange and the tension force at the centroid of the reinforcement.

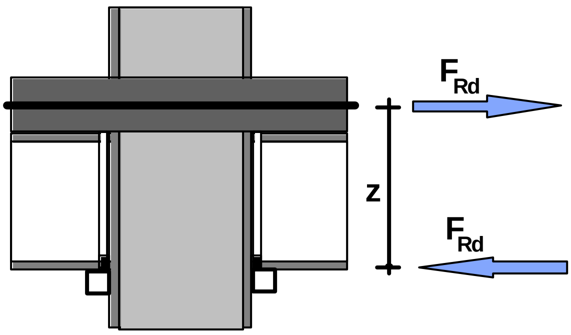


Figure 37 Contact plate joint with one row of reinforcement

The design moment resistance of the joint $M_{j,Rd}$ is dependent on the design resistance F_{Rd} of the weakest joint component; for this joint, the relevant components are assumed to be the reinforcement in tension, the column web in compression, the beam flange and web in compression or the column web panel in shear. So:

$$M_{j,Rd} = F_{Rd} \cdot z \qquad (19)$$

where z is the lever arm of the internal forces. [1] 8.2.4.1

For connections with contact plates, the centre of compression should be assumed to be in line with the mid-thickness of compression flange.

For connections with contact plates and only one row of reinforcement active in tension, the lever arm z should be taken as the distance from the centre of compression to the row of reinforcement in tension.

For connections with contact plates and two rows of reinforcement active in tension the lever arm z should be taken as the distance from the centre of compression to a point midway between these two rows, provided that the two rows have the same cross-sectional area.(See Figure 38)

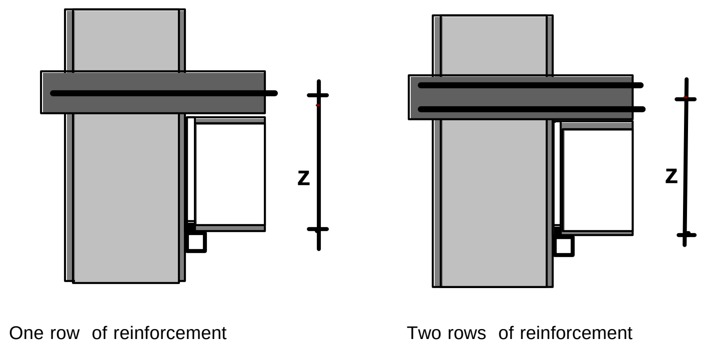


Figure 38 Determination of the lever arm z

For connections with other types of steelwork connections the lever arm z should be taken as equal to z_{eq} obtained using the method given in 5.3.3.1 of prEN 1993-1-8 [2]. [2]5.3.3.1

For the assumption to be correct, detailing rules need to ensure that, under unbalanced loading, failure does not occur by crushing of concrete against the column section. These are given in 3.7.1.

<u>Joints with steelwork connection effective in tension</u>:
A composite flush end-plate connection, such as that shown in Figure 39, is an example of such a joint. Tension arising from bending is resisted by the combined action of the reinforcement and the upper part of the steelwork connection. As there is more than one row of components in the tension region, the distribution is now more complex. It is assumed that as the moment increases the reinforcement bars reach their design resistance before the top row of bolts. High-ductility reinforcement, as defined in EN 1992-1 [10], is to be used and therefore redistribution of internal forces can take place. Thus each bolt-row in turn may reach its resistance, commencing with the top row. [10]

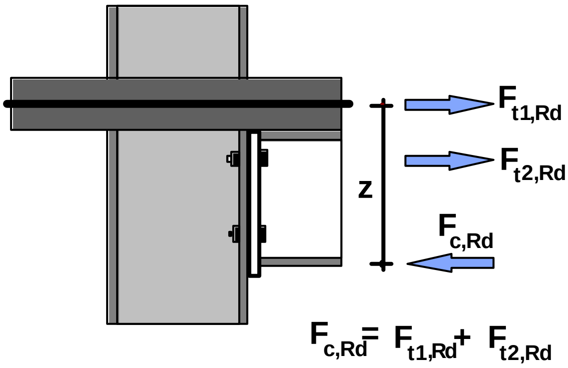


Figure 39 Composite flush end-plate joint

Within the bolt-rows, three types of internal force distribution may be identified:

- An elastic distribution;
- A plastic distribution;
- An elasto-plastic distribution.

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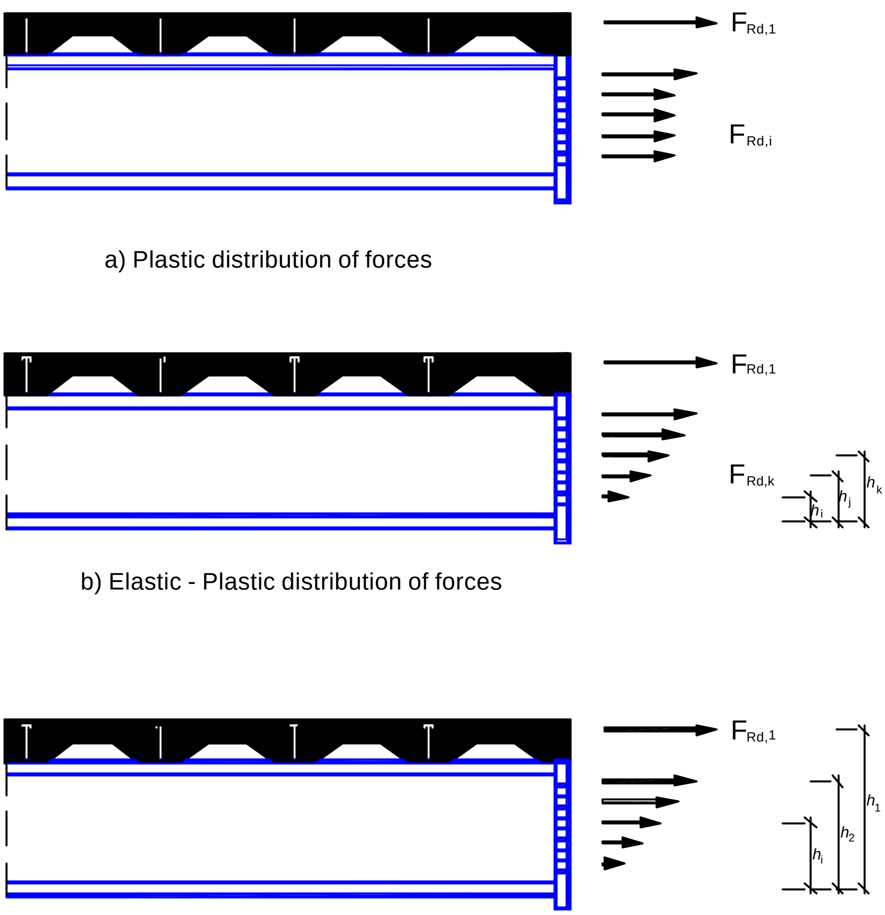
The appropriate one will depend on the deformation capacity of the joint components. The distributions are illustrated in Figure 40.



a) Plastic distribution of forces

b) Elastic - Plastic distribution of forces

c) Elastic distribution of forces

Figure 40 Internal force distribution of reinforcement and bolt rows

In Figure 40, a plastic redistribution of the internal forces takes place progressively from the upper bolt-row towards the lower ones because of their sufficient deformation capacity. prEN 1993-1-8 [2] considers that a bolt-row possesses sufficient capacity to allow this when:

- $F_{Rd,i}$ is associated to the failure of the beam web in tension or
- $F_{Rd,i}$ is associated to the failure of the bolt-plate assembly and
- $F_{Rd,i} = 1,9\ B_{t,Rd}$ where $B_{t,Rd}$ is the tension resistance of the bolt-plate assembly.

The design moment resistance is given by:

$$M_{j,Rd} = \sum F_{Rd,i}\ h_i \qquad (20)$$

where the summation includes the reinforcement in tension.

In some cases (Figure 40b) , the plastic redistribution of forces is interrupted, because of the lack of deformation capacity in the last bolt-row which has reached its design resistance. ($F_{Rd,i} > 1,9\ B_{t,Rd}$ and linked to the failure of the bolts or of the bolt-plate assembly). In the bolt-rows located lower than bolt-row k, the forces are then linearly distributed according to their distance to the centre of compression i.e. the centre of the lower beam flange. The design moment resistance then equals:

$$M_{j,Rd} = \sum F_{Rd,i}\, h_i + (F_{Rd,i} / h_k) \sum h_j^{\,2} \tag{21}$$

where the first summation includes the reinforcement and all bolt-rows down to row k, and the second summation is over the bolt-rows below row k.
In the elastic distribution (Figure 40c), the forces in the bolt-rows are proportional to the distance to the centre of compression. This distribution applies to joint configurations with rather stiff end-plates and column flanges. The above formula applies with k corresponding to the top bolt-row.

The three distributions may be interrupted because the compression force F_c (Figure 39) is limited by the design resistance of the beam flange and web in compression, the column web in compression or the column web panel in shear. The moment resistance M_{Rd} is then evaluated with similar formulae to those given above but now only a limited number of bolt-rows or amount of reinforcement is taken into consideration.

These are determined such that:

$$\sum_{\ell=1,n} F_\ell = F_{c,Rd} \tag{22}$$

where:

n is the number of the last row permitted to transfer a tensile force;

F_l is the tensile force in row number l;

$F_{c,Rd}$ is the least design resistance of the beam flange and web in compression, the column web in compression and (if appropriate) the column web panel in shear.

[1] 8.3.3

3.3 Resistance of basic components

3.3.1 General

In Table 5 a list of components covered by prEN 1993-1-8 [2] is shown.

For composite joints, the following additional basic components are relevant:

- **Longitudinal slab reinforcement in tension**
- **Contact plate in compression**

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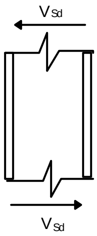
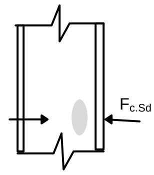
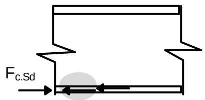
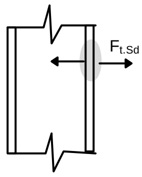
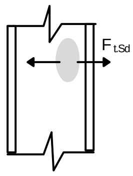
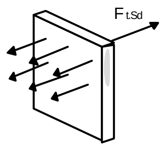
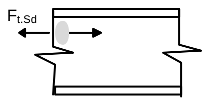
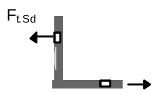
[2] 5.2.6

N°	Component	
1	Column web panel in shear	
2	Column web in compression	
3	Beam flange and web in compression	
4	Column flange in bending	
5	Column web in tension	
6	End-plate in bending	
7	Beam web in tension	
8	Flange cleat in bending	

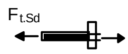
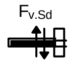
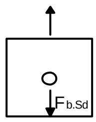
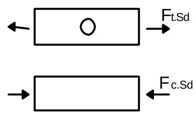
9	Bolts in tension	
10	Bolts in shear	
11	Bolts in bearing (on beam flange, column flange, end-plate or cleat)	
12	Plate in tension or compression	

Table 5 List of components covered by prEN 1993-1-8 [2]

[1]8.3.3.1

3.3.2 Column web panel in shear

For a single sided joint, or for a double sided joint in which the beam depths are similar, the shear resistance $V_{wp,Rd}$ of the steel column web panel, subject to a shear force $V_{wp,Sd}$ should be obtained using:

$$V_{wp,a,Rd} = \frac{0{,}9\, f_{y,wc}\, A_{vc}}{\sqrt{3}\,\gamma_{M0}} \qquad (23)$$

where:

$V_{wp,a,Rd}$ is the design shear resistance of the steel column web panel, see prEN 1993-1-8 [2] 5.2.6.1;

A_{vc} is the shear area of the column, see EN 1993-1 [11] 5.4.6. [11] 5.4.6

The resulting shear force $V_{wp,Sd}$ in the web panel should be obtained using:

$$V_{wp,Sd} = (M_{b1,Sd} - M_{b2,Sd}) / z - (V_{c1,Sd} - V_{c2,Sd}) / 2 \qquad (24$$

see Figure 41

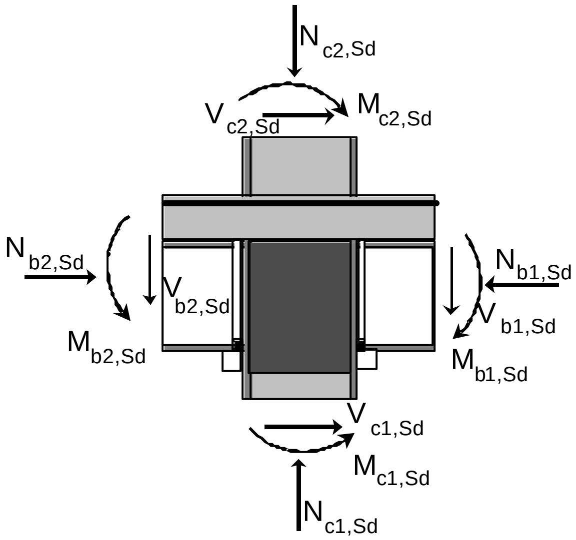


Figure 41 Forces and moments acting on a joint

Where the steel column web is encased in concrete the design shear resistance of the panel, $V_{wp,Rd}$, may be increased to:

$$V_{wp,Rd} = V_{wp,a,Rd} + V_{wp.c.Rd} \qquad (25)$$

For a single-sided joint, or a double-sided joint in which the beam depths are similar, the design shear resistance of concrete encasement to the column web panel $V_{wp,c,Rd}$ is given by:

$$V_{wp,c,Rd} = \nu\ (0{,}85\ f_{ck} / ?_c)\ A_c \sin ? \qquad (26$$

where:

$$A_c = [0{,}8\ (h_c - 2t_{fc}) \cos ?]\ [b_c - t_{wc}] \qquad (27)$$

$$? = \arctan [(h_c - 2\ t_{fc})/z]$$

b_c is the width of the flange of the column section;
h_c is the depth of the column section;
t_{fc} and t_{wc} are defined in Figure 42;
z is the lever arm.

The reduction factor ν to allow for the effect of longitudinal compression in the column on the design resistance of the column web panel in shear is given by:

$$\nu = 0{,}55[1+2(N_{Sd} / N_{pl,Rd})] = 1{,}1 \qquad (28$$

where: [1] 6.8.3.2
N_{Sd} is the design axial compressive force in the column;
$N_{pl,Rd}$ is the design plastic resistance of the column´s cross-section including the encasement, see also EN 1994-1-1 [1] 6.8.3.2.

[1] Figure 8.2

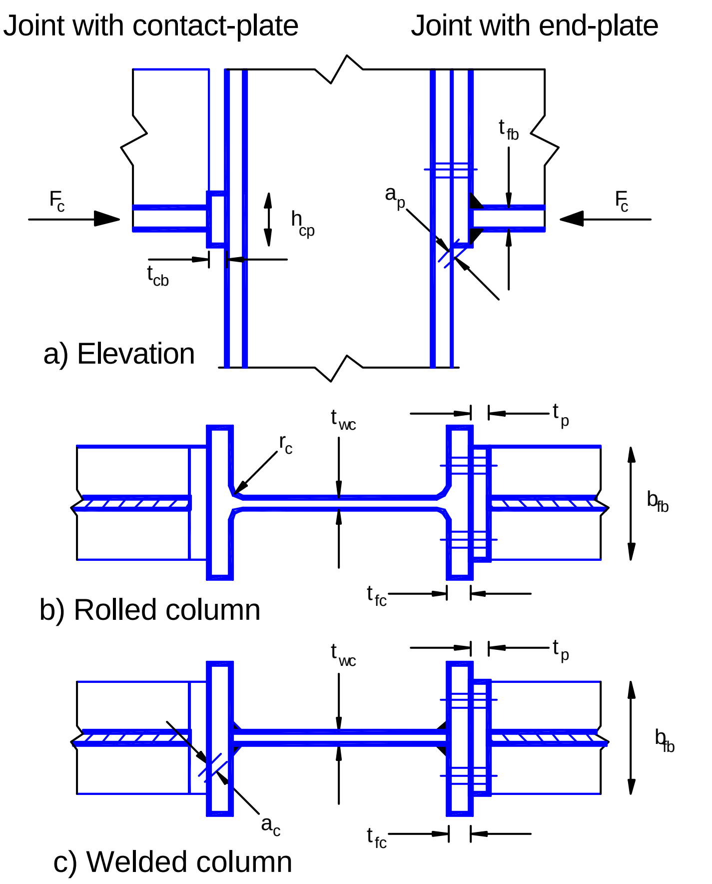


Figure 42 Transverse compression on a column

[1] 8.3.3.2

3.3.3 Column web in compression

[2] 5.2.6.2

The resistance of an unstiffened steel column web subject to transverse compression $F_{c,wc,a,Rd}$ should be determined in accordance with prEN 1993-1-8 [2] 5.2.6.2, using an effective width $b_{eff,c,wc}$.

For a contact plate connection the effective width $b_{eff,c,wc}$ of column web in compression, see Figure 42, should be obtained using:

$$b_{eff,c,wc} = t_{fb} + 5\,(t_{fc} + s) + s_p \qquad (29)$$

in which:

- for a rolled I or H section: $s = r_c$
- for a welded I or H section: $s = \sqrt{2}a_c$ (30)

s_p is the length obtained by dispersion at 45° through the contact plate (at least t_{cp} and, provided that the depth of contact plate below the flange is sufficient, up to 2 t_{cp}).
For a bolted end-plate connection the effective width $b_{eff,c,wc}$ of column web in compression should be obtained using:

$$b_{eff,c,wc} = t_{fb} + 2\sqrt{2}\, a_p + 5(t_{fc} + s) + s_p \qquad (31)$$

in which:

s_p is the length obtained by dispersion at 45° through the end-plate (at least t_p and, provided that the length of end-plate below the flange is sufficient, up to 2 t_p).

Where an unstiffened column web is encased in concrete, the design resistance in transverse compression, $F_{c,wc,Rd}$ may be increased to:

$$F_{c,wc,Rd} = F_{c,wc,a,Rd} + F_{c,wc,c,Rd} \qquad (32)$$

where:
$F_{c,wc,a,Rd}$ is the design resistance of the unstiffened steel column web subject to transverse compression;
$F_{c,wc,c,Rd}$ is the design resistance to transverse compression of the concrete encasement to the column web.
The design resistance to transverse compression of concrete encasement to a column web $F_{c,wc,c,Rd}$ is given by:

$$F_{c,wc,c,Rd} = k_{wc,c}\ t_{eff,c}\ (b_c - t_{wc})\ (0{,}85\, f_{ck} / ?_c) \qquad (33)$$

The effective thickness of concrete $t_{eff,c}$ is given by:

$$t_{eff,c} = l_o + 5t_{fc} \qquad (34)$$

For a contact plate connection, the load introduction length l_o (see Figure 42) is given by:

$$l_o = t_{fb} + s_p \qquad (35)$$

where s_p is defined above.

For a bolted end-plate connection, the load introduction length l_o is given by:

$$l_o = t_{fb} + 2\sqrt{2}\, a_p + s_p \qquad (36)$$

where s_p is defined above.

Where in a steel web the maximum longitudinal compressive stress $s_{com,Ed}$ due to axial force and bending moment in the columns exceeds 0,5 $f_{y,wc}$ in the web (adjacent to the root radius for a rolled section or the toe of the weld for a welded section), its effect on the resistance of the column web in compression should be allowed for by multiplying the value of $F_{c,wc,Rd}$ by a reduction factor $k_{wc,c}$ given in prEN 1993-1-8 [2] 5.2.6.2(8). The factor $k_{wc,c}$ to allow for the effect of longitudinal compression in the column on the design resistance of the concrete encasement to transverse compression $F_{c,wc,c,Rd}$ is given by: [2] 5.2.6.2(8)

$$k_{wc,c} = 1{,}3 + 3{,}3\, s_{com,c,Ed} / (f_{ck} / ?_c) \leq 2{,}0 \qquad (37)$$

where $s_{com,c,Ed}$ is the longitudinal compressive stress in the encasement due to the design axial force N_{Sd}.

3.3.4 Column web in tension

[2] 5.2.6.3

The resis tance of an unstiffened column web subject to transverse tension should be determined from:

$$F_{t,wc,Rd} = \frac{\omega b_{eff,t,wc}\ t_{wc}\ f_{y,wc}}{\gamma_{M0}} \tag{38}$$

where ? is a reduction factor to allow for the possible effects of shear in the column web panel.

For a bolted connection, the effective width $b_{eff,t,wc}$ of column web in tension should be taken as equal to the effective length of equivalent T-stub representing the column flange. (See prEN 1993-1-8 [2] 5.2.2 as well as SSEDTA 1 [3] Module 5.
The reduction factor ? allow for the possible effects of shear in the column web panel should be determined from prEN 1993-1-8 [2] Table 5.2.

[2] 5.2.2
[3] Module 5

3.3.5 Column flange in bending

See prEN 1993-1-8 [2] 5.2.6.4

[2] 5.2.6.4

3.3.6 End-plate in bending

See prEN 1993-1-8 [2] 5.2.6.5

[2] 5.2.6.5

3.3.7 Beam flange and web in compression

[1] 8.3.3.3

For a composite joint, the compression resistance of a beam´s steel flange and the adjacent compression zone of the beam´s steel web, may be assumed to act at the level of the centre of compression. It may be assumed to be given with sufficient accuracy by:

$$F_{c,fb,Rd} = M_{c,Rd} / (h - t_{fb}) \tag{39}$$

where:

h is the depth of the connected beam´s steel section;
$M_{c,Rd}$ is the moment resistance of the steel section, reduced if necessary to allow for shear, see also EN 1993-1 [11] 5.4.5 and 5.4.7;
t_{fb} is the thickness of the connected beam´s steel flange (See Figure 42).

[11] 5.4.5
[11] 5.4.7

3.3.8 Beam web in tension

In a bolted end-plate connection, the tension resistance of the beam web should be obtained using:

$$F_{t,wb,Rd} = b_{eff,t,wb}\ t_{wb}\ f_{y,wb}\ /\ ?_{M0} \tag{40}$$

The effective width $b_{eff,t,wb}$ of the beam web in tension should be taken as equal to the effective length of the equivalent T-stub representing the end-plate in bending, obtained from prEN 1993-1-8 [2] 5.2.6.5 for an individual bolt-row or a bolt-group.

[2] 5.2.6.5

[1] 8.3.3.4

3.3.9 Longitudinal slab reinforcement in tension

The design tension resistance of a row r of reinforcing bars is given by:

$$F_{tr,s,Rd} = A_{r,s}\, f_{sk} / ?_s \qquad (41)$$

where:

$A_{r,s}$ is the cross-sectional area of the longitudinal reinforcement in row r within the total effective width of concrete flange b_{eff}, as given by EN 1994-1-1 [1]5.2.2; [1] 5.2.2
f_{sk} is the characteristic strength of the reinforcement;
$?_s$ is the partial safety factor for reinforcement.

EN 1994-1-1 [1] of 5.2.1(4) is applicable. [1] 5.2.1(4)

For ductile failure the total cross-sectional area of longitudinal reinforcement $\sum_r A_{r,s}$ should not exceed the limit given in EN 1994-1-1 [1] 8.6.1 [1] 8.6.1

3.3.10 Contact plate in compression

The design compression resistance of a contact plate is given by the lesser of:

$$F_{c,cp,Rd} = b_{cp}\, h_{cp}\, f_y \,/\, ?_{M0} \qquad (42)$$

where:

b_{cp} is the breadth of the contact plate;
h_{cp} is the height of the contact plate;
$?_{M0}$ is the partial safety factor for structural steel;

and:

$$F_{c,cp,Rd} = b_{cp}\,(t_{fb} + s_p)\, f_y \,/\, ?_{M0} \qquad (43)$$

where:

s_p is the length obtained by dispersion at 45° through the contact plate.

[1] 8.3.4

3.4 Design moment resistance of joints with full shear connection

3.4.1 General

[2] 5.2.7

The methods for determining the design moment resistance of a joint $M_{j,Rd}$ do not take account of any co-existing axial force N_{Sd} in the connected member. They should not be used if the axial force in the connected member exceeds 10% of the plastic resistance $N_{pl,Rd}$ of its cross-section. The shear connection should be designed in accordance with EN 1994-1-1 [1] 6.7.2.

3.4.2 Beam-to-column joints with contact plate

The design moment resistance $M_{j,Rd}$ of a beam-to-column joint may be determined from:

$$M_{j,Rd} = \sum_r h_r\, F_{tr,Rd} \qquad (44)$$

where:

$F_{tr,Rd}$ is the effective design tension resistance of row r of reinforcing bars;
h_r is the distance from row r to the centre of compression;
r is the number of a particular row.

The centre of compression should be assumed to be in line with the mid-thickness of the compression flange of the connected member.

The effective design tension resistance $F_{tr,Rd}$ for each row of reinforcing bars should be determined in sequence, starting with row 1, the row furthest from the centre of compression.

When determining the value of $F_{tr,Rd}$ of row r of reinforcing bars, all other rows of reinforcement closer to the centre of compression should be omitted.

The effective design tension resistance $F_{tr,Rd}$ of row r of reinforcing bars should be taken as its design tension resistance $F_{tr,s,Rd}$ as an individual row, (See also 3.3.9), reduced if necessary to satisfy the conditions specified below.

The effective tension resistance $F_{r,Rd}$ of row r of reinforcing bars should, if necessary, be reduced below the given value in order to ensure that, when account is taken of all rows up to and including r: [2] 5.2.7.2(7)

The total resistance $\sum F_{t,Rd} \leq V_{wp,Rd} / ß$ (ß see Table 4) (45)

The total resistance $\sum F_{t,Rd}$ does not exceed the smallest of:

The resistance of the column web in compression $F_{c,wc,Rd}$ (see 3.3.3)
The resistance of the beam flange and web in compression $F_{c,fb,Rd}$ (see 3.3.7)
The resistance of the contact plate in compression $F_{c,cp,Rd}$ (see 3.3.10)

3.4.3 Beam-to-column joints with partial-depth end-plate

The design moment resistance may be determined in accordance with 3.4.2 therefore it is necessary that the contribution of the bolt-rows to the design moment resistance may be neglected.

3.4.4 Beam-to-column joints with bolted end-plate steelwork

The design moment resistance $M_{j,Rd}$ of a beam-to-column joint may be determined from:

$$M_{j,Rd} = \sum_r h_r \; F_{tr,Rd} \tag{46}$$

where:

$F_{tr,Rd}$ is the effective design tension resistance of row r of reinforcing bars or bolts;
h_r is the distance from row r of reinforcing bars or bolts to the centre of compression;
r is the number of a particular row.

For bolted end-plate connections, the centre of compression should be assumed to be in line with the mid-thickness of the compression flange of the connected member.

The effective design tension resistance $F_{tr,Rd}$ for each row of reinforcing bars or bolts should be determined in sequence, starting with row 1, the row furthest from the centre of compression.
When determining the value of $F_{tr,Rd}$ for row r of reinforcing bars, all other rows of reinforcement closer to the centre of compression and all bolt-rows should be omitted.

The effective design tension resistance $F_{tr,Rd}$ of row r of reinforcing bars should be taken as its design tension resistance $F_{tr,s,Rd}$ as an individual row, (see 3.3.9) reduced if necessary to satisfy the conditions specified below.
When determining the value of $F_{tr,Rd}$ for bolt-row r all other bolt-rows closer to the centre of

compression should be omitted.

The tension resistance $F_{tr,Rd}$ of bolt-row r as an individual bolt-row should be taken as the smallest value of the tension resistance for an individual bolt-row of the following basic components:

- The column web in tension $F_{t,wc,Rd}$ (see prEN 1993-1-8 [2] 5.2.6.3) [2] 5.2.6.3
- The column flange in bending $F_{t,fc,Rd}$ (see prEN 1993-1-8 [2] 5.2.6.4) [2] 5.2.6.4
- The end-plate in bending $F_{t,ep,Rd}$ (see prEN 1993-1-8 [2] 5.2.6.5) [2] 5.2.6.5
- The beam web in tension $F_{t,wb,Rd}$ (see prEN 1993-1-8 [2] 5.2.6.8) [2] 5.2.6.8

The effective tension resistance $F_{tr,Rd}$ of row r of reinforcing bars or bolts should, if necessary, be reduced below the given value $F_{t,Rd}$ in order to ensure that, when account is taken of all rows up to and including r:
the total resistance [2] 7.3.3

$$\sum F_{t,Rd} \leq \frac{V_{wp,Rd}}{\beta} \qquad (47)$$

with ß from Table 4;
the total resistance $\sum F_{t,Rd}$ does not exceed the smaller of:

the resistance of the column web in compression $F_{c,wc,Rd}$
(see prEN 1993-1-8 [2] 5.2.6.2) [2] 5.2.6.2

the resistance of the beam flange and web in compression $F_{c,fb,Rd}$
(see prEN 1993-1-8 [2] 5.2.6.7) [2] 5.2.6.7

The effective tension resistance $F_{tr,Rd}$ of bolt-row r should, if necessary, be reduced below the value of $F_{t,Rd}$ in order to ensure that the sum of the resistances taken for the bolt-rows up to and including bolt-row r that form part of the same group of bolt-rows, does not exceed the resistance of that group as a whole. This should be checked for the following basic components.

- The column web in tension $F_{t,wc,Rd}$ (see prEN 1993-1-8 [2] 5.2.6.3) [2] 5.2.6.3
- The column flange in bending $F_{t,fc,Rd}$ (see prEN 1993-1-8 [2] 5.2.6.4) [2] 5.2.6.4
- The end-plate in bending $F_{t,ep,Rd}$ (see prEN 1993-1-8 [2] 5.2.6.5) [2] 5.2.6.5
- The beam web in tension $F_{t,wb,Rd}$ (see prEN 1993-1-8 [2] 5.2.6.8) [2] 5.2.6.8

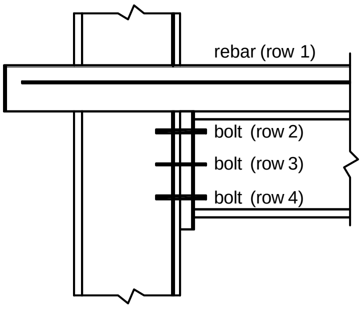
Where the effective tension resistance $F_{tx,Rd}$ of one of the previous bolt-rows x is greater than $1{,}9B_{t,Rd}$ then the effective tension resistance $F_{tr,Rd}$ for bolt-row r should be reduced, if necessary, in order to ensure that:

$$F_{tr,Rd} = F_{tx,Rd}\ h_r / h_x \qquad (48)$$

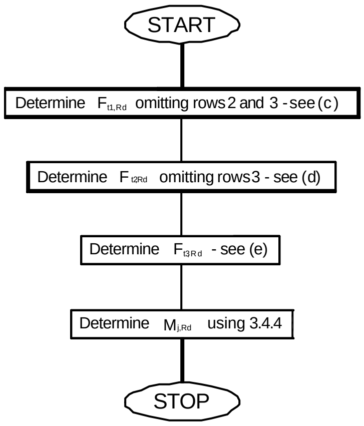
where:
h_x is the distance from bolt-row x to the centre of compression;
x is the bolt-row farthest from the centre of compression that has a tension resistance greater than $1{,}9\ B_{t,Rd}$.

The same method may be applied to bolted beam-to-beam joints with welded end-plates by omitting the items relating to the column.
The method given in prEN 1993-1-8 [2] 5.2.7.2 for a single-sided beam-to-column joint is described in the flow chart below (Figure 43). [2] 5.2.7.2

[2] Figure 5.30

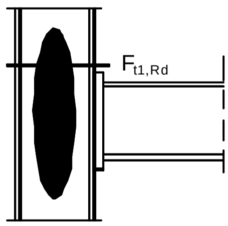
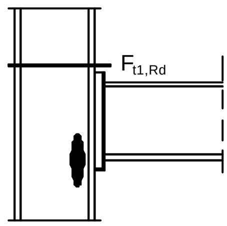
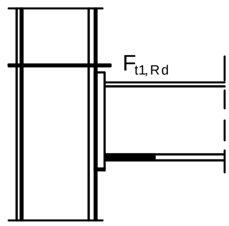
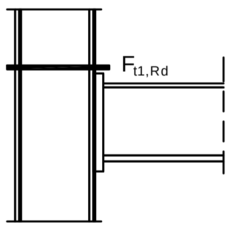


(a) Arrangement of joint



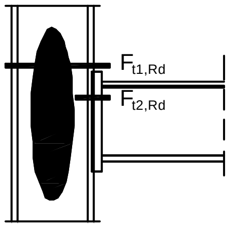
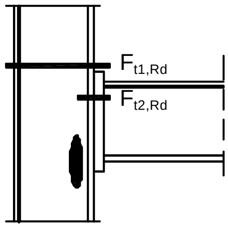
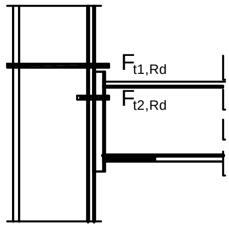
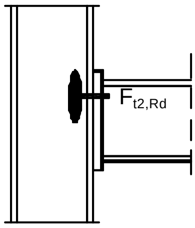
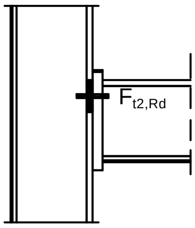
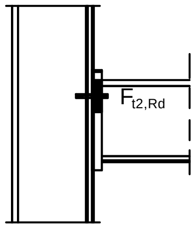
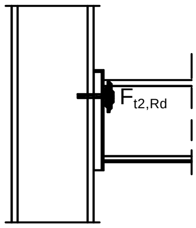
(b) Flow chart

Note: If the resistance of any bolt-row exeeds 1,9 $B_{t,Rd}$ then 3.4.4 also applies

Shear and compression limits, see 3.4.4	 Web panel in shear, see [1] 8.3.3.1 $F_{t1Rd} \le V_{wp,Rd}/\beta$	 Column web in compression, see [1] 8.3.3.2 $F_{t1,Rd} \le F_{c,wc,Rd}$	 Beam flange and web in compression, see [1] 8.3.3.3 $F_{t1Rd} \le F_{c,fb,Rd}$
Tension resistance for rebar (row 1) see 3.4.4	 Rebar in tension see [1] 8.3.3.4 $F_{t1Rd} < F_{tr,s,Rd}$		

In order to satisfy 3.4.4 take F_{t1Rd} as the smallest of: $V_{wp,Rd}/\beta$: $F_{c,wc,Rd}$: $F_{c,fb,Rd}$: $F_{tr,S,Rd}$

(c) Determination of force $F_{t1,Rd}$ in row 1, omitting rows 2 and 3

Shear and compression limits, see 3.4.4	 Web panel in shear, see [1] 8.3.3.1 $F_{t1,Rd} + F_{t2,Rd} \leq V_{wp,Rd}/\beta$	 Column web in compression, see [1] 8.3.3.2 $F_{t1,Rd} + F_{t2,Rd} \leq F_{c,wc,Rd}$	 Beam flange and web in compression, see [1] 8.3.3.3 $F_{t1,Rd} + F_{t2,Rd} \leq F_{c,fc,Rd}$	
Tension resistance for bolt-row 2, see 3.4.4	 Column web in tension, see [2] 5.2.6.3 $F_{t2,Rd} \leq F_{t2,wc,Rd}$	 Column flange in bending, see [2] 5.2.6.4 $F_{t2,Rd} \leq F_{t2,fc,Rd}$	 End plate in bending, see [2] 5.2.6.5 $F_{t2,Rd} \leq F_{t2,ep,Rd}$	 Beam web in tension, see [2] 5.2.6.8 $F_{t2,Rd} \leq F_{t2,wb,Rd}$
In order to satisfy 3.4.4 take $F_{t1,Rd}$ as the smallest of: $V_{wp,Rd}/\beta - F_{c,wc,Rd}$; $F_{c,fb,Rd} - F_{t1,Rd}$; $F_{c,fb,Rd} - F_{t1,Rd}$; $F_{t2,wc,Rd}$; $F_{t2,fc,Rd}$; $F_{t2,ep,Rd}$; $F_{t2,wb,Rd}$				
(d) Determination of force $F_{t2,Rd}$ in bolt-row 2, omitting bolt-row 3				

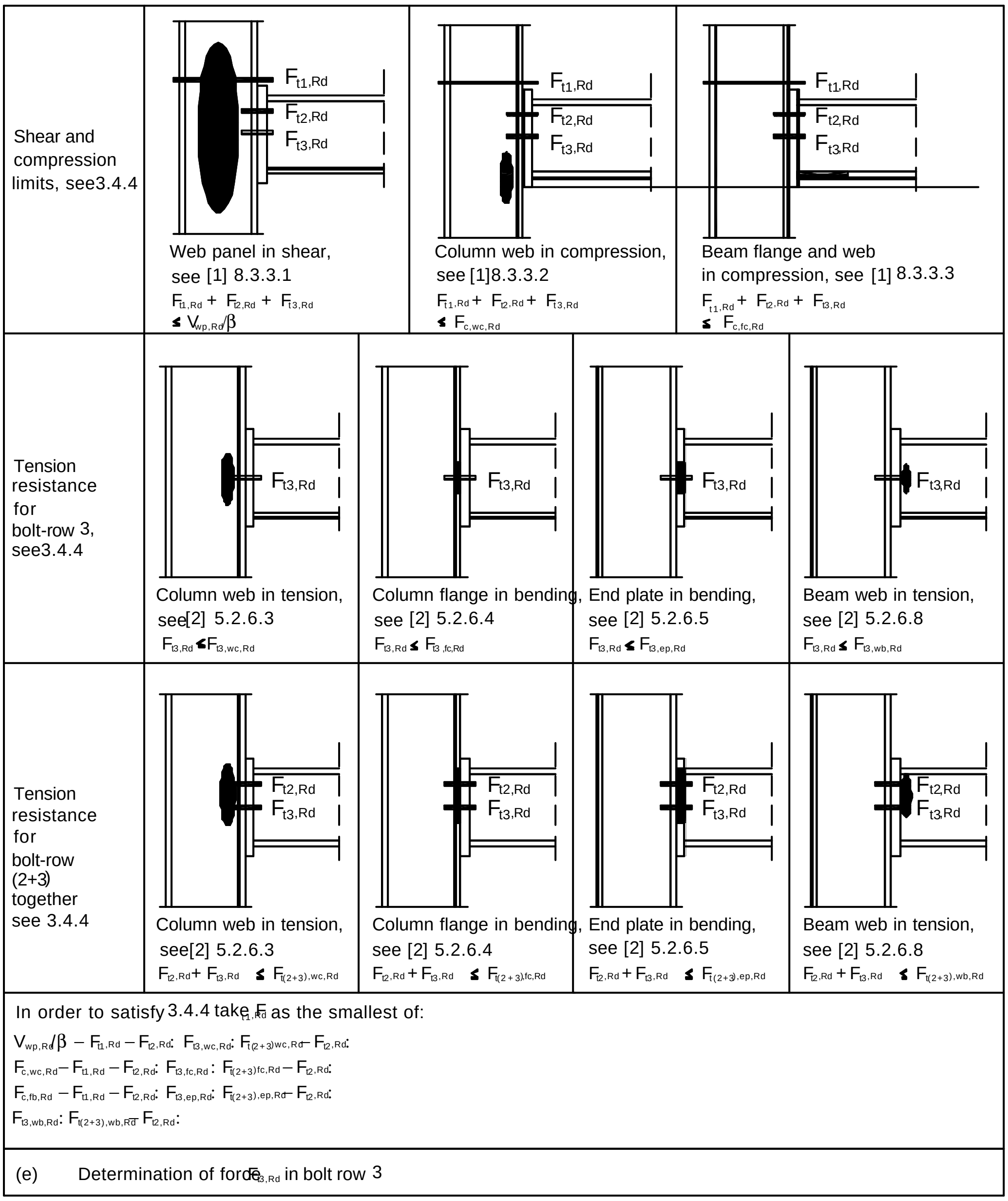


Figure 43 Unstiffened end-plate joint with two bolt-rows in tension

3.5 Rotational stiffness

3.5.1 Basic model

[2] 5.3

The rotational stiffness of a joint should be determined from the flexibilities of its basic components, each represented by its elastic stiffness coefficient k_i. These elastic stiffness coefficients are of general application. The numbering of stiffness coefficients is consistent with that in prEN 1993-1-8 [2]. The elastic translational stiffness of a component i is obtained by multiplying k_i with E_a.

For connections with more than one layer of components in tension, the stiffness coefficients k_i for the related basic components should be combined (see 3.5.2.2).
In a bolted connection with more than one bolt-row in tension, as a simplification, the contribution of any bolt-row may be neglected, provided that the contributions of all other bolt-rows closer to the centre of compression are also neglected. The number of bolt–rows retained need not necessarily be the same for the determination of the moment resistance.
Provided that the axial force N_{Sd} in the connected member does not exceed 10% of the resistance $N_{pl,Rd}$ of its cross-section, the rotational stiffness S_j of a joint, for a moment $M_{j,Sd}$ less than the moment resistance $M_{j,Rd}$ of the joint, may be obtained with sufficient accuracy from:

$$S_j = \frac{E_a z^2}{\mu \sum_i \frac{1}{k_i}} \tag{49}$$

where:

E_a	modulus of elasticity of steel;
k_i	stiffness coefficient for basic joint component i;
z	lever arm, see Figure 38;
μ	stiffness ratio $S_{j,ini} / S_j$, see below;
$S_{j,ini}$	initial rotational stiffness of the joint, given by the expression above with μ=1,0.

The stiffness ratio μ should be determined from the following:

- if $M_{j,Sd} = 2/3\, M_{j,Rd}$ $\quad \mu = 1$ (50)
- if $2/3\, M_{j,Rd} < M_{j,Sd} = M_{j,Rd}$ $\quad \mu = (1{,}5\, M_{j,Sd} / M_{j,Rd})^{\psi}$ (51)

in which the coefficient ? is obtained from Table 8.2 of EN 1994-1-1 [1] or Table 6 below.

Type of connection	Value of ?
Contact plate	1,7
Bolted end-plate	2,7

Table 6 Value of the coefficient ?

3.5.2 Initial stiffness $S_{j,ini}$

The initial rotational stiffness $S_{j,ini}$ is derived from the elastic translational stiffness of the joint components. The elastic behaviour of each component is represented by a spring. The force-deformation relationship of this spring is given by:

$$F_i = E\, k_i\, w_i \tag{52}$$

F_i	the force in the spring i;
E	the modulus of elasticity of structural steel;
k_i	the translational stiffness coefficient of the spring i;
w_i	the deformation of spring i.

Separate rotational stiffness can be derived for the connection and the web panel in shear. In simplified joint modelling a value for the overall joints is all that is required. The derivation of this is now explained.

3.5.2.1 Connections with one layer of components in tension

Contact plate joint:

Figure 44 shows the spring model for a contact plate joint in which tensile forces arising from bending are carried only by one layer of reinforcement.

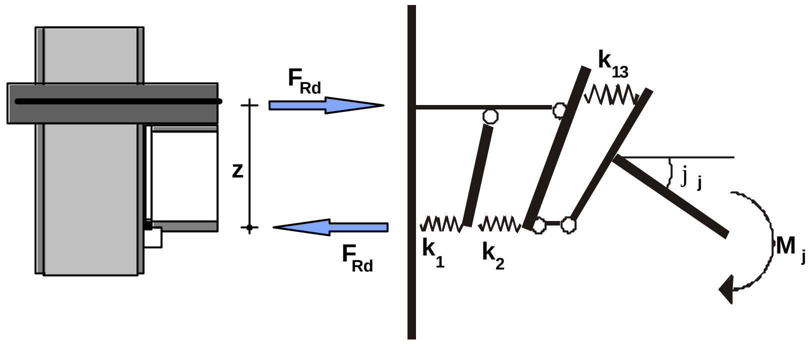


Figure 44 Spring model for a contact plate composite joint

Here, k_1 represents the column web panel in shear, k_2 the unstiffened column web subject to compression from the contact plate and k_{13} the longitudinal reinforcement bars in tension. The contact plate itself is assumed to have infinite stiffness.

The force in each spring is equal to F. The moment M_j acting in the spring model is equal to $F \cdot z$, where z is the distance between the centroid of the reinforcement in tension and the centre of compression (assumed located in the centre of the lower beam flange). The rotation ϕ_j in the joint is equal to $(w_1+w_2+w_{13})/z$. In other words:

$$S_{j,ini} = \frac{M}{\varphi_j} = \frac{Fz}{\frac{\Sigma w_i}{z}} = \frac{Fz^2}{\frac{F}{E}\Sigma\frac{1}{k_i}} = \frac{Ez^2}{\Sigma\frac{1}{k_i}} \qquad (53)$$

3.5.2.2 Connections with more than one layer of components in tension

Joints with steelwork connection effective in tension:

Figure 45a) shows the spring model adapted for a more complicated case where tensile forces arising from bending are carried not only by a layer of reinforcement but also by a bolt-row in tension within the steelwork connection. The reinforcement is assumed to behave like a bolt-row in tension, but with different deformation characteristics. It is assumed that the deformations are proportional to the distance to the point of compression, but that the elastic forces in each row are dependent on the stiffness of the components.

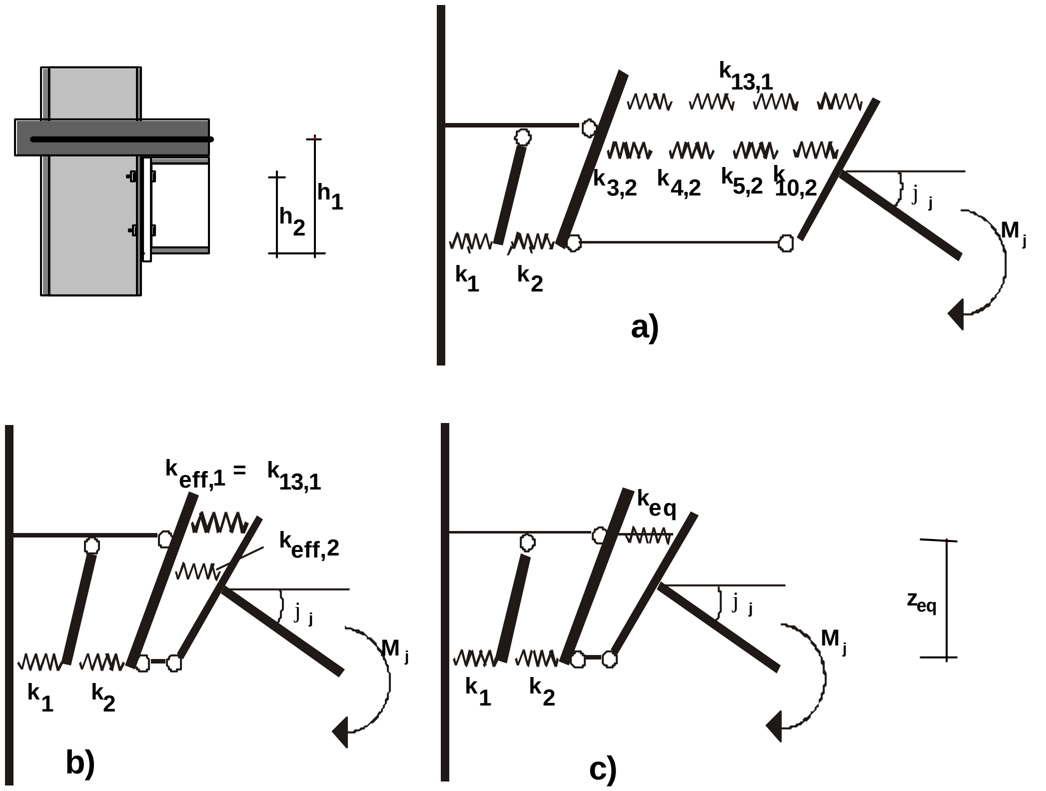


Figure 45 Spring model for a composite flush end-plate connection

Figure 45b) shows how the deformations per bolt-row of the end-plate in bending, the bolts in tension, the column flange in bending and the column web in tension are added to form an effective spring per bolt-row, with an effective stiffness coefficient $k_{eff,r}$ (r is the index of the row number). Figure 45c) indicates how these effective springs at each level are replaced by an equivalent spring acting at a corresponding lever arm z. The stiffness coefficient of this equivalent spring is k_{eq} and this can be directly applied in the preceding equation for the stiffness $S_{j,ini}$.

The formulae given below to determine $k_{eff,r}$, z and k_{eq} can be derived from the sketches of Figure 45. The basis for these formulae is that the moment-rotation behaviour of each of the systems in Figure 45 is equal. An additional condition is that the compressive force in the lower rigid bar is equal in each of these systems.

For composite connections with more than one layer of components in tension, the basic components related to all of these layers should be represented by a single equivalent stiffness coefficient k_{eq} determined from:

$$k_{eq} = \frac{\sum_r k_{eff,r}\, h_r}{z_{eq}} \qquad (54)$$

where:

h_r is the distance between layer r and the centre of compression;

$k_{eff,r}$ is the effective stiffness coefficient for layer r taking into account the stiffness k_i for

the basic components listed below as appropriate;
z_{eq} is the equivalent lever arm.

For a composite joint with more than a single layer of reinforcement considered effective in tension, the above provisions are applicable provided that the layers are represented by a single layer of equivalent cross-sectional area and equivalent distance to the centre of compression.

The effective stiffness coefficient $k_{eff,r}$ for bolt-row r should be determined from:

$$k_{eff,r} = \frac{1}{\sum_i \frac{1}{k_{i,r}}} \tag{55}$$

where:
$k_{i,r}$ is the stiffness coefficient representing component i relative to bolt-row r.

The equivalent lever arm z_{eq} should be determined from:

$$z_{eq} = \frac{\sum_r k_{eff,r}\ h_r^{\,2}}{\sum_r k_{eff,r}\ h_r} \tag{56}$$

In the case of a bolt-row r of a beam-to-column joint with an end-plate connection, $k_{eff,r}$ should be based upon (and replace) the stiffness coefficients k_i for:

- The column web in tension (k_3)
- The column flange in bending (k_4)
- The end-plate in bending (k_5)
- The bolts in tension (k_{10}).

In the case of a bolt-row r of a beam-to-beam joint with bolted end-plates, $k_{eff,r}$ should be based upon (and replace) the stiffness coefficients k_i for:

- The end-plates in bending (k_5)
- The bolts in tension (k_{10}).

In calculating the stiffness of joint configurations with bolted end-plate connections, the stiffness coefficients k_i for the basic components listed in Table 8 should be taken into account.

3.5.3 Stiffness coefficients for basic joint components

In stiffness calculations, the stiffness coefficients k_i for basic components taken into account should be as listed in Table 7 for joint configurations with contact plate connections, and in Table 8 for bolted end-plate connections.

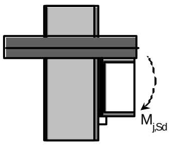
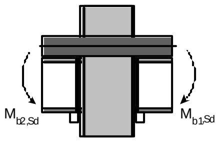
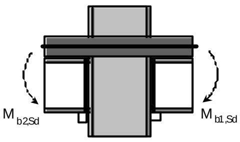
Beam to column joint	**Stiffness coefficients k_i to be taken into account**
 Single sided	k_1 , k_2 , k_{13}
 double sided- Moments equal and opposite	k_2, k_{13}
 double sided- Moments unequal	k_1 , k_2 , k_{13}

Table 7 Stiffness coefficients for joints with contact plate connections

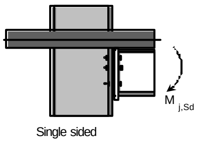
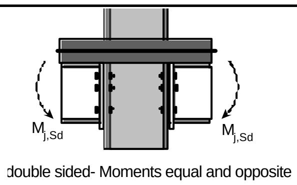
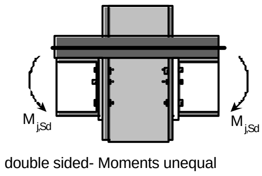
Beam to column joint with bolted end-plate connections	Stiffness coefficients k_i to be taken into account
	k_1, k_2, k_{eq}
	k_2, k_{eq}
	k_1, k_2, k_{eq}

Table 8 Stiffness coefficients for joints with bolted end-plate connections

The basic joint components are identified and listed in this lecture. Other basic components are identified in prEN 1993-1-8 [2] 5.3.2. [2] 5.3.2 [1] 8.4.2

For an unstiffened column web panel in shear, the stiffness coefficient k_1 for a single-sided joint, or a double-sided joint in which the beam depths are similar, should be obtained from the following formulae.

For a joint with a steelwork contact plate or a partial depth end-plate: [1]8.4.2 (2)

$$k_1 = 0{,}87\ \frac{0{,}38\,A_{vc}}{\beta z} \qquad (57)$$

where:

A_{vc} is the shear area of the column, see EN 1993-1 [11] 5.4.6; [11] 5.4.6
z is the lever arm from EN 1994-1-1 [1] 8.2.4; [1] 8.2.4
ß is the transformation parameter from Table 4 and prEN 1993-1-8 [2] 7.3.3. [2] 7.3.3

For a joint with a flush end-plate: [1] 8.4.2 (3)

$$k_1 = \frac{0{,}38\,A_{vc}}{\beta z} \qquad (58)$$

For an unstiffened column web in compression, in a contact plate connection, the stiffness coefficient k_2 should be obtained from: [1] 8.4.2 (4)

$$k_2 = \frac{0{,}2\ b_{eff,c,wc}\ t_{wc}}{d_c} \qquad (59)$$

where:
$b_{eff,c,wc}$ is the effective width of the column web in compression from prEN 1993-1-8 [2] 5.2.6.2; [2] 5.2.6.2
d_c is the clear depth of the column web;
t_{wc} is the thickness of the column web.

For an unstiffened column web in compression, in a bolted end-plate connection, the stiffness coefficient k_2 should be obtained from:

[1] 8.4.2 (5)

$$k_2 = \frac{0{,}7\ b_{eff,c,wc}\ t_{wc}}{d_c} \qquad (60)$$

Where concrete web encasement is used, the stiffness coefficients for the relevant basic joint components k_1 and k_2 should be increased as described below. [1] 8.4.2 (6)

For a column web in shear in a joint with a steelwork contact plate connection, the stiffness coefficient k_1 should be obtained from: [1] 8.4.2 (7)

$$k_1 = 0{,}87\,\frac{0{,}38\,A_{vc}}{\beta z} + 0{,}06\,\frac{E_{cm}}{E_a}\,\frac{b_c\,h_c}{\beta z} \qquad (61)$$

where:
E_a is the modulus of elasticity for structural steel;
E_{cm} is the modulus of elasticity for concrete;
b_c is the breadth of the column´s steel section;
h_c is the height of the column´s steel section.
For a column web in shear in a joint with a flush end-plate connection, the stiffness coefficient k_1 should be obtained from:

[1] 8.4.2 (8)

$$k_1 = \frac{0{,}38\,A_{vc}}{\beta z} + 0{,}06\,\frac{E_{cm}}{E_a}\,\frac{b_c\,h_c}{\beta z} \qquad (62)$$

For a column web in compression in a contact plate connection, the stiffness coefficient k_2 should be obtained from:

[1] 8.4.2 (9)

$$k_2 = \frac{0{,}2\,b_{effc,c,wc}\,t_{wc}}{d_c} + \left(\frac{0{,}13\,b_{el}\,b_c}{h_c}\right)\left(\frac{E_{cm}}{E_a}\right) \qquad (63)$$

where:

$$b_{el} = t_{fb} + 2(t_{fc} + s) + s_p \qquad (64)$$

[2] 5.2.6.2

and s and s_p are defined in prEN 1993-1-8 [2] 5.2.6.2.

For a column web in compression in a bolted end-plate connection, the stiffness coefficient k_2 should be obtained from:

[1] 8.4.2 (10)

$$k_2 = \frac{0{,}7\,b_{effc,c,wc}\,t_{wc}}{d_c} + \left(\frac{0{,}5\,b_{el}\,b_c}{h_c}\right)\left(\frac{E_{cm}}{E_a}\right) \qquad (65)$$

For a column web in tension, in a stiffened or unstiffened bolted connection with a single bolt-row in tension, the stiffness k_3 should be obtained from:

$$k_3 = \frac{0{,}7\, b_{eff,t,wc}\, t_{wc}}{d_c} \tag{66}$$

where:
$b_{eff,t,wc}$ is the effective width of the column web in tension (See 3.3.4). For a joint with a single bolt-row in tension, $b_{eff,t,wc}$ should be taken as equal to the smallest of the effective lengths l_{eff} (individually or as a part of a group of bolt-rows).

For a column flange in bending, for a single bolt-row in tension the stiffness coefficient k_4 should be obtained from:

$$k_4 = \frac{0{,}85\, l_{eff}\, t_{fc}^{\,3}}{m^3} \tag{67}$$

where:
l_{eff} is the smallest of the effective lengths (individually or as part of a bolt group) for this bolt-row for an unstiffened column flange or a stiffened column flange;
m is defined in Figure 46;
t_{fc} is the thickness of the column flange.

For an end-plate in bending, for a single bolt-row in tension the stiffness coefficient k_5 should be obtained from:

$$k_5 = \frac{0{,}85\, l_{eff}\, t_p^{\,3}}{m^3} \tag{68}$$

where:
l_{eff} is the smallest of the effective lengths (individually or as part of bolt-rows);
m is defined in Figure 46.
For a beam flange and web in compression, the stiffness coefficient k_7 should be taken equal to infinity. This component need not be taken into account when calculating the rotational stiffness S_j.

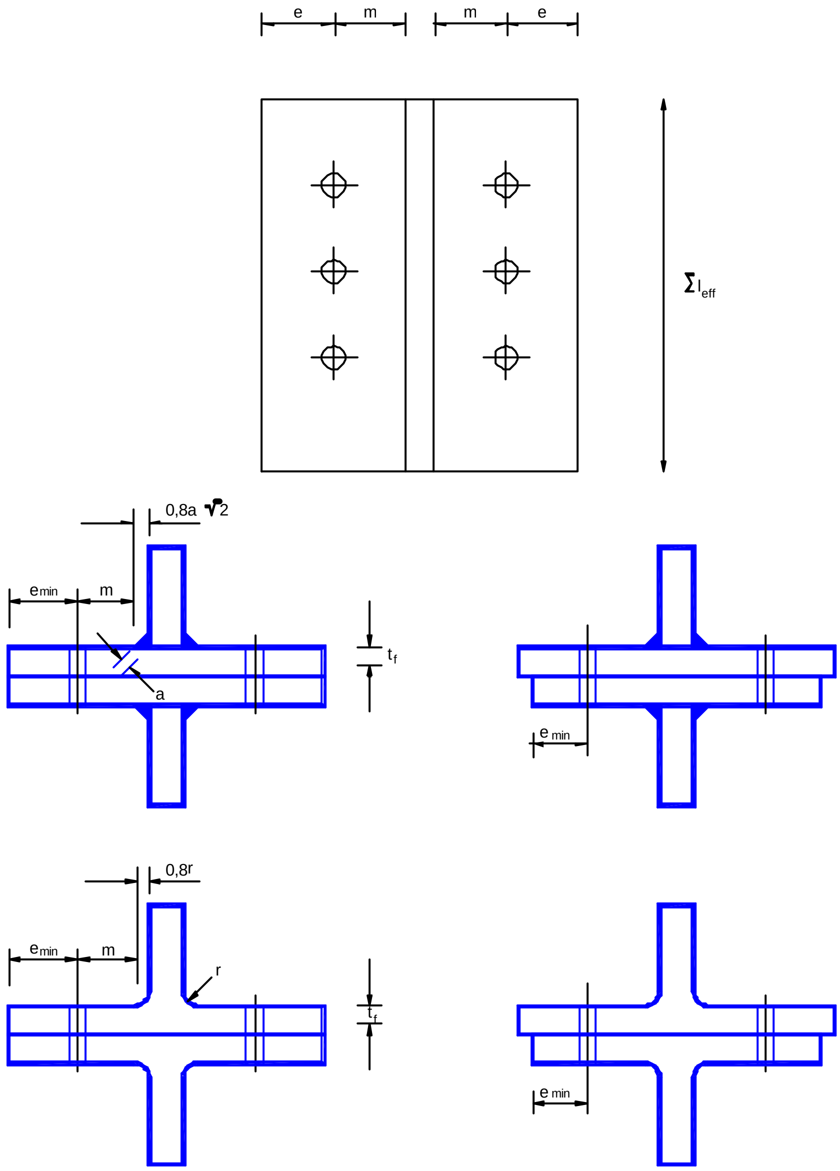


Figure 46 Dimensions of an equivalent T-stub flange

For a beam web in tension, the stiffness coefficient k_8 should be taken as equal to infinity and needs also not be taken into account when calculating the rotational stiffness S_j.

For bolts in tension, the stiffness coefficient k_{10} for a single bolt-row should be obtained from:

$$k_{10}=1{,}6\ A_s\ /L_b \tag{69}$$

where:

A_s is the tensile stress area of the bolt;

L_b is the bolt elongation length, taken as equal to the grip length (total thickness of material and washers), plus half the sum of the height of the bolt head and the height of the nut.

For longitudinal reinforcing bars in tension, the stiffness coefficient k_{13} for a row r should be obtained from formulae below:

For a double-sided joint configuration under balanced loading, $M_{b1,Sd} = M_{b2,Sd}$ (see Figure 41) [1] 8.4.2 (12)

$$k_{13} = \frac{2A_{r,s}}{h_c}\frac{E_s}{E_a} \tag{70}$$

where:

$A_{r,s}$ is the cross-sectional area of the longitudinal reinforcement within the total effective width of concrete flange b_{eff}, as given by EN 1994-1-1 [1] 5.2.2; [1] 5.2.2

E_s is the modulus of elasticity for steel reinforcement;

h_c is the height of the column´s steel section.

For a beam-to-beam joint, the breadth of the flange of the supporting primary beam replaces the height of the column section.

For a single-sided joint configuration: [1] 8.4.2 (13)

$$k_{13} = \frac{A_{r,s}}{h_c(1+K_\beta)}\frac{E_s}{E_a} \tag{71}$$

where:

$K_ß$=2,6.

For a double-sided joint configuration under unbalanced loading, with $M_{b1,Sd} > M_{b2,Sd}$ (see Figure 41)

for the more heavily loaded joint: [1] 8.4.2 (14)

$$k_{13} = \frac{A_{r,s}}{h_c\left(\frac{1+\beta}{2}+K_\beta\right)}\left(\frac{E_s}{E_a}\right) \tag{72}$$

where:

$$K_ß = ß\,(4{,}3\,ß^2 - 8{,}9\,ß + 7{,}2) \tag{73}$$

for the less heavily loaded joint:

$$k_{13} = \frac{A_{r,s}}{h_c\left(\frac{1-\beta}{2}\right)}\left(\frac{E_s}{E_a}\right) \tag{74}$$

For a contact plate in compression, the stiffness coefficient k_{14} should be taken as equal to infinity. This component need not be taken into account when calculating the rotational stiffness S_j. [1] 8.4.2 (15)

3.5.4 Deformation of the shear connection

Unless account is taken of deformation of the shear connection by a more exact method, the influence of slip on the stiffness of the joint should be determined by following: [1] 8.4.3

For:

- a double-sided joint configuration under balanced loading;
- a single-sided joint configuration;
- the more heavily loaded joint in a double-sided configuration under unbalanced loading:

the stiffness coefficient k_{13} (see EN 1994-1-1 [1] 8.4.2) should be multiplied by the reduction factor k_r [1] 8.4.2

$$k_r = \frac{1}{1 + \dfrac{E_s\, k_{13}}{K_{sc}}} \qquad (75)$$

where:

$$K_{sc} = \frac{N\, k_{sc}}{\nu - \dfrac{\nu - 1}{1 + \xi}\dfrac{h_s}{d_s}} \qquad (76)$$

h_s is the distance between the longitudinal reinforcing bars in tension and the centre of compression;
d_s is the distance between longitudinal reinforcing bars in tension and the centroid of the beam´s steel section;

$$\xi = \frac{E_a\, I_a}{d_s^{\,2}\, E_s\, A_s} \qquad (77)$$

$$\nu = \left[\left[1 + \xi\right] N\, k_{sc}\, \ell\, d_s^2 \,/\, (E_a\, I_a)\right]^{0,5} \qquad (78)$$

I_a is the second moment of area of the beam´s steel section;
l is the length of the beam in hogging bending adjacent to the joint, which in a braced frame may be taken as 15% of the length of the span;
N is the number of shear connectors distributed over the length l;
k_{sc} is the stiffness of one shear connector.

The stiffness of the shear connector k_{sc} may be taken as $(0,7\, P_{Rk}/s)$, where: [1] 8.4.3 (3)

P_{Rk} is the characteristic resistance of the shear connector
s is the slip, determined from push tests in accordance with EN 1994-1-1 [1] 11.2 at a load of $0,7\, P_{Rk}$. [1] 11.2

Alternatively, for a solid slab or for a composite slab in which the reduction factor k_t is unity, see EN 1994-1-1 [1] 6.7.5.2, the following approximate values may be assumed for k_{sc}: [1] 8.4.3 (4)

for 19 mm diameter headed studs: 100 kN/mm
for cold-formed angles of 80 mm to 100 mm height: 70 kN/mm.

For a composite joint with more than a single layer of reinforcement considered effective in tension, the above formulae are applicable provided that the layers are represented by a single layer of equivalent cross-sectional area and equivalent distances from the centre of compression and the centroid of the beam´s steel section. [1] 8.4.3 (5)

3.6 Rotation capacity

It is not usual for designers to calculate either the required or available rotation capacity of structural elements. Rotation capacity is essential though if redistribution of bending moments is assumed in the global analysis. For members, well-known classification systems are used to ensure that adequate rotation capacity is available.
With semi-continuous construction, rotation capacity may be required of the joints rather than the members. As a result, prEN 1993-1-8 [2] gives guidance on joint ductility.

Component models can be used to calculate the rotation capacity of a joint, provided the limiting deformation capacity of each active component is known. For steel joints though it is often sufficient to rely on the observed behaviour of critical joint components. Thus for bolted joints the prEN 1993-1-8 [2] permits the designer to assume sufficient rotation capacity for plastic global analysis provided that the moment resistance of the joint is governed by the resistance of one of the following:

- The column web panel in shear;
- The column flange in bending;
- The beam end-plate in bending.

In the latter two cases, the thickness of the flange or the end-plate must also be limited to avoid fracture of the bolts.
For composite joints, yielding of the slab reinforcement in tension is the main source of predictable deformation capacity. The rotation capacity corresponding to this failure mode can be calculated from a simplified component model.

When plastic global analysis is used, the partial-strength joints should have sufficient rotation capacity. Where necessary, see EN 1994-1-1 [1] 8.2.3.3, full-strength joints should also have sufficient rotation capacity. [1] 8.5 (1) [1] 8.2.3.3

When elastic global analysis is used, joints should have sufficient rotation capacity if the conditions given in EN 1994-1-1 [1] 8.2.3.2 are not satisfied. [1] 8.5 (2) [1] 8.2.3.2

A joint with a bolted connection, in which the moment resistance $M_{j,Rd}$ is governed by the resistance of bolts in shear, should not be assumed to have sufficient rotation capacity for plastic global analysis.

In the case of members of steel grades S235, S275 and S355, the provisions given below may be used for joints in which the axial force N_{Sd} in the connected member does not exceed 10% of the resistance $N_{pl.Rd}$ of its cross-section. However, these provisions should not be applied in the case of members of steel grades S420 and S460.

A beam-to-column joint in which the moment resistance of the joint $M_{j,Rd}$ is governed by the resistance of the column web panel in shear, may be assumed to have sufficient rotation capacity for plastic global analysis. [1] 8.5 (3)
The steelwork parts of a composite joint with a bolted connection with end-plates may be assumed to have sufficient rotation capacity for plastic analysis, provided that both of the [1] 8.5 (4)

following conditions are satisfied:

- The moment resistance of the steelwork connection is governed by the resistance of either:
    - The column flange in bending;
    - The beam end-plate in bending.
- The thickness t of either the column flange or beam end-plate satisfies:

$$t \leq 0{,}36\ d\sqrt{\frac{f_{ub}}{f_y}} \qquad (79)$$

where:

d is the nominal diameter of the bolts;
f_{ub} is the ultimate tensile strength of the bolts;
f_y is the yield strength of the relevant basic component.

The rotation capacity of a composite joint may be determined by testing. Alternatively, appropriate calculation models may be used. [1] 8.5 (5)

The design rotation capacity determined from a tested structure or element should be adjusted to take account of possible variations of the properties of materials from specified characteristic values. [1] 8.5 (6)

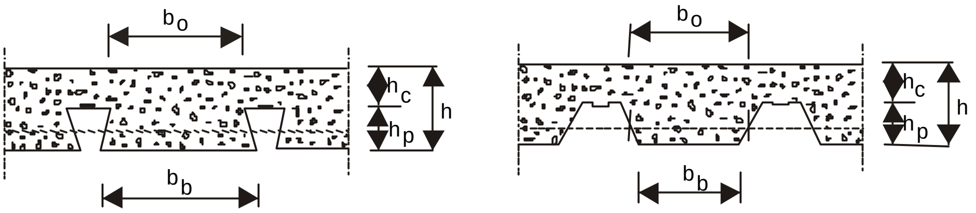
3.7 Detailing

3.7.1 Longitudinal slab reinforcement

For ductile failure, the total cross-sectional area of longitudinal reinforcement A_s should not exceed the following limit: [1] 8.6 1 (1)

$$A_s \leq \frac{1{,}10\ (0{,}85\, f_{ck} / \gamma_c)\, b_c\ d_{eff}}{\beta\, f_{sk} / \gamma_s} \qquad (80)$$

where:
for a solid slab d_{eff} is the overall depth of the slab;
for a composite slab $d_{eff} = h_c$ (see Figure 47).



re-entrant trough profile   Open trough profile

Figure 47 Sheet and slab dimensions

Longitudinal reinforcement for a composite joint shall be positioned so that the distance e_L from the axis of the column web to the centre of gravity of the longitudinal reinforcement placed each side of the column is within the following limits: [1] 8.6 1 (2)

$$0{,}7\ b_c = e_L = 2{,}5\ b_c \qquad (81)$$

where:

b_c is the width of the column´s steel section.

3.7.2 Transverse slab reinforcement

Transverse slab reinforcement shall be provided adjacent to the column.

Transverse reinforcement shall be positioned so that the distance e_T from the face of the column´s steel section to the centre of gravity of the transverse reinforcement placed each side of the column is within the following limits: [1] 8.6 1 (2)

$$e_L = e_T = 1{,}5\, e_L \tag{82}$$

Sufficient transverse reinforcement should be provided so that the design tensile resistance of the transverse reinforcement placed each side of the column fulfils the condition: [1] 8.6 1 (3)

$$\frac{A_T\, f_{sk,T}}{\gamma_s} \geq \frac{\beta}{2\tan\delta}\frac{A_s\; f_{sk}}{\gamma_s} \tag{83}$$

where:

A_T is the area of transverse reinforcement placed each side of the column;

$f_{sk,T}$ is the characteristic tensile strength of that reinforcement, and:

$$\tan\delta = 1{,}35\left(\frac{e_T}{e_L} - 0{,}20\right) \tag{84}$$

[1] 8.6 1 (3)

3.7.3 Anchorage of reinforcement

Bars shall be anchored in accordance with EN 1992-1 [10] so as to develop their design tension resistance.

For a single-sided joint configuration, the longitudinal slab reinforcement in tension shall be anchored sufficiently well beyond the span of the beam to enable the design tension resistance to be developed.

4 Summary

4.1 General

<u>Following configurations and connection types are described</u>

- Double-sided beam-to-column joint configurations
- Single-sided beam-to-column joint configurations

<u>Types of steelwork connections:</u>

- Connections with contact plates
- Connections with partial-depth end-plates
- Connections with flush end-plates

4.2 Field of application of the calculation procedures

- Static loading
- Small normal force N in the beam

($N/N_{pl} < 0,1$ where N_{pl} is the squash load of the beam)

- Strong axis beam-to-column joints
- In the case of beam-to-beam joints and two-sided beam-to-column joints, only a slight difference in beam depth on each side is possible
- Steel grade: S235 to S355
- H and I hot-rolled cross-sections
- Flush end-plates: one bolt-row in tension zone
- All bolt property classes as in prEN 1993-1-8 [2], it is recommended to use 8.8 or 10.9 grades of bolts
- One layer of longitudinal reinforcement in the slab
- Encased or bare column sections

4.3 Summary of key steps

The calculation procedures are based on the component method which requires three steps:

- Definition of the active components for the studied joint;
- Evaluation of the stiffness coefficients (k_i) and/or strength ($F_{Rd,i}$) characteristics of each individual basic component;
- Assembly of the components to evaluate the stiffness (S_j) and/or resistance (M_e, M_{Rd}) characteristics of the whole joint.

The stiffness (k_i) and the design resistance ($F_{Rd,i}$) of each component are evaluated from analytical models. The assembly is achieved as follows:

<u>Initial stiffness:</u>

$$S_{j,ini} = \frac{E_a z^2}{\sum_{i=1,n} \frac{1}{k_i}} \tag{85}$$

where:

z relevant lever arm;
n number of relevant components;
E_a steel elastic modulus.

<u>Nominal stiffness:</u>

$S_j = \frac{S_{j,ini}}{1,5}$ for beam-to-column joints with contact plates

$S_j = \frac{S_{j,ini}}{2,0}$ for beam-to-column joints with flush end –plates

<u>Plastic design moment resistance:</u>

$$F_{Rd} = \min [F_{Rd,i}] \tag{86}$$

$$M_{Rd} = F_{Rd} \cdot z \tag{87}$$

<u>Elastic design moment resistance:</u>

$$M_e = 2/3\, M_{Rd} \tag{88}$$

4.4 Conclusion

Conventionally joints have been treated either as pinned or as fully rigid due to a lack of more realistic guidance in view of modelling. In reality both assumptions may be inaccurate and uneconomic and do only represent the boundaries of the real moment-rotation behaviour. They may lead to a wrong interpretation of the structural behaviour in terms of load resistance and deflections. So whereas up to now the joint construction expensively has been adopted to the possibilities of calculation, the new approach is to develop efficient joint types first and to take their realistic behaviour into consideration within the global frame analysis afterwards.

In contrast to the idealising assumptions of beam-to-column connections as hinges or full restraints with the main attention to their resistance, the interest of recent research activities lies in the assessment of the whole non-linear joint response with the three main characteristics.

- **Initial rotational stiffness**
- **Moment resistance**
- **Rotation capacity**

The moment-rotation behaviour of joints can be represented in structural analysis in a practical way:
The joint representation covers all necessary actions to come from a specific joint configuration to its reproduction within the frame analysis. These actions are the:

- **Joint characterisation:** determination of the joint response in terms of M-f curves reflecting the joint behaviour in view of bending moments and shear (in case of moment imbalance).
- **Joint classification:** if aiming at conventional modelling the actual joint behaviour may be compared with classification limits for stiffness, strength and ductility; if the respective requirements are fulfilled a joint then may be modelled as a hinge or as a rigid restraint.
- **Joint idealisation:** for semi-continuous joint modelling the M- f behaviours have to be taken into consideration; depending on the desired accuracy and the type of global frame analysis the non-linear curves may be simplified as bi- or tri linear approximations.
- **Joint modelling:** reproduction (computational model) of the joint´s M- f behaviour within the frame modelling for global analysis.

An analytical description of the behaviour of a joint has to cover all sources of deformabilities, local plastifications, plastic redistribution of forces within the joint itself and local instabilities. Due to the multitude of influencing parameters, a macroscopic inspection of the complex joint by subdividing it into "components" has proved to be most appropriate. In comparison with the finite element method, these components, which can be modelled by translational spring with non-linear force-deformation response, are exposed to internal forces and not to stresses.
The procedure of the COMPONENT METHOD can be expressed in three steps:

- Component **identification**
  determination of contributing components in compression, tension and shear in view of connecting elements and load introduction into the column web panel.
- Component **characterisation**
  determination of the component´s individual force-deformation response with the help of analytical mechanical models, component tests of FE-simulations.
- Component **assembly**
  assembly of all contributing translational component springs to overall rotational joint springs according to the chosen component mode.

Basic components of a joint are described and detailed design provisions for resistance and rotational stiffness are given.

Structural Steelwork Eurocodes
Development of
a Trans-National Approach

Course: Eurocode 4

Lecture 9 : Composite joints
Annex B1

Annex B1: **Calculation procedures and worked examples**
Joints with contact plate connection in single- or double-sided configurations

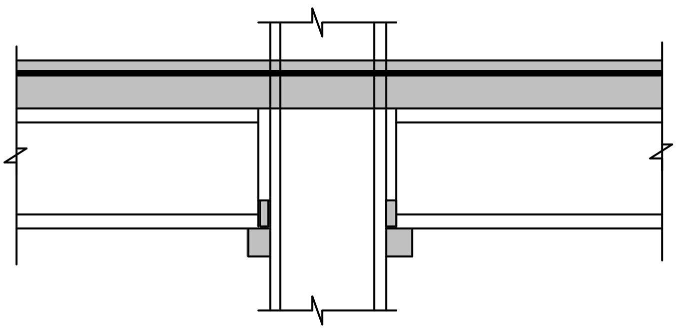
References:

- COST C1: Composite steel-concrete joints in frames for buildings: Design provisions
  Brussels, Luxembourg 1999

STIFFNESS AND RESISTANCE CALCULATION

FOR BEAM-TO-COLUMN COMPOSITE JOINTS

WITH CONTACT PLATES



Calculation procedure

for rotational and shear characteristics

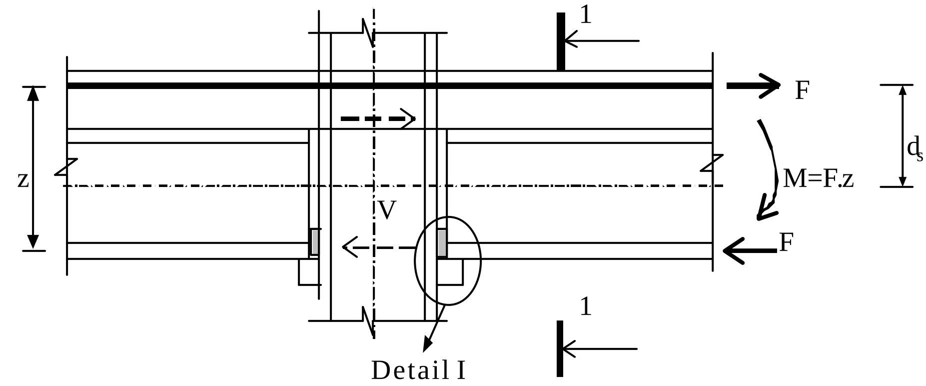
Mechanical characteristics

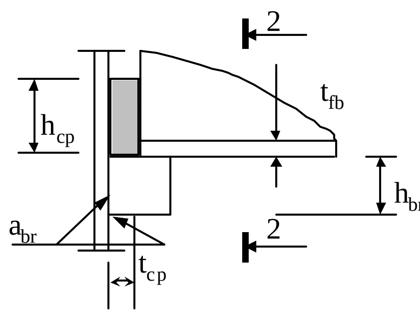
Components	Component strength	Modulus	Partial safety factors
Beam web	f_{ywb}	E_a	Structural steel :
Beam flange	f_{yfb}	E_a	$\gamma_{Mo} = \boxed{1.1}$
Column web	f_{ywc}	E_a	
Column flange	f_{yfc}	E_a	Steel reinforcement :
Bracket	f_{ybr}	E_a	$\gamma_s = \boxed{1.15}$
Contact plate	f_{ycp}	E_{cp}	
Reinforcement	f_{sk}	E_s	Concrete :
Concrete of slab	f_{cks}	$E_{cm.s}$	$\gamma_c = \boxed{1.5}$
Concrete encased column	f_{ckc}	$E_{cm.c}$	Welds :
			$\gamma_{Mw} = \boxed{1.25}$

If hot-rolled profiles : $f_{ywc} = f_{yfc}$ and $f_{ywb} = f_{yfb}$

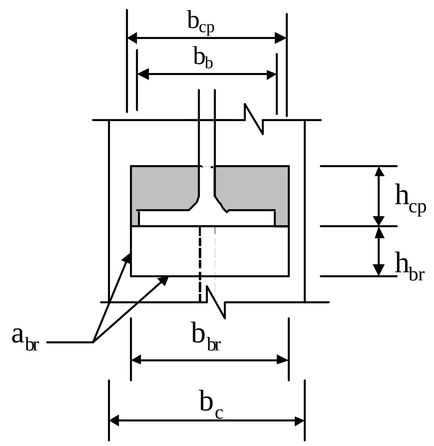
Geometrical characteristics

Joint



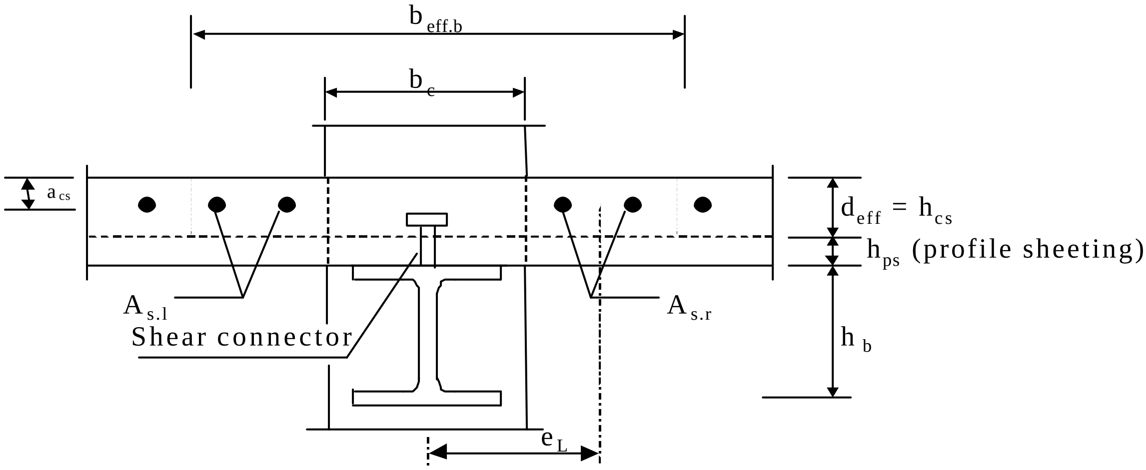


Detail I
(Contact plate detail)



Section 2-2
(side view of contact plate)

Beam

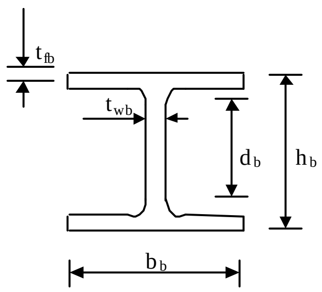


Section 1-1
Composite section with profile steel sheeting concrete slab

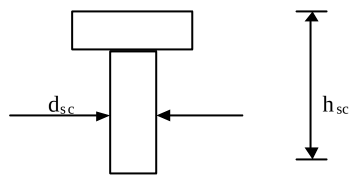
$A_s = A_{s.l} + A_{s.r}$: reinforcement area in the joint .
$\mu\% = A_s / [(b_{eff.b} - b_c) d_{eff}]$: reinforcement ratio .
in which : $A_{s.l}$; $A_{s.r}$: area of longitudinal reinforcements on the left and the right of composite beam section, respectively.

e_L = distance from the axis of the column web to the centre of gravity of the longitudinal reinforcement placed each side of the column
e_L should fulfill the following conditions : $0.7\,b_c \le e_L \le 2.5\,b_c$

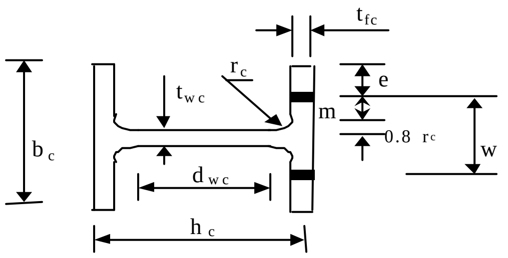
e_T = distance from the face of the column steel section to the centre of gravity of the transverse reinforcement placed each side of the column
e_T should fulfill the following conditions : $e_L \le e_T \le 1.5\,e_L$



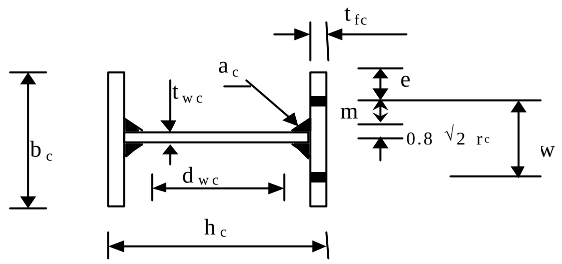
Shear connector



Column



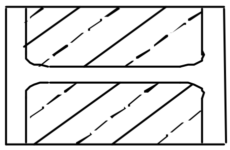
Unencased column-Rolled section



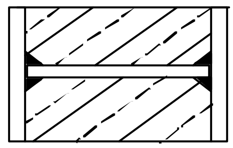
Unencased column-Welded section

$A_{vc} = A_c - 2\, b_c\, t_{fc} + (\, t_{wc} + 2\, r_c\,)\, t_{fc}$
A_c : column section area

$A_{vc} = (\, h_c - 2\, t_{fc}\,)\, t_{wc}$



Encased column-Rolled section



. $s = r_c$: for a rolled section

. $s = a_c \sqrt{2}$: for a welded section

Encased column-Welded section

	STIFFNESS	RESISTANCE
Column web panel in shear	$k_1 = k_{a,wp.s} + k_{c,wp.s}$ in which : $k_{a,wp.s} = \frac{0.33 A_{vc}}{b\ z}$ **.If unencased column :** $k_{c,wp.s} = 0$ **.If encased column :** $k_{c,wp.s} = \frac{0.06\, b_c\ h_c}{b\ z} \frac{E_{cm.c}}{E_a}$	$F_{Rd,1} = \frac{V_{wp,Rd}}{b}$ with $V_{wp,Rd} = V_{a,wp,Rd} + V_{c,wp,Rd}$ in which : $V_{a,wp,Rd} = \frac{0.9\ A_{vc}\ f_{ywc}}{\sqrt{3}\ g_{Mo}}$ **.If unencased column :** $V_{c,wp,Rd} = 0$ **.If encased column :** $V_{c,wp,Rd} = u\ A_c \sin q\ \frac{0.85\ f_{ck\,c}}{g_c}$ where : $A_c = 0.8\,(h_c - 2\,t_{fc})(b_c - t_{wc})\cos q$ $q = \arctan\left(\frac{h_c - 2t_{fc}}{z}\right)$ $u = 0.55\left[1 + 2\frac{N_{Sd}}{N_{pl,Rd}}\right] \leq 1.1$ .N_{Sd} : is the design axial compressive force in the column. .$N_{pl,Rd}$: is the design plastic resistance of encased column cross-section.

	$\beta =$ 1 : for single-sided joint configurations. 0 : for double-sided joint configurations symmetrically loaded. 1 : for double-sided joint configurations non-symmetrically loaded with balanced moments.	
Column web in compression	$k_2 = k_{a.wc.c} + k_{c.wc.c}$ $k_{a.wc.c} = \dfrac{0.2 b_{eff,c,wc} t_{wc}}{d_{wc}}$ **.If unencased column :** $k_{c.wc.c} = 0$ **.If encased column :** $k_{c.wc.c} = \dfrac{0.13\, b_{el}\, b_c}{h_c} \dfrac{E_{cm.c}}{E_a}$	$F_{Rd,2} = F_{a,wc.c,Rd} + F_{c,wc.c,Rd}$ In which : $F_{a,wc.c,Rd} = k_{wc,a} \omega_c b_{eff,c,wc} t_{wc} f_{ywc}/\gamma_{Mo}$ $if\ \bar{\lambda}_p \le 0.67$ $F_{a,wc.c,Rd} = k_{wc,a} \omega_c b_{eff,c,wc} t_{wc} \left[\dfrac{1}{\bar{\lambda}_p}\left(1 - \dfrac{0.22}{\bar{\lambda}_p}\right)\right] f_{ywc}/\gamma_{Mo}$ $if\ \bar{\lambda}_p > 0.67$ $k_{wc,a} = \min\left[1.0\,;1.25 - 0.5\dfrac{\sigma_{com,a,Ed}}{f_{ywc}}\right]$
		.If unencased column : $\bar{\lambda}_p = 0.932\sqrt{\dfrac{b_{eff,c,wc} d_{wc} f_{ywc}}{E_a\ t_{wc}^2}}$ $if\ \beta = 1:\ \ \omega_c = \dfrac{1}{\sqrt{1 + 1.3(b_{eff,c,wc}\ t_{wc} / A_{vc})^2}}$ $if\ \beta = 0:\ \omega_c = 1$ And : $F_{c,wc.c,Rd} = 0$ **.If encased column :** $\omega_c = 1\ and\ \ \bar{\lambda}_p = 0$ and $F_{c,wc.c,Rd} = 0.85 k_{wc,c} t_{eff,c} (b_c - t_{wc}) f_{ckc}/\gamma_c$ *with* $k_{wc,c} = \min[1.3 + 3.3\sigma_{com,c,Ed}/(f_{ckc}/\gamma_c)\ ;2.0]$ $\sigma_{com,a,Ed}$: normal stresses in column web at the root of fillet radius or of the weld. .$\sigma_{com,c,Ed}$: is the longitudinal compressive stress in the encasement due to axial force N_{Ed} .

	$\ell_o = \min\ [\ t_{fb} + t_{cp}\ ;\ h_{cp}\ ]$ $t_{eff,c} = \ell_o + 5\ t_{fc}$ $b_{eff,c,wc} = \ell_o + 5(s + t_{fc})$ $b_{el} = \ell_o + 2(s + t_{fc})$	
Beam flange in compression	$k_7 = \infty$	$F_{Rd7} = M_{c,Rd} / (h_b - t_{fb})$.$M_{c,Rd}$: is steel beam design moment resistance
Longitudi-nal slab reinforce-ment in tension	$k_{13} = k_{s.t} k_r$ where : $k_{s,t} = \dfrac{A_s}{h_c\ (\dfrac{1+b}{2} + K_b\ )} \dfrac{E_s}{E_a}$ $K_b = b\ (4.3 b^2 - 8.9 b + 7.2)$ • For single-sided joint configuration: *b = 1* • For double-sided joint configuration symmetrically loaded with balanced moments: *b = 0* for both sides • For double-sided joint configuration non-symmetrically loaded with balanced moments: + for the more heavily loaded side : β = 1 + for the less heavily loaded side : β = 0 and $k_{s,t} = \infty$ $k_r = \dfrac{1}{1 + \dfrac{E_s\ k_{s.t}}{K_{sc}}}$ $with\ K_{sc} = \dfrac{N\, k_{sc}}{u - \dfrac{u-1}{1+x} \dfrac{z}{d_s}}$ $x = \dfrac{E_a\ I_a}{d_s^2\ E_s\ A_s}$ $u = \sqrt{\dfrac{(1+x)\, N\, k_{sc}\, l_{eff.b}\ d_s^2}{E_a\ \ I_a}}$ .k_{sc} : is the stiffness of one shear connector 100 kN/mm for 19mm headed studs	$F_{Rd,13} = \dfrac{A_s\ f_{sk}}{g_s}$ with : $A_s \geq 0.004\ d_{eff}\ (b_{eff.b} - b_c)$ $A_s \leq \dfrac{1.1(0.85\ f_{ck\ s} / g_c) b_c\ d_{eff}}{b\ \ (f_{sk} / g_s)}$.$b_{eff.b}$: effective width of the composite beam cross-section. A minimum amount of transverse reinforcement A_T should be placed each side of the column : $A_T \geq 0.5 b A_s / tgd$ In which : $tgd = 1.35(e_T/e_L - 0.2)$

	$l_{eff.b}$: is the length of beam in hogging bending zone. I_a : steel beam second moment of inertia N : is the number of shear connectors distributed over the length $l_{eff.b}$.	
Contact plate in compression	$k_{14} = \infty$	No resistance check is required but the following conditions should be fulfilled : • $b_{cp} \geq min\ (b_c ; b_b)$ • $h_{cp} \geq t_{fb}$ • $f_{ycp} \geq f_{yfb}$
JOINT	.Initial stiffness : $S_{j,ini} = \dfrac{E_a\ z^2}{\sum_i \dfrac{1}{k_i}}$ $with\ \ i = 1,\ 2,\ 7\ ,13\ ,\ 14$. Nominal stiffness : $S_j = S_{j,ini} / 1.5$	$F_{Rd} = \min\ [\ F_{Rd,i}\ ]$ $with\ i = 1,\ 2,\ 7,\ 13,\ 14$ .Elastic moment resistance : $M_{e,Rd} = \dfrac{2}{3} F_{Rd}\, z$ .Plastic design moment resistance : $M_{Rd} = F_{Rd}\ z$

SHEAR RESISTANCE

1. Shear resistance of the fillet welds :

$$V_{Rd,1} = f_{vw.d}\ a_{br}\ l_{br}$$

In which $l_{br} = 2\ h_{br} + b_{br}$: total length of fillet welds connecting the bracket to the column

$$f_{vw.d} = \frac{f_u / \sqrt{3}}{b_w\ g_{Mw}}$$

In which f_u is the lower of the ultimate stresses of the column flange and the bracket

b_w should be taken as follows (linear interpolation between the values if needed):

Steel grade	Ultimate tensile strength f_u	b_w
S235	360 N/mm²	0.8
S275	430 N/mm²	0.85
S355	510 N/mm²	0.9

2. Shear resistance of the bracket :

$$V_{Rd,2} = b_{br}\ h_{br}\ (f_{ybr} / \sqrt{3})\ / g_{Mo}$$

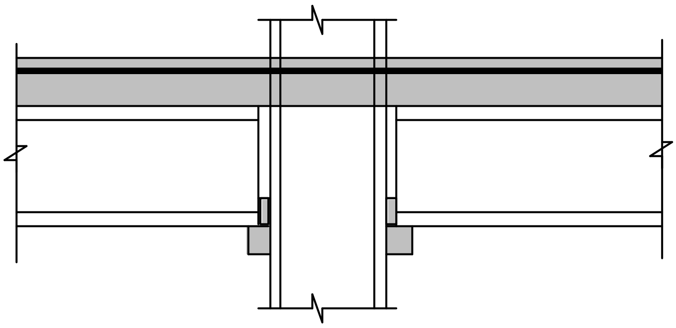
3. Shear resistance of the joint :

$$V_{Rd} = \min [V_{Rd,1} ; V_{Rd,2}]$$

STIFFNESS AND RESISTANCE CALCULATION

FOR BEAM-TO-COLUMN COMPOSITE JOINTS

WITH CONTACT PLATES

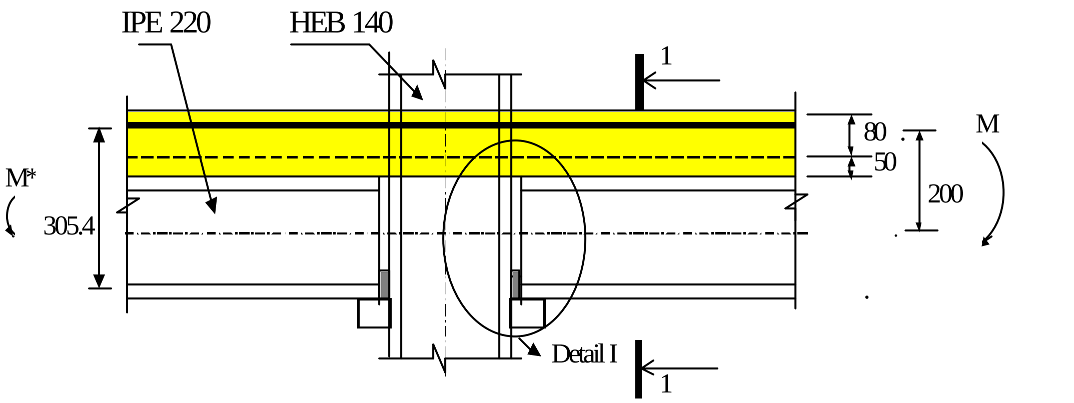


Worked example for the evaluation of

the rotational characteristics

A- DATA FOR CALCULATION

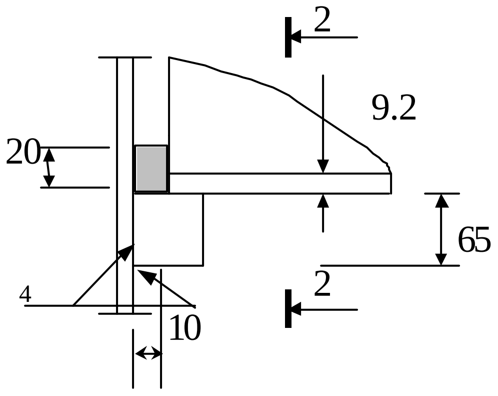
- Main data for joint and loading



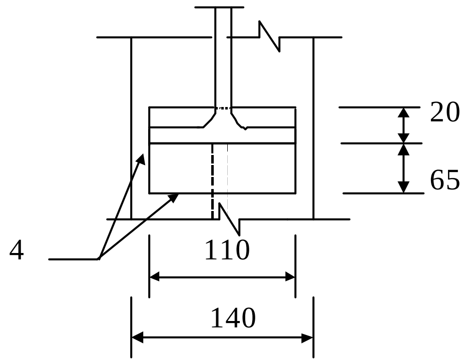
Calculation of the right joint with M > M*

- Dimensions of contact plate:

h_{cp}	b_{cp}	t_{cp}	h_{br}	b_{br}	t_{br}	a_{br}
20	110	10	65	110	15	4



Detail I
(contact plate detail)



Section 2-2
(side view of contact plate)

- Other various data

Grade for all steel components except rebars: S235
Percentage of reinforcement in the effective slab: 0.7%
Connectors : - Stiffness: 100 kN/mm
- Spacing: 100 mm
Reinforcement bars: f_{sk} = 460 N/mm²
Concrete: f_{ck} = 20 N/mm² and E_{cm} = 29000 N/mm²

B- PRELIMINARY CALCULATIONS

Column:

$d_{wc} = h_c - 2\ t_{fc} - 2\ r_c = 140 - 2 \times 12 - 2 \times 12 = 92$ mm

$A_{vc} = A_c - 2\ b_c\ t_{fc} + (t_{wc} + 2\ r_c)\ t_{fc} = 4295.6 - 2 \times 140 \times 12 + (\ 7 + 2 \times 12) \times 12 = 1307.6\ mm^2$.

$s = r_c = 12$ mm (for a hot-rolled section)

Beam:

$d_s = 0.5\ h_b + h_{cs} + h_{ps} - a_{cs} = 0.5 \times 220 + 80 + 50 - 40 = 200$ mm

$z = h_b - 0.5\ t_{fb} + h_{cs} + h_{ps} - a_{cs} = 220 - 0.5 \times 9.2 + 80 + 50 - 40 = 305.4$ mm

$$M_{c.Rd}(class\ 1) = \frac{W_{pby}\ f_{yb}}{g_{Mo}} = \frac{285406 \times 235}{1.1} = 60973100.517\ N.mm \cong 60.973\ kN.m$$

Concrete slab:

$d_{eff} = h_{cs} = 80$ mm

$b_{eff.b} = 3\ h_b = 3 \times 220 = 660$ mm

$l_{eff.b} = 4\ h_b = 4 \times 220 = 880$ mm

$$m = 0.7\% \;\rightarrow A_s = m \; d_{eff} \left(b_{eff.b} - b_c\right) = \frac{0.7}{100} 80 \times (660 - 140) = 291\,\text{mm}^2$$

Shear connectors:

k_{sc} = 100 kN/mm^2 *(assumption)*

$$N = \frac{l_{eff.b}}{spacing} + 1 = \frac{880}{100} + 1 \cong 10 \quad (assumption)$$

C- CALCULATION OF THE STIFFNESS AND RESISTANCE PROPERTIES

Component 1: Column web panel in shear

a) Resistance:

- *If unencased column:*

$$V_{a,wp,Rd} = \frac{0.9 \; A_{vc} \; f_{ywc}}{\sqrt{3}\; g_{Mo}} = \frac{0.9 \times 1307.6 \times 235}{\sqrt{3} \times 1.1} = 145155 \;\; \text{N} = 145.155 \;\; \text{kN}$$

- *If encased column:*

$$u = 0.55 \left[1 + 2\frac{N_{Sd}}{N_{pl,Rd}}\right] = 0.626 \; (hyp)$$

$$q = \arctan\left(\frac{h_c - 2t_{fc}}{z}\right) = \arctan\left(\frac{140 - 2 \times 12}{305.4}\right) = 20.8^{\text{o}}$$

$$A_c = 0.8(h_c - 2t_{fc})(b_c - t_{wc})\cos q = 0.8 \times (140 - 2 \times 12) \times (140 - 7) \times \cos\left(20.8^{o}\right) = 11538 \;\text{mm}^2$$

$$V_{c,wp,Rd} = \nu \, A_c \sin\theta \, \frac{0.85\, f_{ck\,c}}{\gamma_c} = 0.626 \times 11538 \times \sin\left(20.8^{o}\right) \times \frac{0.85 \times 20}{1.5} = 29068 \ \text{N} = 29.068 \ \text{kN}$$

- *Total resistance:*

  - *unencased column:*

$$F_{Rd,1} = \frac{V_{wp,Rd}}{\beta} = \frac{V_{a,wp,Rd}}{\beta} = \frac{145.155}{1} = 145.155 \ kN$$

  - *encased column:*

$$F_{Rd,1} = \frac{V_{wp,Rd}}{\beta} = \frac{V_{a,wp,Rd} + V_{c,wp,Rd}}{\beta} = \frac{145.155 + 29.068}{1} = 174.223 \ \text{kN}$$

b) Stiffness:

- *If unencased column:*

$$k_{a,wp.s} = \frac{0.33\, A_{vc}}{\beta \;\; z} = \frac{0.33 \;\; \times 1307.6}{1 \times 305.4} = 1.415 \ \ \text{mm}$$

- *If encased column:*

$$k_{a,wp.s} = \frac{0.33\, A_{vc}}{\beta \;\; z} = \frac{0.33 \;\; \times 1307.6}{1 \times 305.4} = 1.415 \ \ \text{mm}$$

$$k_{c,wp.s} = \frac{0.06 \;\, b_c \;\, h_c}{\beta \; z} \frac{E_{cm.c}}{E_a} = \frac{0.06 \times 140 \times 140}{1 \times 305.4} \;\; \frac{29}{210} = 0.532 \ \text{mm}$$

- *Total stiffness:*

  - *unencased column:*

    $k_1 = k_{a,wp.s} = 1.415$ mm

  - *encased column:*

    $k_1 = k_{a,wp.s} + k_{c,wp.s} = 1.415 + 0.532 = 1.95$ mm

Component 2: Column web in compression

a) Resistance:

$$\ell_o = \min\ [\ t_{fb} + t_{cp}\ ;\ h_{cp}\ ] = \min\ \left[9.2 + 10\ ;\ 20 \right] = 19.2\ mm$$

$$t_{eff\ .c} = \ell_o + 5\ t_{fc} = 19.2 + 5 \times 12 = 79.2\ mm$$

$$b_{eff\ ,c,wc} = \ell_o + 5\,(s + t_{fc}) = 19.2 + 5 \times (12 + 12) = 139.2\ mm$$

$$b_{el} = \ell_o + 2(s + t_{fc}) = 19.2 + 2 \times (12 + 12) = 67.2\ mm$$

$$k_{wc,a} = \min\left[1.0;1.25 - 0.5\frac{s_{com,a,Ed}}{f_{ywc}}\right] = 1 \quad (assumption)$$

- *If unencased column:*

β = 1

$$\bar{l}_p = 0.932\sqrt{\frac{b_{eff,c,wc}\,d_{wc}\,f_{ywc}}{E\,t_{wc}^2}} = 0.932\sqrt{\frac{139.2\times92\times235}{210\times10^3\times7^2}} = 0.504$$

$$b = 1:\quad w_c = \frac{1}{\sqrt{1+1.3\,(b_{eff,c,wc}\,t_{wc}/A_{vc})^2}} = \frac{1}{\sqrt{1+1.3\times\left(\frac{139.2\times7}{1307.6}\right)^2}} = 0.7621$$

$$\bar{l}_p = 0.5055 \le 0.67 \rightarrow F^0_{a,wc.c,Rd} = k_{wc,a} w_c b_{eff,c,wc} t_{wc} f_{ywc}/g_{Mo} = 1\times0.7621\times139.2\times7\times235/1.1 = 158644\ \text{N}$$

$$F^0_{a,wc.c,Rd} = 158.644\,\text{kN}$$

- *If encased column:*

  - Steel column alone:

$$w_c = 1\ and\quad \bar{l}_p = 0$$

$$\bar{l}_p = 0 \le 0.67 \qquad F_{a,wc.c,Rd} = k_{wc,a} w_c b_{eff,c,wc} t_{wc} f_{ywc}/g_{Mo} = 1\times1\times139.2\times7\times235/1.1 = 208167N$$

$$F_{a.wc.c,Rd} = 208.167kN$$

  - Contribution from the encased concrete:

$$k_{wc,w} = \min\left[\ 2.0;1.3+3.3\ \frac{s_{com.c.Ed}}{(f_{ck\ c}/g_c)}\right] = 1.577 \quad (assumption)$$

$$F_{c,wc.c,Rd} = 0.85 k_{wc,c}\ t_{eff,c}\ (b_c - t_{wc})\ f_{ck\ c}/g_c = 0.85\times1.557\times79.2\times(140-7)\times20/1.5$$

$$= 188263\,\text{N} = 188.264\,\text{kN}$$

- *Total resistance:*

  - *unencased column:*

$$\text{F}_{\text{Rd,2}} = \text{F}^0_{\text{a ,wc.c ,Rd}} = 158.644\ \text{kN}$$

- *encased column:*

$F_{Rd,2} = F_{a,wc.c,Rd} + F_{c,wc.c,Rd} = 208.167 + 188.264 = 396.431$ kN

b) Stiffness:

- *If unencased column:*

$$k_{a,wc.c} = \frac{0.2\, b_{eff,c,wc} t_{wc}}{d_{wc}} = \frac{0.2 \times 139.2 \times 7}{92} = 2.118 \text{ mm}$$

- *If encased column:*

$$k_{c,wc.c} = \frac{0.13\, b_{el}\, b_c}{h_c} \frac{E_{cm.c}}{E_a} = \frac{0.13 \times 67.2 \times 140}{140} \frac{29}{210} = 1.2064 \text{ mm}$$

- *Total stiffness:*

  - *unencased column:*

$k_2 = k_{a,wc.c} = 2.118$ mm

  - *encased column:*

$k_2 = k_{a,wc.c} + k_{c,wc.c} = 2.118 + 1.206 = 3.324$ mm

Component 7: Beam flange in compression

a) Resistance:

$$F_{Rd,7} = \frac{M_{c,Rd}}{h_b - t_{fb}} = \frac{60.973 \times 10^3}{220 - 9.2} = 289.246\ kN$$

b) Stiffness:

$$k_7 = \infty$$

Component 13: Longitudinal slab reinforcement in tension

a) Resistance:

$$A_s^{\min} = 0.004\ d_{eff}\ (b_{eff.b} - b_c) = 0.004 \times 80 \times (660 - 140) = 166.4\ mm^2$$

$$A_s^{\max} = \frac{1.1(0.85\, f_{ck\,s} / g_c) b_c\, d_{eff}}{b\ (f_{sk} / g_s)} = \frac{1.1 \times (0.85 \times 20 / 1.5) \times 140 \times 80}{1 \times (460 / 1.15)} = 349.07\ mm^2$$

with $\mu = 0.7\% \rightarrow A_s = 291\ mm^2 < A_s^{max} = 349.07\ mm^2$: **OK**

and $A_s = 291\ mm^2 > A_s^{min} = 166.40\ mm^2$: **OK**

$$F_{Rd,13} = \frac{A_s\ f_{sk}}{g_s} = \frac{291 \times 460}{1.15} = 116400 \quad N = 116.4 \quad kN$$

b) Stiffness:

$\beta = 1$

$$K_b = b\ (4.3b^2 - 8.9b + 7.2) = 1 \times (4.3 \times 1^2 - 8.9 \times 1 + 7.2) = 2.6$$

$$k_{s,t} = \frac{A_s}{h_c\ (\frac{1+b}{2} + K_b)} = \frac{291}{140\left(\frac{1+1}{2} + 2.6\right)} = 0.5774 \quad mm$$

$$x = \frac{E_a\ I_a}{d_s^2\ E_s\ A_s} = \frac{210 \times 10^3 \times 2772 \times 10^4}{200^2 \times 210 \times 10^3 \times 291} = 2.3814$$

$$n = \sqrt{\frac{(1+x)\,N\,k_{sc}\,l_{eff.b}\,d_s^2}{E_a\ I_a}} = \sqrt{\frac{(1+2.3814) \times 10 \times 100 \times 10^3 \times 880 \times 200^2}{210 \times 10^3 \times 2772 \times 10^4}} = 4.5218$$

$$K_{sc} = \frac{N\,k_{sc}}{n - \frac{n-1}{1+n}\frac{z}{d_s}} = \frac{10 \times 100 \times 10^3}{4.5218 - \frac{4.5218-1}{1+2.3814} \times \frac{305.4}{200}} = 341134.2747$$

$$k_r = \frac{1}{1 + \frac{E_s\ k_{s.t}}{K_{sc}}} = \frac{1}{1 + \frac{210 \times 10^3 \times 0.5774}{341134.2747}} = 0.7378$$

$$k_{13} = k_{s,tens} = k_{s,t}\ k_r = 0.5774 \times 0.7378 = 0.4260 \quad mm$$

Component 14: Contact plate in compression

a) Resistance:

Requirements on the dimensions are satisfied.
Therefore no resistance check is needed.

b) Stiffness:

$$k_{14} = \infty$$

D- EVALUATION OF THE MECHANICAL PROPERTIES OF THE JOINT

a) Resistance:

- *If unencased column:*

F_{Rd} = min [$F_{Rd,i}$] = min [145.155 ; 158.644 ; 289.246 ; 116.4] = 116.4 kN

(slab reinfocement in tension → **ductile** ® **O.K.**)

→ Plastic design moment resistance:

$$M_{Rd} = F_{Rd}\, z = 116.4 \times 305.4 = 35549 \text{ kN.mm} = 35.55 \text{ kN.m}$$

® Elastic moment resistance:

$$M_{e,Rd} = \frac{2}{3} M_{Rd} = \frac{2}{3} \times 35.55 = 23.7 \text{ kN.m}$$

- *If encased column:*

F_{Rd} = min [$F_{Rd,i}$] = min [174.223 ; 396.431 ; 289.246 ; 116.4] = 116.4 kN

(slab reinfocement in tension → **ductile** ® **O.K.**)

→ Plastic design moment resistance:

$$M_{Rd} = F_{Rd}\, z = 116.4 \times 305.4 = 35549 \text{ kN.mm} = 35.55 \text{ kN.m}$$

® Elastic moment resistance:

$$M_{e,Rd} = \frac{2}{3} M_{Rd} = \frac{2}{3} \times 35.55 = 23.7 \text{ kN.m}$$

b) Stiffness:

- *If unencased column:*

→ Initial stiffness :

$$S_{j,ini} = \frac{E_a\, z^2}{\sum \frac{1}{k_i}} = \frac{210 \times 10^3 \times 305.4^2}{\frac{1}{1.415} + \frac{1}{2.118} + \frac{1}{\infty} + \frac{1}{0.426}} \cong 5554 \text{ kN.m}$$

→ Nominal stiffness:

$$S_j = S_{j,ini} / 1.5 = 5554/1.5 = 3702 kN.m$$

- *If encased column:*

→ Initial stiffness :

$$S_{j,ini} = \frac{E_a z^2}{\sum \frac{1}{k_i}} = \frac{210 \times 10^3 \times 305.4^2}{\frac{1}{1.950} + \frac{1}{3.324} + \frac{1}{\infty} + \frac{1}{0.426}} \cong 6196 \text{ kN.m}$$

→ Nominal stiffness:

$$S_j = S_{j,ini} / 1.5 = 6196/1.5 = 4131\, kN.m$$

SUMMARY

Unencased Column		Encased Column	
M_{Rd} (kNm)	$S_{j,ini}$ (kNm/rad)	M_{Rd} (kNm)	$S_{j,ini}$ (kNm/rad)
35.55	5554	35.55	6196

Structural Steelwork Eurocodes
Development of
a Trans-National Approach

Course: Eurocode 4

Lecture 9 : Composite joints
Annex B2

Annex B2: **Calculation procedures and worked examples**
Joints with partial-depth end-plate connection in single- or double-sided configurations

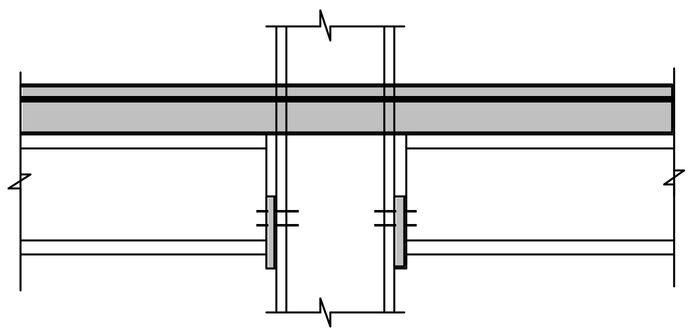
References:

- COST C1: Composite steel-concrete joints in frames for buildings: Design provisions
  Brussels, Luxembourg 1999

STIFFNESS AND RESISTANCE CALCULATION

FOR BEAM-TO-COLUMN COMPOSITE JOINTS

WITH PARTIAL-DEPTH END- PLATES



Calculation procedure

for rotational and shear characteristics

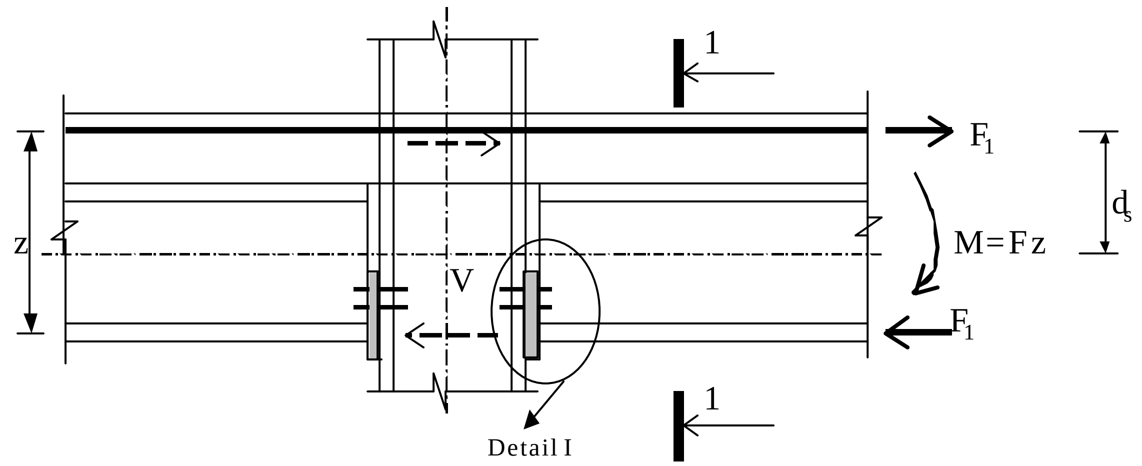
Mechanical characteristics

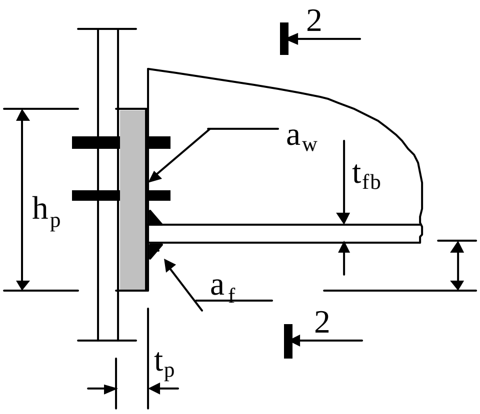
Components	Component strength	Modulus	Partial safety factors
Beam web	f_{ywb}	E_a	Structural steel : $\gamma_{Mo} = \boxed{1.1}$
Beam flange	f_{yfb}	E_a	Steel Reinforcement : $\gamma_s = \boxed{1.15}$
Column web	f_{ywc}	E_a	Concrete : $\gamma_c = \boxed{1.5}$
Column flange	f_{yfc}	E_a	Welds : $\gamma_{Mw} = \boxed{1.25}$
End plate	f_{yp}	E_a	Bolt : $\gamma_{Mb} = \boxed{1.25}$
Bolt	f_{ub}	-	
Reinforcement	f_{sk}	E_s	
Concrete of slab	f_{cks}	$E_{cm.s}$	
Concrete encased column	f_{ckc}	$E_{cm.c}$	

If hot-rolled profiles : $f_{ywc} = f_{yfc}$ and $f_{ywb} = f_{yfb}$

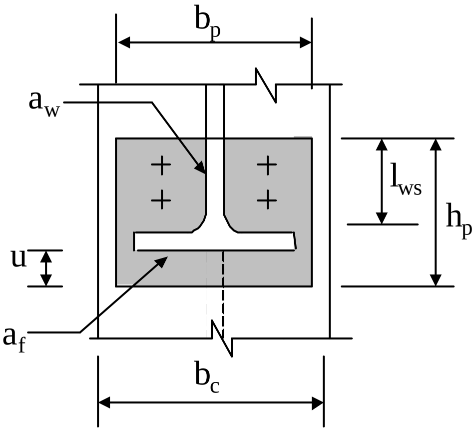
Geometrical characteristics

Joint



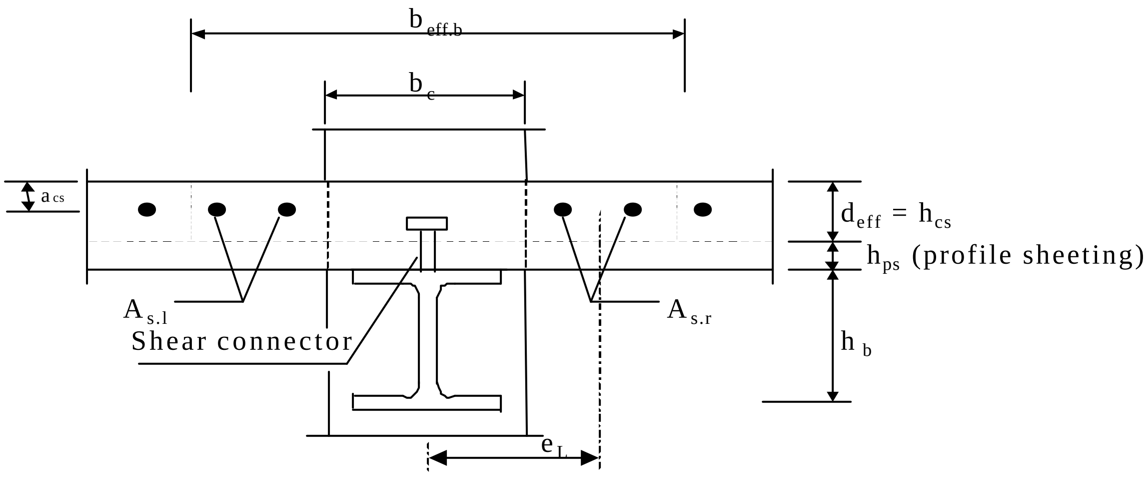


Detail I
(Partial-depth end-plate detail)



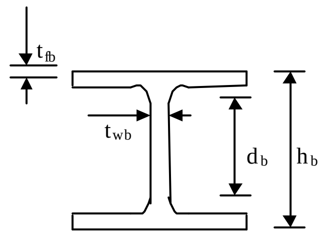
SECTION 2-2
(side view of partial-depth end-plate)

B e a m

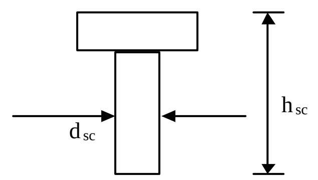


Section 1-1
Composite section with profile steel sheeting concrete slab

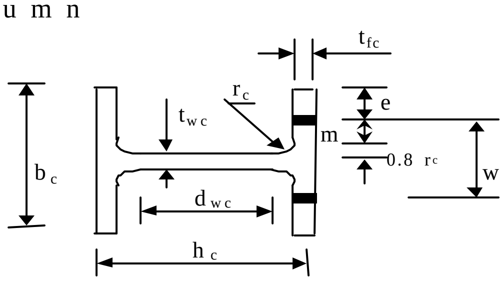
$A_s = A_{s.l} + A_{s.r}$: reinforcement area in the joint .
$\mu\% = A_s / [\,(b_{eff.b} - b_c)\, d_{eff}\,]$: reinforcement ratio .
in which : $A_{s.l}$; $A_{s.r}$: area of longitudinal reinforcements on the left and the right of composite beam section, respectively.
e_L = distance from the axis of the column web to the centre of gravity of the longitudinal reinforcement placed each side of the column
e_L should fulfill the following conditions : $0.7\, b_c \leq e_L \leq 2.5 b_c$
e_T = distance from the face of the column steel section to the centre of gravity of the transverse reinforcement placed each side of the column
e_T should fulfill the following conditions : $e_L \leq e_T \leq 1.5 e_L$



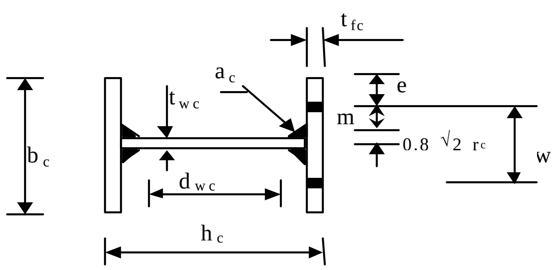
Shear connector



C o l u m n



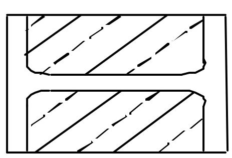
Unencased column-Rolled section



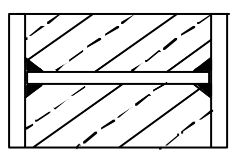
Unencased column-Welded section

$A_{vc} = A_c - 2\, b_c\, t_{fc} + (\,t_{wc} + 2\, r_c\,)\, t_{fc}$
A_c : column section area

$A_{vc} = (\,h_c - 2\, t_{fc}\,)\, t_{wc}$



Encased column-Rolled section



. $s = r_c$: for a rolled section
. $s = a_c \sqrt{2}$: for a welded section

Encased column-Welded section

	STIFFNESS	RESISTANCE
Column web panel in shear	$k_1 = k_{a,wp.s} + k_{c,wp.s}$ in which : $k_{a,wp.s} = \dfrac{0.33 A_{vc}}{\beta\ z}$ **.If unencased column :** $k_{c,wp.s} = 0$ **.If encased column :** $k_{c,wp.s} = \dfrac{0.06\ b_c\ h_c}{\beta\ z} \dfrac{E_{cm.c}}{E_a}$	$F_{Rd,1} = \dfrac{V_{wp,Rd}}{\beta}$ $with\ V_{wp,Rd} = V_{a,wp,Rd} + V_{c,wp,Rd}$ in which : $V_{a,wp,Rd} = \dfrac{0.9\ A_{vc}\ f_{ywc}}{\sqrt{3}\ \gamma_{Mo}}$ **.If unencased column :** $V_{c,wp,Rd} = 0$ **.If encased column :** $V_{c,wp,Rd} = \nu\ A_c \sin\theta\ \dfrac{0.85\ f_{ck\ c}}{\gamma_c}$ where : $A_c = 0.8(h_c - 2t_{fc})(b_c - t_{wc})\cos\theta$ $\theta = \arctan\left(\dfrac{h_c - 2t_{fc}}{z}\right)$ $\nu = 0.55\left[1 + 2\dfrac{N_{Sd}}{N_{pl,Rd}}\right] \leq 1.1$ $.N_{Sd}$: is the design axial compressive force in the column. $.N_{pl,Rd}$: is the design plastic resistance of encased column cross-section.
	β = 1 : for single-sided joint configurations. 0 : for double-sided joint configurations symmetrically loaded. 1 : for double-sided joint configurations non-symmetrically loaded with balanced moments.	
Column web in compression	$k_2 = k_{a.wc.c} + k_{c.wc.c}$ $k_{a.wc.c} = \dfrac{0.7 b_{eff,c,wc} t_{wc}}{d_{wc}}$ **.If unencased column :** $k_{c.wc.c} = 0$ **.If encased column :** $k_{c.wc.c} = \dfrac{0.50\ b_{el}\ b_c}{h_c} \dfrac{E_{cm.c}}{E_a}$	$F_{Rd,2} = F_{a,wc.c,Rd} + F_{c,wc.c,Rd}$ In which : $F_{a,wcc,Rd} = k_{wc,a} \omega_c b_{eff,c,wc} t_{wc} f_{ywc} / \gamma_{Mo}$ $if\ \bar{\lambda}_p \leq 0.67$ $F_{a,wcc,Rd} = k_{wc,a} \omega_c b_{eff,c,wc} t_{wc} \left[\dfrac{1}{\bar{\lambda}_p}\left(1 - \dfrac{0.22}{\bar{\lambda}_p}\right)\right] f_{ywc} / \gamma_{Mo}$ $if\ \bar{\lambda}_p > 0.67$ $k_{wc,a} = \min\left[1.0 ; 1.25 - 0.5\dfrac{\sigma_{com,a,Ed}}{f_{ywc}}\right]$

		.If unencased column : $\overline{l}_p = 0.932 \sqrt{\dfrac{b_{eff,c,wc} d_{wc} f_{ywc}}{E\ t_{wc}^2}}$ $if\ \ b = 1:\ \ w_c = \dfrac{1}{\sqrt{1+1.3\,(b_{eff,c,wc}\, t_{wc} / A_{vc})^2}}$ $if\ \ b = 0:\ w_c = 1$ And : $F_{c,wc.c,Rd} = 0$ **.If encased column :** $w_c = 1\ and\ \ \overline{l}_p = 0$ and $F_{c,wc.c,Rd} = 0.85 k_{wc,c}\, t_{eff,c}\,(b_c - t_{wc})\, f_{ckc} / g_c$ $with$ $k_{wc,c} = \min[1.3 + 3.3\, s_{com,c,Ed} / (f_{ckc} / g_c)\ ;2.0]$ $\sigma_{com,a,Ed}$: normal stresses in column web at the root of fillet radius or of the weld. .$\sigma_{com,c,Ed}$: is the longitudinal compressive stress in the encasement due to axial force N_{Ed} .
	$\ell_o = t_{fb} + a_f\sqrt{2} + \min\ [u\,; a_f\sqrt{2} + t_p\,]$ $t_{eff,c} = \ell_o + 5 t_{fc}$ $b_{eff,c,wc} = \ell_o + 5(s + t_{fc})$ $b_{el} = \ell_o + 2(s + t_{fc})$	
Beam flange in compression	$k_7 = \infty$	$F_{Rd,7} = M_{c,Rd} / (h_b - t_{fb})$.$M_{c,Rd}$: is steel beam design moment resistance
Longitudi-nal slab reinforce-ment in tension	$k_{13} = k_{s,t} k_r$ where : $k_{s,t} = \dfrac{A_s}{h_c\,(\frac{1+b}{2} + K_b)} \dfrac{E_s}{E_a}$ $K_b = b\ \ (4.3\,b^2 - 8.9\,b + 7.2)$ • For single-sided joint configuration : $b = 1$ • For double-sided joint configuration symmetrically loaded with balanced moments : $b = 0$ for both sides	$F_{Rd,13} = \dfrac{A_s\ f_{sk}}{g_s}$ with : $A_s \geq 0.004\ \ d_{eff}\ \ (b_{eff.b} - b_c)$ $A_s \leq \dfrac{1.1(0.85\, f_{ck\,s} / g_c) b_c\, d_{eff}}{b\ \ (f_{sk} / g_s)}$.$b_{eff.b}$: is the effective slab breadth in composite cross section of beam. A minimum amount of transverse reinforcement A_t should be placed each side of the column : $A_t \geq 0.5\, b A_s / tgd$ In which : tg$d$ = *1.35(e_T/e_L - 0.2)*

	• For double-sided joint configuration non- symmetrically loaded with balanced moments: + for the more heavily loaded side : *b = 1* + for the less heavily loaded side : *b = 0 and* $k_{s,t}$ = ¥	
	$k_r = \dfrac{1}{1 + \dfrac{E_s\, k_{s.t}}{K_{sc}}}$ $with\ K_{sc} = \dfrac{N\, k_{sc}}{u - \dfrac{u-1}{1+x}\dfrac{z}{d_s}}$ $x = \dfrac{E_a\, I_a}{d_s^2\, E_s\, A_s}$ $u = \sqrt{\dfrac{(1+x)\, N\, k_{sc}\, l_{eff.b}\, d_s^2}{E_a\ I_a}}$ .k_{sc} : is the stiffness of one shear connector .$l_{eff.b}$: is the length of the beam in hogging bending zone. .I_a : steel beam second moment of inertia .N : is the number of shear connectors dist_ ributed over the length $l_{eff.b}$.	
JOINT	.Initial stiffness : $S_{j,ini} = \dfrac{E_a\, z^2}{\sum_i \dfrac{1}{k_i}}$ $with\, i = 1, 2, 7, 13$. Nominal stiffness : $S_j = S_{j,ini}\, /1.5$	$F_{Rd} = \min\,[\,F_{Rd,i}\,]$ $with\, i = 1, 2, 7, 13$ .Elastic moment resistance : $M_{e,Rd} = \dfrac{2}{3} F_{Rd}\, z$ .Plastic design moment resistance : $M_{Rd} = F_{Rd}\ z$

SHEAR RESISTANCE

1. Shear resistance of the fillet welds :

$$V_{Rd,1} = f_{v\,w.d}\, a_w\, l_{ws}$$

In which : $l_{ws} = 2\ (h_p - u - t_{fb} - r_b\ )$

$$f_{vw.d} = \frac{f_u / \sqrt{3}}{b_w\ g_{Mw}}$$

In which f_u is the lower of the ultimate stresses of the column flange and the bracket

b_w should be taken as follows (linear interpolation between the values if needed):

Steel grade	Ultimate tensile strength f_u	b_w
S235	360 N/mm²	0.8
S275	430 N/mm²	0.85
S355	510 N/mm²	0.9

2. Shear resistance of the bolts :

$$V_{Rd,2} = N_b\, F_{v,Rd}$$

In which : N_b : number of bolts.
$V_{s,Rd}$: shear resistance of one bolt (shear plane passes through threaded part)

$$F_{v,Rd} = \frac{0.6\ f_{ub}\ A_{s,b}}{g_{Mb}} \quad \text{for grades 4.6, 5.6 and 8.8}$$

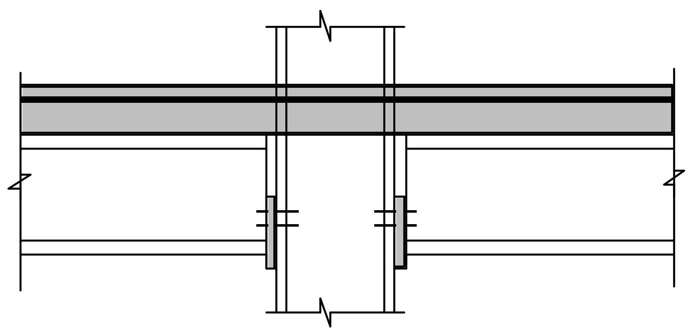
$$F_{v,Rd} = \frac{0.5\ f_{ub}\ A_{s,b}}{g_{Mb}} \quad \text{for grades 4.8, 5.8 and 10.9}$$

$A_{s,b}$: stress area of the bolts

3. Shear resistance of the joint :

$$\boldsymbol{V_{Rd} = \min\ [V_{Rd,1}\ ;\ V_{Rd,2}\ ]}$$

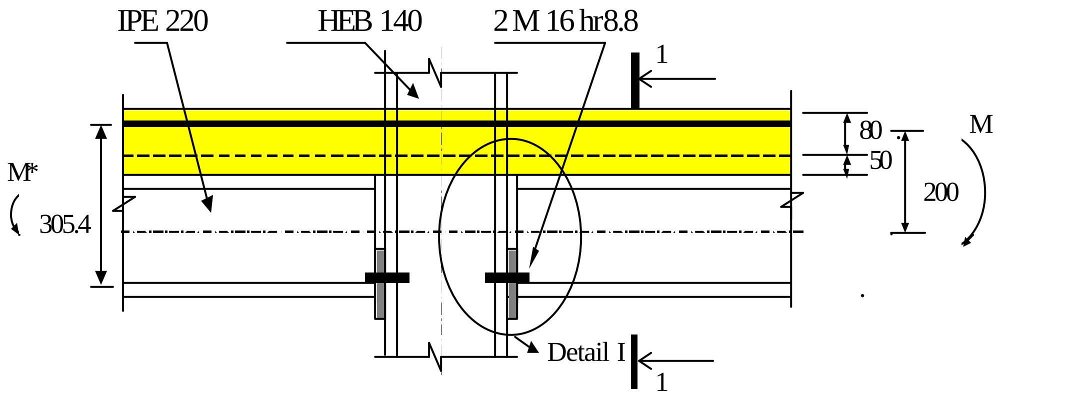
STIFFNESS AND RESISTANCE CALCULATION FOR BEAM-TO-COLUMN COMPOSITE JOINTS WITH PARTIAL-DEPTH END- PLATES



Worked example for the evaluation of the rotational characteristics

A- DATA FOR CALCULATION

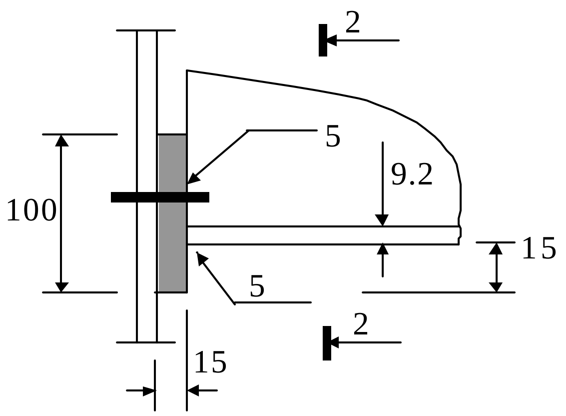
- Main data for joint and loading



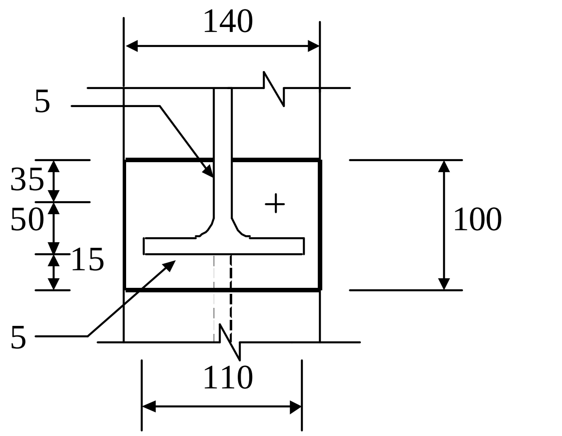
Calculation of the right joint with M > M*

- Dimensions of the partial-depth end-plate

h_p	e_p	u_2	u	b_p	w	w_1	t_p
100	35	50	15	140	80	30	15



Detail _ I
(partial-depth end plate detail)



SECTION 2_2
(sided view of partial depth end plate)

- Other various data

Grade for all steel components except rebars: S235
Percentage of reinforcement in the effective slab: 0.7%
Connectors : - Stiffness: 100 kN/mm
- Spacing: 100 mm
Reinforcement bars: f_{sk} = 460 N/mm²
Concrete: f_{ck} = 20 N/mm² and E_{cm} = 29000 N/mm²

B- PRELIMINARY CALCULATIONS

Column:

$d_{wc} = h_c - 2\ t_{fc} - 2\ r_c = 140 - 2 \times 12 - 2 \times 12 = 92$ mm

$A_{vc} = A_c - 2\ b_c\ t_{fc} + (t_{wc} + 2\ r_c)\ t_{fc} = 4295.6 - 2 \times 140 \times 12 + (7 + 2 \times 12) \times 12 = 1307.6\ mm^2$.

$s = r_c = 12$ mm (for a hot-rolled section)

Beam:

$d_s = 0.5\ h_b + h_{cs} + h_{ps} - a_{cs} = 0.5 \times 220 + 80 + 50 - 40 = 200$ mm

$z = h_b - 0.5\ t_{fb} + h_{cs} + h_{ps} - a_{cs} = 220 - 0.5 \times 9.2 + 80 + 50 - 40 = 305.4$ mm

$$M_{c.Rd}(class\,1) = \frac{W_{pby}\ f_{yb}}{g_{Mo}} = \frac{285406 \times 235}{1.1} = 60973100.517\ N.mm \cong 60.973\ kN.m$$

Concrete slab:

$d_{eff} = h_{cs} = 80$ mm

$b_{eff.b} = 3\ h_b = 3 \times 220 = 660$ mm

$l_{eff.b} = 4\ h_b = 4 \times 220 = 880$ mm

$$m = 0.7\,\% \;\rightarrow A_s = m\ d_{eff}\left(b_{eff.b} - b_c\right) = \frac{0.7}{100} 80 \times (660 - 140) = 291\,\text{mm}^2$$

Shear connectors:

$k_{sc} = 100\ kN/mm^2$ *(assumption)*

$$N = \frac{l_{eff.b}}{spacing} + 1 = \frac{880}{100} + 1 \cong 10 \quad (assumption)$$

C- CALCULATION OF THE STIFFNESS AND RESISTANCE PROPERTIES

Component 1: Column web panel in shear

a) Resistance:

- *If unencased column:*

$$V_{a,wp,Rd}=\frac{0.9\ A_{vc}\ f_{ywc}}{\sqrt{3}\ g_{Mo}}=\frac{0.9\times 1307.6\times 235}{\sqrt{3}\times 1.1}=145155\ \ \mathrm{N}=145.155\ \ \mathrm{kN}$$

- *If encased column:*

$$u=0.55\left[1+2\frac{N_{Sd}}{N_{pl,Rd}}\right]=0.626\ (hyp)$$

$$q=\arctan\left(\frac{h_c-2t_{fc}}{z}\right)=\arctan\left(\frac{140-2\times 12}{305.4}\right)=20.8^{o}$$

$$A_c=0.8(h_c-2t_{fc})(b_c-t_{wc})\cos q=0.8\times(140-2\times 12)\times(140-7)\times\cos\left(20.8^{o}\right)=11538\ \mathrm{mm}^2$$

$$V_{c,wp,Rd}=u\ A_c\ \sin q\ \frac{0.85\ f_{ck\,c}}{g_c}=0.626\times 11538\times\sin\left(20.8^{o}\right)\times\frac{0.85\times 20}{1.5}=29068\ \ \mathrm{N}=29.068\ \mathrm{kN}$$

- *Total resistance:*

  - *unencased column:*

$$F_{Rd,1}=\frac{V_{wp,Rd}}{b}=\frac{V_{a,wp,Rd}}{b}=\frac{145.155}{1}=145.155\ kN$$

  - *encased column:*

$$F_{Rd,1}=\frac{V_{wp,Rd}}{b}=\frac{V_{a,wp,Rd}+V_{c,wp,Rd}}{b}=\frac{145.155+29.068}{1}=174.223\ \mathrm{kN}$$

b) Stiffness:

- *If unencased column:*

$$k_{a,wp.s}=\frac{0.33\,A_{vc}}{b\ \ z}=\frac{0.33\ \times 1307.6}{1\times 305.4}=1.415\ \ \mathrm{mm}$$

- *If encased column:*

$$k_{a,wp.s} = \frac{0.33\, A_{vc}}{b\ \ z} = \frac{0.33\ \times 1307.6}{1 \times 3054} = 1.415\ \text{ mm}$$

$$k_{c,wp.s} = \frac{0.06\ b_c\ \ h_c}{b\ z}\frac{E_{cm.c}}{E_a} = \frac{0.06 \times 140 \times 140}{1 \times 305.4}\ \ \frac{29}{210} = 0.532\ \text{mm}$$

- *Total stiffness:*

  - *unencased column:*

    $k_1 = k_{a,wp.s} = 1.415$ mm

  - *encased column:*

    $k_1 = k_{a,wp.s} + k_{c,wp.s} = 1.415 + 0.532 = 1.95$ mm

Component 2: Column web in compression

a) Resistance:

$$\ell_o = t_{fb} + a_f\sqrt{2} + t_p + \min\,[u\,;a_f\sqrt{2} + t_p\ ] = 9.2 + 5\sqrt{2} + 15 + \min\left[15;5\sqrt{2} + 15\right] = 46.27\ mm$$

$$t_{eff,c} = \ell_o + 5t_{fc} = 46.27 + 5 \times 12 = 106.27\ mm$$

$$b_{eff,c,wc} = \ell_o + 5(s + t_{fc}) = 46.27 + 5 \times (12 + 12) = 166.27\,mm$$

$$b_{el} = \ell_o + 2(s + t_{fc}) = 46.27 + 2 \times (12 + 12) = 94.27\ mm$$

$$k_{wc,a} = \min\left[1.0;1.25 - 0.5\frac{s_{com,a,Ed}}{f_{ywc}}\right] = 1 \quad (assumption)$$

- *If unencased column:*

$\beta = 1$

$$\overline{l}_p = 0.932\sqrt{\frac{b_{eff,c,wc}\, d_{wc}\, f_{ywc}}{E\ t_{wc}^2}} = 0.932\sqrt{\frac{166.27 \times 92 \times 235}{210 \times 10^3 \times 7^2}} = 0.5509$$

$$b = 1:\quad w_c = \frac{1}{\sqrt{1 + 1.3(b_{eff,c,wc}\ t_{wc}\ /\ A_{vc})^2}} = \frac{1}{\sqrt{1 + 1.3 \times \left(\frac{166.27 \times 7}{1307.6}\right)^2}} = 0.7019$$

$$\overline{l}_p = 0.5509 \le 0.67 \rightarrow F^0_{a,wc.c,Rd} = k_{wc,a} w_c b_{eff,c,wc} t_{wc} f_{ywc}/g_{Mo} = 1 \times 0.7621 \times 166.27 \times 7 \times 235/1.1 = 174526.9\text{N}$$

$$F^0_{a,wc.c,Rd} = 174.527\ \text{kN}$$

- *If encased column:*

- Steel column alone:

$$w_c = 1 \; and \quad \overline{l}_p = 0$$

$$\overline{l}_p = 0 \leq 0.67 \qquad F_{a,wc.c,Rd} = k_{wc,a} w_c b_{eff,c,wc} t_{wc} f_{ywc} / g_{Mo} = 1 \times 1 \times 166.27 \times 7 \times 235 / 1.1 = 248651 N$$

$$F_{a.wc.c,Rd} = 248.651 kN$$

- Contribution from the encased concrete:

$$k_{wc,c} = \min\left[2.0; 1.3 + 3.3 \frac{s_{com.c.Ed}}{(f_{ck\,c} / g_c)}\right] = 1.577 \quad (assumption)$$

$$F_{c,wc.c,Rd} = 0.85 k_{wc,c}\, t_{eff,c}\, (b_c - t_{wc})\, f_{ck\,c} / g_c = 0.85 \times 1.557 \times 106.27 \times (140 - 7) \times 20 / 1.5$$
$$= 252613 \text{ N} = 252.613 \text{ kN}$$

- *Total resistance:*

  - *unencased column:*

    $F_{Rd,2} = F^0_{a,wc.c,Rd} = 174.527$ kN

  - *encased column:*

    $F_{Rd,2} = F_{a,wc.c,Rd} + F_{c,wc.c,Rd} = 248.651 + 252.613 = 501.264$ kN

b) Stiffness:

- *If unencased column:*

$$k_{a,wc.c} = \frac{0.7\, b_{eff,c,wc} t_{wc}}{d_{wc}} = \frac{0.7 \times 166.27 \times 7}{92} = 8.856 \text{ mm}$$

- *If encased column:*

$$k_{c,wc.c} = \frac{0.13\, b_{el}\, b_c}{h_c} \frac{E_{cm.c}}{E_a} = \frac{0.13 \times 94.27 \times 140}{140} \frac{29}{210} = 1.692 \text{ mm}$$

- *Total stiffness:*

  - *unencased column:*

  $k_2 = k_{a,wc.c} = 8.856$ mm

  - *encased column:*

  $k_2 = k_{a,wc.c} + k_{c,wc.c} = 8.856 + 1.692 = 10.548$ mm

Component 7: Beam flange in compression

a) Resistance:

$$F_{Rd,7} = \frac{M_{c,Rd}}{h_b - t_{fb}} = \frac{60.973 \times 10^3}{220 - 9.2} = 289.246\ kN$$

b) Stiffness:

$$k_7 = \infty$$

Component 13: Longitudinal slab reinforcement in tension

a) Resistance:

$$A_s^{min} = 0.004\ d_{eff}\ (b_{eff.b} - b_c) = 0.004 \times 80 \times (660 - 140) = 166.4\ mm^2$$

$$A_s^{max} = \frac{1.1(0.85\ f_{ck\ s} / g_c) b_c\ d_{eff}}{b\ (f_{sk} / g_s)} = \frac{1.1 \times (0.85 \times 20 / 1.5) \times 140 \times 80}{1 \times (460 / 1.15)} = 349.07\ mm^2$$

with μ = 0.7% $\rightarrow$ A_s = 291 mm^2 < A_s^{max} = 349.07 mm^2 : **OK**

and A_s = 291 mm^2 > A_s^{min} = 166.40 mm^2 : **OK**

$$F_{Rd,13} = \frac{A_s\ f_{sk}}{g_s} = \frac{291 \times 460}{1.15} = 116400 \quad \text{N} = 116.4 \quad \text{kN}$$

b) Stiffness:

β = 1

$$K_b = b\ (4.3 b^2 - 8.9 b + 7.2) = 1 \times (4.3 \times 1^2 - 8.9 \times 1 + 7.2) = 2.6$$

$$k_{s,t} = \frac{A_s}{h_c\ (\frac{1 + b}{2} + K_b)} = \frac{291}{140 \left(\frac{1+1}{2} + 2.6 \right)} = 0.5774 \quad \text{mm}$$

$$x = \frac{E_a\ I_a}{d_s^2\ E_s\ A_s} = \frac{210 \times 10^3 \times 2772 \times 10^4}{200^2 \times 210 \times 10^3 \times 291} = 2.3814$$

$$n = \sqrt{\frac{(1+x) N k_{sc}\ l_{eff.b}\ d_s^2}{E_a\ I_a}} = \sqrt{\frac{(1 + 2.3814) \times 10 \times 100 \times 10^3 \times 880 \times 200^2}{210 \times 10^3 \times 2772 \times 10^4}} = 4.5218$$

$$K_{sc} = \frac{N\,k_{sc}}{n - \frac{n-1}{1+n}\frac{z}{d_s}} = \frac{10 \times 100 \times 10^3}{4.5218 - \frac{4.5218-1}{1+2.3814} \times \frac{305.4}{200}} = 341134.2747$$

$$k_r = \frac{1}{1 + \frac{E_s\,k_{s.t}}{K_{sc}}} = \frac{1}{1 + \frac{210 \times 10^3 \times 0.5774}{341134.2747}} = 0.7378$$

$$k_{13} = k_{s,tens} = k_{s,t}\ k_r = 0.5774 \times 0.7378 = 0.4260 \quad \text{mm}$$

D- EVALUATION OF THE MECHANICAL PROPERTIES OF THE JOINT

a) Resistance:

- *If unencased column:*

F_{Rd} = min [$F_{Rd,i}$] = min [145.155 ; 174.527 ; 289.246 ; 116.4] = 116.4 kN

(slab reinforcement in tension ® **ductile** ® **O.K.**)

→ Plastic design moment resistance:

$$M_{Rd} = F_{Rd}\, z = 116.4 \times 305.4 = 35549 \text{ kN.mm} = 35.55 \text{ kN.m}$$

® Elastic moment resistance:

$$M_{e,Rd} = \frac{2}{3} M_{Rd} = \frac{2}{3} \times 35.55 = 23.7 \text{ kN.m}$$

- *If encased column:*

F_{Rd} = min [$F_{Rd,i}$] = min [174.223 ; 501.264 ; 289.246 ; 116.4] = 116.4 kN

(slab reinforcement in tension ® **ductile** ® **O.K.**)

→ Plastic design moment resistance:

$$M_{Rd} = F_{Rd}\, z = 116.4 \times 305.4 = 35549 \text{ kN.mm} = 35.55 \text{ kN.m}$$

® Elastic moment resistance:

$$M_{e,Rd} = \frac{2}{3} M_{Rd} = \frac{2}{3} \times 35.55 = 23.7 \text{ kN.m}$$

b) Stiffness:

- *If unencased column:*

→ Initial stiffness :

$$S_{j,ini=} = \frac{E_a\, z^2}{\sum \frac{1}{k_i}} = \frac{210 \times 10^3 \times 305.4^2}{\frac{1}{1.415} + \frac{1}{8.856} + \frac{1}{\infty} + \frac{1}{0.426}} \cong 6184 \text{ kN.m}$$

→ Nominal stiffness:

$$S_j = S_{j,ini} / 1.5 = 6184/1.5 = 4123 kN.m$$

- *If encased column:*

→ Initial stiffness :

$$S_{j,ini} = \frac{E_a z^2}{\sum \frac{1}{k_i}} = \frac{210 \times 10^3 \times 305.4^2}{\frac{1}{1.950} + \frac{1}{10.548} + \frac{1}{\infty} + \frac{1}{0.426}} \cong 6628 \text{ kN.m}$$

→ Nominal stiffness:

$$S_j = S_{j,ini} / 1.5 = 6628/1.5 = 4419\ kN.m$$

SUMMARY

Unencased Column		Encased Column	
M_{Rd} (kNm)	$S_{j,ini}$ (kNm/rad)	M_{Rd} (kNm)	$S_{j,ini}$ (kNm/rad)
35.55	6184	35.55	6628

Structural Steelwork Eurocodes

Development of
a Trans-National Approach

Course: Eurocode 4

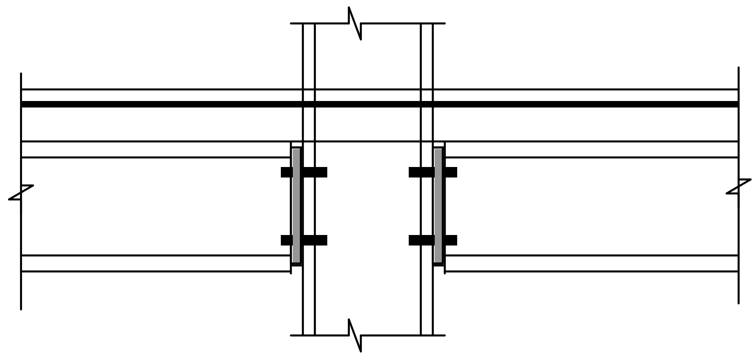
Lecture 9 : Composite joints
Annex B3

Annex B3: Calculation procedures and worked examples
Joints with flush-end-plate connection in single- or double-sided configurations

References:

- COST C1: Composite steel-concrete joints in frames for buildings: Design provisions
  Brussels, Luxembourg 1999

STIFFNESS AND RESISTANCE CALCULATION FOR BEAM-TO-COLUMN COMPOSITE JOINTS WITH FLUSH END- PLATES



Calculation procedure

for rotational and shear characteristics

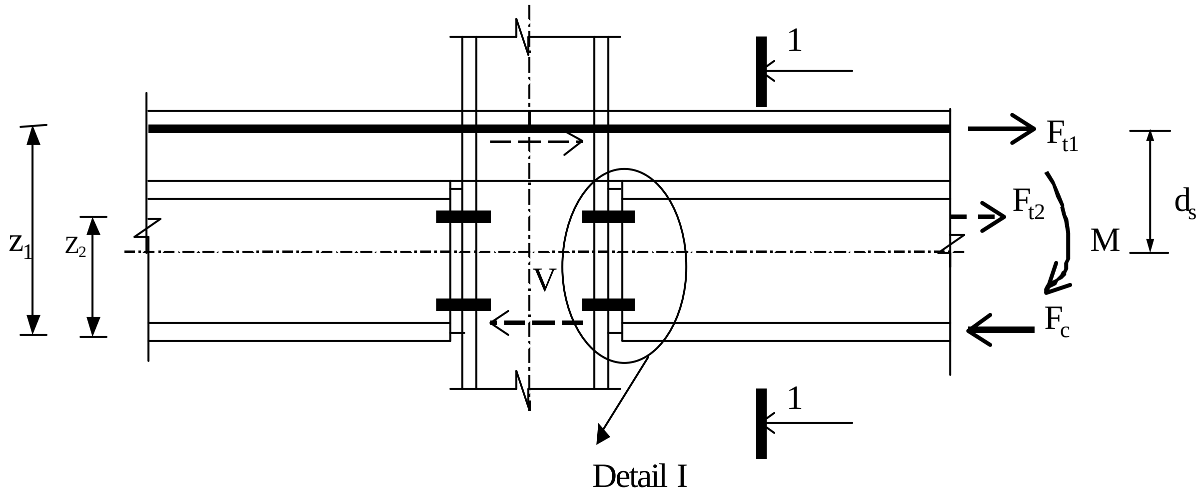
Mechanical characteristics

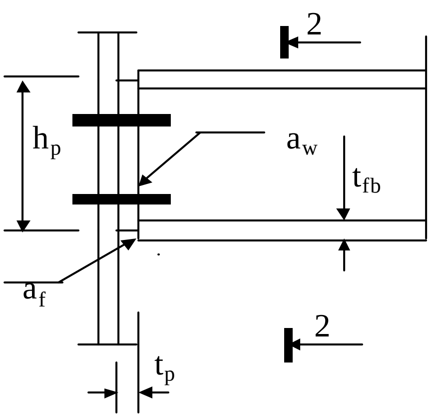
Components	Component strength	Modulus	Partial safety factors
Beam web	f_{ywb}	E_{sb}	Structural steel :
Beam flange	f_{yfb}	E_{sb}	$\gamma_{Mo} = \boxed{1.1}$
Column web	f_{ywc}	E_{sc}	Steel Reinforcement :
Column flange	f_{yfc}	E_{sc}	$\gamma_s = \boxed{1.15}$
End plate	f_{yp}	E_{sp}	Concrete :
Welding	f_{uw}	-	$\gamma_c = \boxed{1.5}$
Bolt	f_{ub}	-	Welding :
Reinforcement	f_{sk}	E_s	$\gamma_{Mw} = \boxed{1.25}$
Concrete of slab	f_{cks}	E_{cs}	Bolt :
Concrete encased column	f_{ckc}	E_{cc}	$\gamma_{Mb} = \boxed{1.25}$

If hot-rolled profiles : $f_{ywc} = f_{yfc}$ and $f_{ywb} = f_{yfb}$

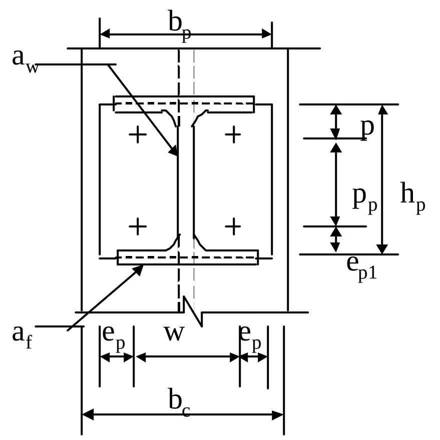
Geometrical characteristics

Joint



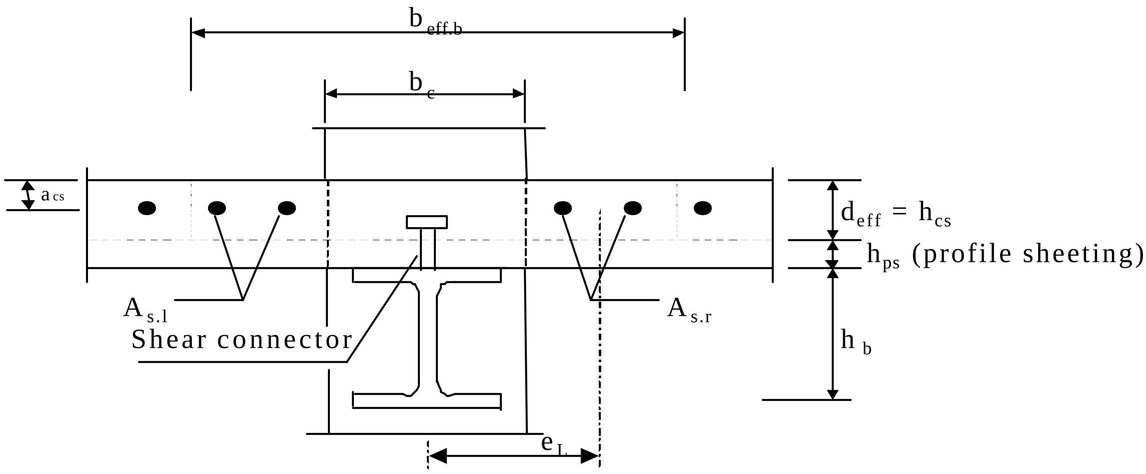


Detail _ I
(flush end-plate detail)



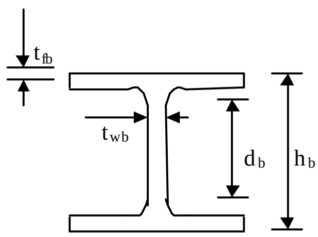
SECTION 2-2
(side view of flush end-plate)

Beam

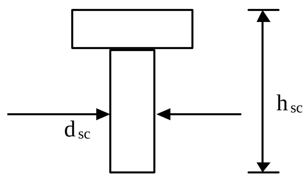


Section 1-1
Composite section with profile steel sheeting concrete slab

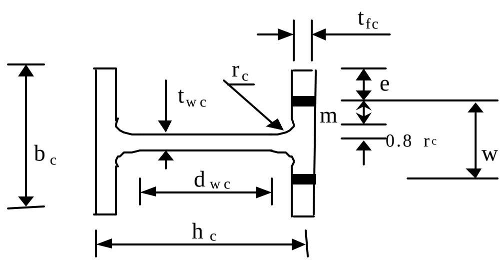
$A_s = A_{s.l} + A_{s.r}$: reinforcement area in the joint .
$\mu\% = A_s / [(b_{eff.b} - b_c) d_{eff}]$: reinforcement ratio .
in which : $A_{s.l}$; $A_{s.r}$: area of longitudinal reinforcements on the left and the right of composite beam section, respectively.
e_L = distance from the axis of the column web to the centre of gravity of the longitudinal reinforcement placed each side of the column
e_L should fulfill the following conditions : $0.7 b_c \le e_L \le 2.5 b_c$
e_T = distance from the face of the column steel section to the centre of gravity of the transverse reinforcement placed each side of the column
e_T should fulfill the following conditions : $e_L \le e_T \le 1.5 e_L$



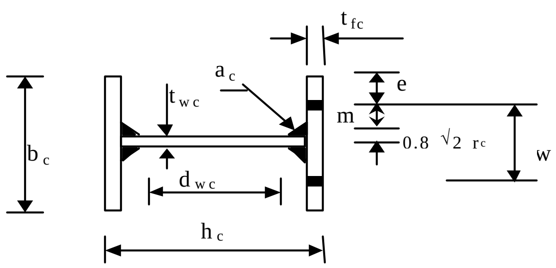
Shear connector



Column



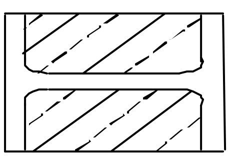
Unencased column-Rolled section



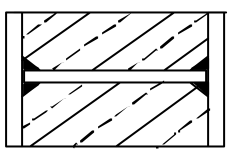
Unencased column-Welded section

$A_{vc} = A_c - 2 b_c t_{fc} + (t_{wc} + 2 r_c) t_{fc}$
A_c : column section area

$A_{vc} = (h_c - 2 t_{fc}) t_{wc}$



Encased column-Rolled section



. $s = r_c$: for a rolled section
. $s = a_c \sqrt{2}$: for a welded section

Encased column-Welded section

	STIFFNESS	RESISTANCE

Column web in compression

STIFFNESS

$$k_2 = k_{a.wc.c} + k_{c.wc.c}$$

$$k_{a.wc.c} = \frac{0.7 b_{eff,c,wc} t_{wc}}{d_{wc}}$$

.If unencased column : $k_{c.wc.c} = 0$

.If encased column :

$$k_{c.wc.c} = \frac{0.5\, b_{el}\, b_c}{h_c} \frac{E_{cm.c}}{E_a}$$

RESISTANCE

$$F_{Rd,2} = F_{a,wc.c,Rd} + F_{c,wc.c,Rd}$$

In which :

$$F_{a,wc.c,Rd} = k_{wc,a} \omega_c b_{eff,c,wc} t_{wc} f_{ywc} / \gamma_{Mo}$$

$$if \quad \bar{\lambda}_p \leq 0.67$$

$$F_{a,wc.c,Rd} = k_{wc,a} \omega_c b_{eff,c,wc} t_{wc} \left[\frac{1}{\bar{\lambda}_p} \left(1 - \frac{0.22}{\bar{\lambda}_p} \right) \right] f_{ywc} / \gamma_{Mo}$$

$$if \quad \bar{\lambda}_p > 0.67$$

$$k_{wc,a} = \min \left[1.0 \,; 1.25 - 0.5 \frac{\sigma_{com,a,Ed}}{f_{ywc}} \right]$$

.If unencased column :

$$\bar{\lambda}_p = 0.932 \sqrt{\frac{b_{eff,c,wc}\, d_{wc}\, f_{ywc}}{E\, t_{wc}^2}}$$

$$if \quad \beta = 1 : \quad \omega_c = \frac{1}{\sqrt{1 + 1.3 \left(b_{eff,c,wc}\, t_{wc} / A_{vc} \right)^2}}$$

$$if \quad \beta = 0 : \quad \omega_c = 1$$

And :

$$F_{c,wc.c,Rd} = 0$$

.If encased column : $\omega_c = 1 \; and \; \bar{\lambda}_p = 0$

and

$$F_{c,wc.c,Rd} = 0.85 k_{wc,c}\, t_{eff,c}\, (b_c - t_{wc})\, f_{ckc} / \gamma_c$$

$$with \; k_{wc,c} = \min[1.3 + 3.3 \sigma_{com,c,Ed} / (f_{ckc} / \gamma_c) \; ; 2.0 \,]$$

$\sigma_{com,a,Ed}$: normal stresses in column web at the root of fillet radius or of the weld.

.$\sigma_{com,c,Ed}$: is the longitudinal compressive stress in the encasement due to axial force N_{Ed} .

$$\ell_o = t_{fb} / 2 + a_f \sqrt{2} + t_p$$

$$t_{eff,c} = \ell_o + 5 t_{fc}$$

$$b_{eff,c,wc} = \ell_o + 5(s + t_{fc})$$

$$b_{el} = \ell_o + 2(s + t_{fc})$$

β = 1 : for single-sided joint configuration.

0 : for double-sided joint configurations symmetrically loaded.

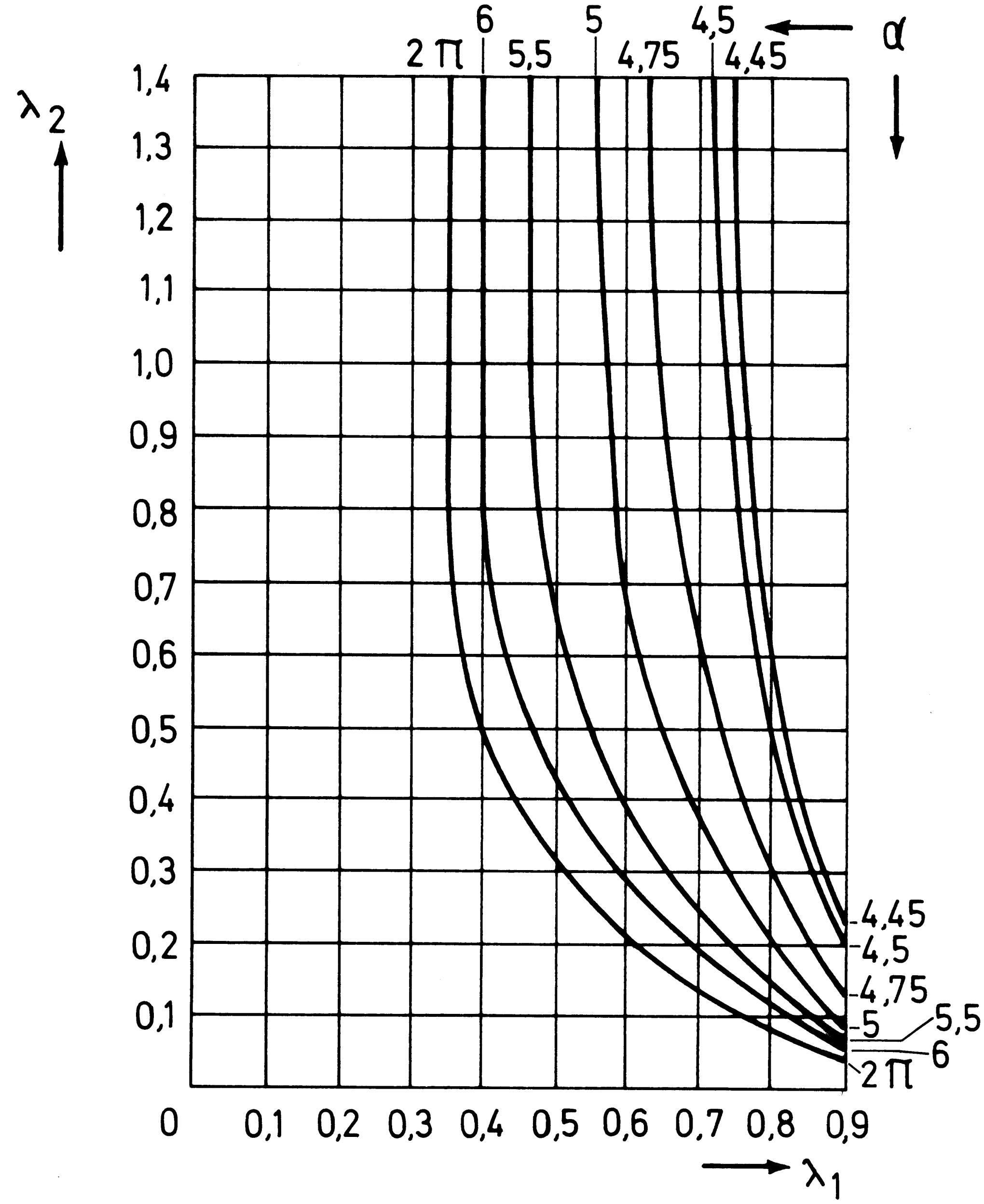
1 : for double-sided joint configurations non-symmetrically loadedwith balanced moments.

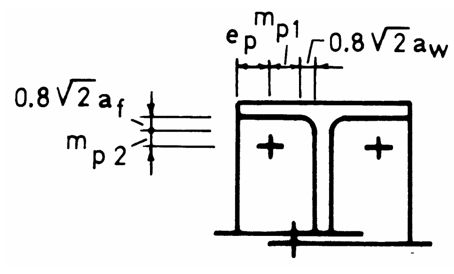
Column web in tension	$k_3 = \dfrac{0.7 b_{eff,t,wc}\, t_{wc}}{d_{wc}}$	$F_{Rd,3} = w_t\, b_{eff,t,wc}\, t_{wc}\, f_{ywc} / g_{Mo}$ with : $if\ \ b = 1 :\ \ w_t = \dfrac{1}{\sqrt{1 + 1.3 (b_{eff,t,wc}\, t_{wc} / A_{vc})^2}}$ $if\ \ b = 0 :\ w_t = 1$
	$b_{eff,t,wc} = \min\ [2p\, m\, ; 4m + 1.25\, e\,]$	
Column flange in bending	$k_4 = \dfrac{0.85 l_{eff,t,fc}\, t_{fc}^3}{m^3}$	$F_{Rd,4} = min\ [\ F_{fc.Rd,t1}\ ;\ F_{fc.Rd,t2}\ ]$ $F_{fc.Rd,t1} = \dfrac{(8n - 2e_w)\ l_{eff,t,fc}\ m_{pl.fc}}{2mn - e_w (m + n)} k_{fc}$ $F_{fc.Rd,t2} = \dfrac{2 l_{eff,t,fc}\, m_{pl.fc}\, k_{fc} + 2 B_{t,Rd}\, n}{m + n}$ If $s_{n,fc} \leq$ 180 (N/mm²) : $k_{fc} = 1$ If $s_{n,fc} >$ 180 (N/mm²) : $k_{fc} = min\ [\ 1\ ;\ (2\, f_{yfc} - 180 - s_{com,fc,Ed}) / (2\, f_{yfc} - 360)\ ]$ $n = \min\ [e ; 1.25 m ; e_p]$ $m_{pl.fc} = 0.25\ t_{fc}^2\ \ f_{yfc}\ /\ g_{Mo}$ $e_w = d_w / 4$ d_w : diameter of the washer, or of the width across points of the bolt head or nut, as relevant $s_{com,,fc,Ed}$: normal stress at mid-thickness of the column flange.
	$l_{eff,t,fc} = b_{eff,t,wc}$	
End-plate in bending	$k_5 = \dfrac{0.85\, l_{eff,p}\ \ t_p^3}{m_{p1}^3}$	$F_{Rd,5} = min\ [\ F_{ep,Rd,1}\ ;\ F_{ep,Rd,2}\ ]$ $F_{ep.Rd,1} = \dfrac{(8 n_p - 2 e_w)\ l_{eff,p}\ \ m_{pl.p}}{2 m_{p1}\, n_p - e_w (m_{p1} + n_p)}$ $F_{ep.Rd,2} = \dfrac{2 l_{eff,p}\, m_{pl.p} + 2 B_{t,Rd}\, n_p}{m_{p1} + n_p}$ $n_p = \min\ [e_p ; 1.25 m_{p1} ; e\,]$ $m_{pl.fc} = 0.25\ t_p^2\ \ f_{yp}\ \ /\ g_{Mo}$
	$l_{eff,p} = min\ [\ 2\, p\ \ m_{p1}\ ;\ a\ m_{p1}\ ]$ *with* m_{p1} *and* a *defined in the annexed table*	

Beam flange in compression	$k_7 = \infty$	$F_{Rd,7} = M_{c,Rd}/(h_b - t_{fb})$.$M_{c,Rd}$: is steel beam design moment resistance
Beam web in tension	$k_8 = \infty$	$F_{Rd,8} = b_{eff,t,wb}\ t_{wb}\ f_{ywb}\ /\ g_{Mo}$
	$b_{eff,t,wb} = l_{eff.p}$	
Upper bolt-row in tension	$k_{10} = 1.6 \frac{A_{s,bolt}}{L_b}$ $L_b = t_p + t_{fb} + 1/2(h_n + h_h)$ h_n : height of the bolt nut h_h : height of the bolt head	$F_{Rd,10} = 2\ B_{t,Rd}$ $B_{t,Rd} = \frac{0.9\ f_{ub}\ A_{s,b}}{g_{Mb}}$ **A $_{s,b}$: *stress area of the bolts***
Longitudi-nal slab reinforce-ment in tension	$k_{13} = k_{s.t} k_r$ where : $k_{s,t} = \frac{A_s}{h_c\ (\frac{1+b}{2} + K_b)} \frac{E_s}{E_a}$ $K_b = b\ \ (4.3\,b^2 - 8.9\,b + 7.2)$ • For single-sided joint configuration : *b = 1* • For double-sided joint configuration symmetrically loaded with balanced moments : *b = 0* for both sides • For double-sided joint configuration non-symmetrically loaded with balanced moments: + for more heavily loaded side : *b = 1* + for less heavily loaded side : *b = 0* and $k_{s,t}$ = ¥	$F_{Rd,13} = \frac{A_s\ f_{sk}}{g_s}$ with : $A_s \geq 0.004\ d_{eff}\ (b_{eff.b} - b_c)$ $A_s \leq \frac{1.1(0.85\ f_{ck\ s}/g_c)b_c\ d_{eff}}{b\ (f_{sk}/g_s)}$.$b_{eff.b}$: is the effective slab breadth in composite cross section of beam. A minimum amount of transverse reinforcement A_t should be placed each side of the column : $A_t \geq 0.5\,bA_s\ / tgd$ In which : tg$d$ = *1.35(e_T/e_L - 0.2)*

	$$k_r = \frac{1}{1 + \frac{E_s\, k_{s.t}}{K_{sc}}}$$ $$with\ K_{sc} = \frac{N\, k_{sc}}{u - \frac{u-1}{1+x}\frac{z_1}{d_s}}$$ $$x = \frac{E_a\, I_a}{d_s^2\, E_s\, A_s}$$ $$u = \sqrt{\frac{(1+x)\, N\, k_{sc}\, l_{eff.b}\, d_s^2}{E_a\, I_a}}$$ .k_{sc} : is the stiffness of one shear connector .$l_{eff.b}$: is the length of beam in hogging bending zone. .Ia : is the steel beam second moment of inertia .N : is the number of shear connectors distributed over the length $l_{eff.b}$.	
Column web panel in shear	$$k_t = \frac{1}{\sum \frac{1}{k_i}} \quad with \quad i = 3, 4, 5, 8, 10$$ $$k_{eq} = \frac{k_{13} z_1 + k_t z_2}{z_{eq}}$$ $$z_{eq} = \frac{k_{13} z_1^2 + k_t\, z_2^2}{k_{13} z_1 + k_t\, z_2}$$ $$k_1 = k_{a,wp.s} + k_{c,wp.s}$$ in which : $$k_{a,wp.s} = \frac{0.38 A_{vc}}{b\, z_{eq}}$$ **.If unencased column :** $k_{c,wp.s} = 0$ **.If encased column :** $$k_{c,wp.s} = \frac{0.06\, b_c\, h_c}{b\, z_{eq}} \frac{E_{cm.c}}{E_a}$$	$$F_{Rd,1} = \frac{V_{wp,Rd}}{b} \quad with\ V_{wp,Rd} = V_{a,wp,Rd} + V_{c,wp,Rd}$$ in which : $$V_{a,wp,Rd} = \frac{0.9\, A_{vc}\, f_{ywc}}{\sqrt{3}\, g_{Mo}}$$ **.If unencased column :** $V_{c,wp,Rd} = 0$ **.If encased column :** $$V_{c,wp,Rd} = u\, A_c \sin q\, \frac{0.85\, f_{ck\,c}}{g_c}$$ where : $$A_c = 0.8(h_c - 2t_{fc})(b_c - t_{wc})\cos q$$ $$q = \arctan\left(\frac{h_c - 2t_{fc}}{z}\right)$$ $$u = 0.55\left[1 + 2\frac{N_{Sd}}{N_{pl,Rd}}\right] \le 1.1$$ N_{Sd} : is the design axial compressive force in the column . .$N_{pl,Rd}$: is the design plastic resistance of encased column cross-section.

		$F_{t,Rd} = min\ [\ F_{Rd,,i}\ ]\quad with\ i= 3, 4, 5, 8, 10$ $F_{c.Rd} = min\ [\ F_{Rd,2}\ ;\ F_{Rd,7}\ ]$ • ***If*** $\boldsymbol{F_{c,Rd} > F_{Rd,13}}$ **:** $F_{Rdo} = min\ [\ F_{c,Rd}\ ;\ F_{Rd,13} + Ft,_{Rd}\ ]$ $z = \frac{1+F_o\, c_o^2}{1+F_o\, c_o}\, z_1 \quad with\ F_o = \frac{F_{Rdo}}{F_{Rd,13}} - 1$ $c_o = \frac{z_2}{z_1}$ • ***If*** $\boldsymbol{F_{c,Rd}}$ £ $\boldsymbol{F_{Rd,13}}$ **:** $z = z_1$
JOINT	**.Initial stiffness :** $S_{j,ini} = \frac{E_a\, z_{eq}^2}{\left(\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_{eq}}\right)}$ **. Nominal stiffness :** $S_j = S_{j,ini} / 2$	$\mathrm{F_{Rd}} = \mathrm{min}\ [\ \mathrm{F_{Rd,1}}\ ,\ \mathrm{F_{c,Rd}}\ ,\ \mathrm{F_{Rd,13}} + \mathrm{F_{t,Rd}}\ ]$ **.Elastic moment resistance :** $M_{e,Rd} = \frac{2}{3} F_{Rd}\ z$ **.Plastic design moment resistance :** $M_{Rd} = F_{Rd}\ z$





$$l_1 = \frac{m_{p1}}{m_{p1} + e_p}$$

$$l_2 = \frac{m_{p2}}{m_{p1} + e_p}$$

SHEAR RESISTANCE

1. Shear resistance of the fillet welds :

$$V_{Rd,1} = f_{v\,w.d}\, a_w\, l_{ws}$$

In which : $l_{ws} = 2\,[\,(h_b - 2\,(\,t_{fb} + s\,)]$

$$f_{v\,w.d} = \frac{f_u / \sqrt{3}}{b_w\ g_{Mw}}$$

In which f_u is the lower of the ultimate stresses of the column flange and the bracket

b_w should be taken as follows (linear interpolation between the values if needed):

Steel grade	Ultimate tensile strength f_u	b_w
S235	360 N/mm²	0.8
S275	430 N/mm²	0.85
S355	510 N/mm²	0.9

2. Shear resistance of the bolts :

$$V_{Rd,2} = 2\,(1+ z\,)\,F_{v,Rd}$$

In which : $F_{v,Rd}$: shear resistance of one bolt (shear plane passes through threaded part)

$$F_{v,Rd} = \frac{0.6\ f_{ub}\ A_{s,b}}{g_{Mb}}$$ for grades 4.6, 5.6 and 8.8

$$F_{v,Rd} = \frac{0.5\, f_{ub}\ A_{s,b}}{g_{Mb}}$$ for grades 4.8, 5.8 and 10.9

ζ = 0.28 if the upper bolt row is subjected to tension forces because of M
= 1.00 if the upper bolt row is not subjected to tension forces because of M

$A_{s,b}$ = stress area of the bolts

3. Shear resistance of the joint* :

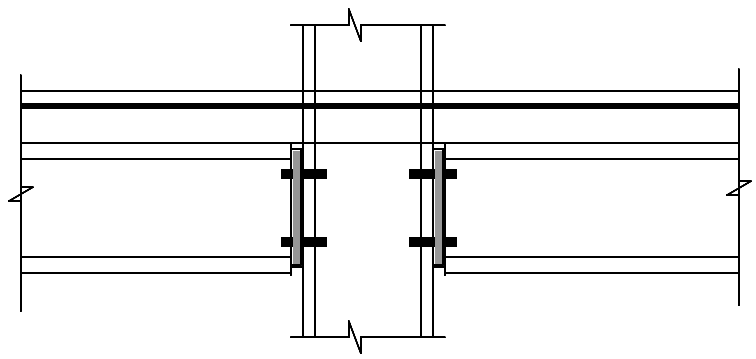
$$\boldsymbol{V_{Rd} = min\,[V_{Rd,1}\ ;\ V_{Rd,2}\,]}$$

* : The bearing resistance of the column flange and of the end-plate is supposed not to be predominant

STIFFNESS AND RESISTANCE CALCULATION

FOR BEAM-TO-COLUMN COMPOSITE JOINTS

WITH FLUSH END- PLATES

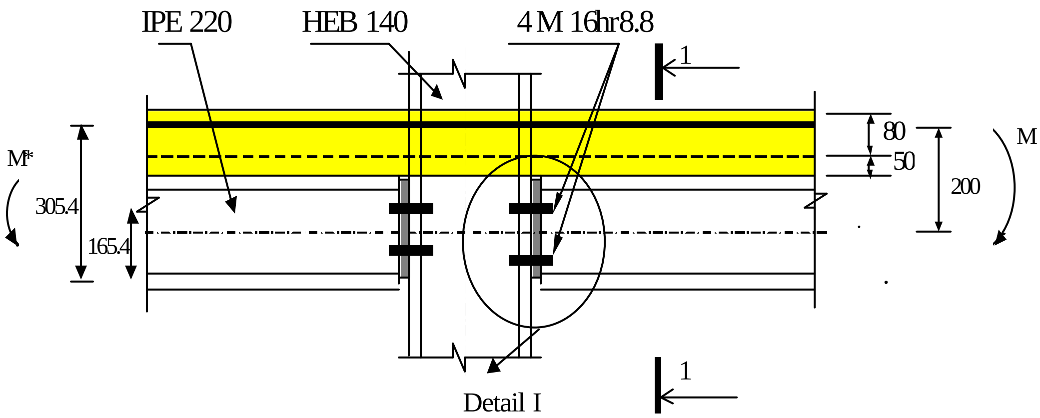


Worked example for the evaluation of

the rotational characteristics

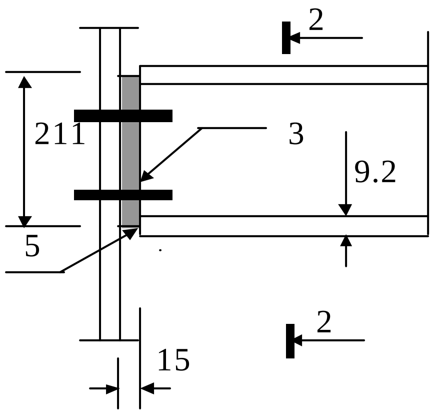
A- DATA FOR CALCULATION

- Main data for joint and loading

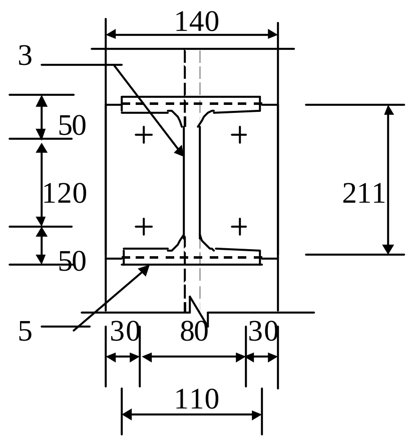


Calculation of the right joint with M > M*

- Dimensions of the end-plate



Detail _ I
(flush end-plate detail)



SECTION 2_2
(side view of flush end-plate)

- Other various data

Grade for all steel components except rebars: S235
Percentage of reinforcement in the effective slab: 0.7%

Connectors : - Stiffness: 100 kN/mm
- Spacing: 100 mm

Reinforcement bars: f_{sk} = 460 N/mm²
Concrete: f_{ck} = 20 N/mm² and E_{cm} = 29000 N/mm²

B- PRELIMINARY CALCULATIONS

Column:

$d_{wc} = h_c - 2\ t_{fc} - 2\ r_c = 140 - 2 \times 12 - 2 \times 12 = 92$ mm

$A_{vc} = A_c - 2\ b_c\ t_{fc} + (t_{wc} + 2\ r_c)\ t_{fc} = 4295.6 - 2 \times 140 \times 12 + (7 + 2 \times 12) \times 12 = 1307.6$ mm^2.

$$m = \frac{w - t_{wc}}{2} - 0.8\, r_c = \frac{80 - 7}{2} - 0.8 \times 12 = 26.9\, mm$$

$$e = \frac{b_c - w}{2} = \frac{140 - 80}{2} = 30\, mm$$

$$m_{pl,fc} = 0.25 t_{fc}^2 \frac{f_{yfc}}{g_{Mo}} = 0.25 \times 12^2 \frac{235}{1.1} = 7690.9\, Nmm/mm$$

$s = r_c = 12$ mm (for hot-rolled section)

Beam:

$d_s = 0.5\ h_b + h_{cs} + h_{ps} - a_{cs} = 0.5 \times 220 + 80 + 50 - 40 = 200$ mm

$z_1 = h_b - 0.5\ t_{fb} + h_{cs} + h_{ps} - a_{cs} = 220 - 0.5 \times 9.2 + 80 + 50 - 40 = 305.4$ mm

$z_2 = p_p + e_{p1} - 0.5\ t_{fb} = 120 + 50 - 0.5 \times 9.2 = 165.4$ mm

$$c_o = \frac{z_2}{z_1} = \frac{165.4}{305.4} = 0.5416$$

$$M_{c.Rd}(class\,1) = \frac{W_{pby}\ f_{yb}}{g_{Mo}} = \frac{285406 \times 235}{1.1} = 60973100.517\, N.mm \cong 60.973\, kN.m$$

End plate:

$$m_{p1} = \frac{w - t_{wb}}{2} - 0.8\sqrt{2}\, a_w = \frac{80 - 5.9}{2} - 0.8 \times \sqrt{2} \times 3 = 33.66\, mm$$

$$m_{p2} = p - t_{fb} - 0.8\sqrt{2}\, a_f = 50 - 9.2 - 0.8 \times \sqrt{2} \times 5 = 35.14\, mm$$

$$e_p = \frac{b_p - w}{2} = \frac{140 - 80}{2} = 30\, mm$$

$$l_1 = \frac{m_{p1}}{m_{p1} + e_p} = \frac{33.66}{33.66 + 30} = 0.5287$$

$$l_1 = \frac{m_{p2}}{m_{p1} + e_p} = \frac{35.14}{33.66 + 30} = 0.5521$$

$\alpha = 5.4871$

$$m_{pl,p} = 0.25 t_p^2 \frac{f_{yp}}{g_{Mo}} = 0.25 \times 15^2 \frac{235}{1.1} = 12017 N.mm/mm$$

Bolts:

$$F_{t.Rd} = \frac{0.9 \ f_{ub} \ A_{s,b}}{g_{Mb}} = \frac{0.9 \times 800 \times 157 \times 10^{-3}}{1.25} = 90.4 \, kN$$

$$F_{v.Rd} \left(shear \ plane \ passes \ through \ threaded \ part \right) = \frac{0.6 \ f_{ub} \ A_{s,b}}{g_{Mb}} = \frac{0.6 \times 800 \times 157 \times 10^{-3}}{1.25} = 60.3 \, kN$$

$L_b = t_{fc} + t_p + 0.5\,(h_h + h_n) = 12 + 15 + 0.5 \times (10 + 13) = 38.5$ mm

$d_w = 26.75$ mm

$$e_w = \frac{d_w}{4} = \frac{26.75}{4} = 6.69 \, mm$$

Concrete slab:

$d_{eff} = h_{cs} = 80$ mm

$b_{eff.b} = 3\,h_b = 3 \times 220 = 660$ mm

$l_{eff.b} = 4\,h_b = 4 \times 220 = 880$ mm

$$m = 0.7\,\% \ \rightarrow A_s = m \ d_{eff} \left(b_{eff.b} - b_c \right) = \frac{0.7}{100} 80 \times \left(660 - 140 \right) = 291 \,\mathrm{mm}^2$$

Shear connectors:

$k_{sc} = 100$ kN/mm^2 *(assumption)*

$$N = \frac{l_{eff.b}}{spacing} + 1 = \frac{880}{100} + 1 \cong 10 \quad (assumption)$$

C- CALCULATION OF THE STIFFNESS AND RESISTANCE PROPERTIES

Component 2: Column web in compression

a) Resistance:

$$\ell_o = t_{fb}/2 + a_f\sqrt{2} + t_p = 9.2/2 + 5\sqrt{2} + 15 = 26.671\,mm$$
$$t_{eff,c} = \ell_o + 5\,t_{fc} = 26.671 + 5\times 12 = 86.671\,mm$$
$$b_{eff,c,wc} = \ell_o + 5\,(s + t_{fc}) = 26.671 + 5\times(12+12) = 146.671\,mm$$
$$b_{el} = \ell_0 + 2\,(s + t_{fc}) = 26.671 + 2\times(12+12) = 74.671\,mm$$

$$k_{wc} = \min\left[1.0 ; 1.25 - 0.5\frac{\sigma_{com,a,Ed}}{f_{ywc}}\right] = 1 \quad (\textit{assumption})$$

- *If unencased column:*

$\beta = 1$

$$\bar{\lambda}_p = 0.932\sqrt{\frac{b_{eff,c,wc}\,d_{wc}\,f_{ywc}}{E\,t_{wc}^2}} = 0.932\sqrt{\frac{146.671\times 92\times 235}{210\times 10^3\times 7^2}} = 0.5174$$

$$\beta = 1: \quad \omega_c = \frac{1}{\sqrt{1+1.3(b_{eff,c,wc}\,t_{wc}/A_{vc})^2}} = \frac{1}{\sqrt{1+1.3\times\left(\frac{146.671\times 7}{1307.6}\right)^2}} = 0.7451$$

$$\bar{\lambda}_p = 0.5174 \leq 0.67 \rightarrow F^0_{a,wc.c,Rd} = k_{wc,a}\omega_c b_{eff,c,wc} t_{wc} f_{ywc}/\gamma_{Mo} = 1\times 0.7451\times 146.671\times 7\times 235/1.1 = 163421\text{ N}$$
$$F^0_{a,wc.c,Rd} = 163.421\,\text{kN}$$

- *If encased column:*

  - Steel column alone:

$$\omega_c = 1 \textit{ and } \bar{\lambda}_p = 0$$
$$\bar{\lambda}_p = 0 \leq 0.67 \rightarrow F_{a,wc.c,Rd} = k_{wc,a}\omega_c b_{eff,c,wc} t_{wc} f_{ywc}/\gamma_{Mo}$$

$$F_{a,wc.c,Rd} = 1\times 1\times 146.671\times 7\times\frac{235}{1.1} \approx 219340\ N = 219.34\ kN$$

  - Contribution from the encasing concrete:

$$k_{wc,c} = \min\left[1.3 + 3.3\frac{\sigma_{com.c.Ed}}{(f_{ck\,c}/\gamma_c)} ; 2.0\right] = 1.577 \quad (\textit{assumption})$$

$$F_{c,wc.c,Rd} = 0.85 k_{wc,c}\,t_{eff,c}\,(b_c - t_{wc})\,f_{ck\,c}/\gamma_c = 0.85\times 1.557\times 86.671\times(140-7)\times 20/1.5$$
$$= 206023\text{ N} = 206.023\,\text{kN}$$

- *Total resistance:*

  - *unencased column:*

$$F_{Rd,2} = F^{0}_{a,wc.c,Rd} = 163.421 \text{ kN}$$

  - *encased column:*

$$F_{Rd,2} = F_{a,wc.c,Rd} + F_{c,wc.c,Rd} = 219.34 + 206.023 = 425.363 \text{ kN}$$

b) Stiffness:

- *If unencased column:*

$$k_{a,wc.c} = \frac{0.7 b_{eff,c,wc} t_{wc}}{d_{wc}} = \frac{0.7\times146.671\times7}{92} = 7.812 \text{ mm}$$

- *If encased column:*

$$k_{c,wc.c} = \frac{0.5\, b_{el}\, b_c}{h_c}\,\frac{E_{cm.c}}{E_a} = \frac{0.5\times74.671\times140}{140}\,\frac{29}{210} = 5.156 \text{ mm}$$

- *Total stiffness:*

  - unencased column:

$$k_2 = k_{a,wc.c} = 7.812 \text{ mm}$$

  - encased column:

$$k_2 = k_{a,wc.c} + k_{c,wc.c} = 7.812 + 5.156 = 11.968 \text{ mm}$$

Component 3: Column web in tension

a) Resistance:

$$b_{eff,t,wc} = \min\ [\,2p\,m\,; 4m + 1.25\,e\,] = \min\left[2\times p\times26.9\,;\,4\times26.9 + 1.25\times30\right]$$
$$b_{eff,t,wc} = \min\left[169.018\,;145.1\right] = 145.1\ mm$$

$$b = 1 \quad \rightarrow w_t = \frac{1}{\sqrt{1 + 1.3(b_{eff,t,wc}\, t_{wc} / A_{vc})^2}} = \frac{1}{\sqrt{1 + 1.3\times\left(\frac{145.1\times7}{1307.6}\right)^2}} = 0.7486$$

$$F_{Rd,3} = w_t\, b_{eff,t,c}\, t_{wc}\, f_{ywc} / g_{Mo} = 0.7486\times145.1\times7\times235/1.1 = 162439 \text{ N} = 162.439 \text{ kN}$$

b) Stiffness:

$$k_3 = \frac{0.7 b_{eff,t,wc}\, t_{wc}}{d_{wc}} = \frac{0.7 \times 145.1 \times 7}{92} = 7.728 \quad \text{mm}$$

Component 4: Column flange in bending

a) Resistance:

$l_{eff,t,fc} = b_{eff,t,wc}$ = 145.1 mm

$$n = \min \ [e; 1.25 m; e_p] = \min \left[30 \ ; 1.25 \times 26.9 ; 30\right] = 30 \ mm$$

$$k_{fc} = \min \ \left[1; \frac{2 f_{yfc} - 180 - s_{n,fc}}{2 f_{yfc} - 360} \right] = 1 \qquad (assumption)$$

$$F_{fc.Rd,t1} = \frac{(8n - 2e_w)\, l_{eff,t,fc} \ m_{pl,fc}}{2mn - e_w\,(m+n)} k_{fc} = \frac{(8 \times 30 - 2 \times 6.69) \times 145.1 \times 7690.9}{2 \times 26.9 \times 30 - 6.69 \times (26.9 + 30)} \times 1 = 205050 \ \text{N}$$

$$F_{fc.Rd,t2} = \frac{2 l_{eff,t,fc}\, m_{pl,fc}\, k_{fc} + 2 B_{t,Rd}\, n}{m+n} = \frac{2 \times 145.1 \times 7690.9 \times 1 + 2 \times 90.4 \times 10^3 \times 30}{26.9 + 30} = 134550 \ \text{N}$$

$F_{Rd,4} = min\ [\ F_{fc.Rd,t1}\ ;\ F_{fc.Rd,t2}\ ]$ = min [205050 ; 134550] = 134550 N = 134.550 kN

b) Stiffness:

$$k_4 = \frac{0.85\, l_{eff,t,fc}\, t_{fc}^3}{m^3} = \frac{0.85 \times 145.1 \times 12^3}{26.9^3} = 10.949 \ \text{mm}$$

Component 5: End plate in bending

a) Resistance:

$l_{eff,p} = \min\ [\ 2\ \pi\ m_{p1}\ ;\ \alpha\ m_{p1}\ ] = \min\ [\ 2 \times \pi \times 33.66\ ;\ 5.4871 \times 33.66\ ]$

$l_{eff,p}$ = min [211.5 ; 184.696] = 184.696 mm

$$n_p = \min \ [e_p ; 1.25 m_{p1} ; e] = \min \ \left[30 ; 1.25 \times 33.66 ; 30\right] = 30\, mm$$

$$F_{ep.Rd,1} = \frac{(8 n_p - 2 e_w)\, l_{eff,p} \ m_{pl,p}}{2 m_{p1} n_p - e_w\,(m_{p1} + n_p)} = \frac{(8 \times 30 - 2 \times 6.69) \times 184.696 \times 12017}{2 \times 33.66 \times 30 - 6.69 \times (33.66 + 30)} = 315586 \ \text{N} = 315.586 \ \text{kN}$$

$$F_{ep.Rd,2} = \frac{2 l_{eff,p}\, m_{pl,p} + 2 B_{t,Rd}\, n_p}{m_{p1} + n_p} = \frac{2 \times 184.696 \times 12017 + 2 \times 90.4 \times 10^3 \times 30}{33.66 + 30} = 154965 \ \text{N} = 154.965 \ \text{kN}$$

$F_{Rd,5} = min\ [\ F_{ep,Rd,1}\ ;\ F_{ep,Rd,2}\ ]$ = min [315.586 ; 154.965] = 154.965 kN

b) Stiffness:

$$k_5 = \frac{0.85 l_{eff,p}\ t_p^3}{m_{p1}^3} = \frac{0.85 \times 184.696 \times 15^3}{33.66^3} = 13.897 \ \text{mm}$$

Component 7: Beam flange in compression

a) Resistance:

$$F_{Rd,7} = \frac{M_{c,Rd}}{h_b - t_{fb}} = \frac{60.973 \times 10^3}{220 - 9.2} = 289.246\ kN$$

b) Stiffness:

$$k_7 = \infty$$

Component 8: Beam web in tension

a) Resistance:

$b_{eff,t,wb}$ = $l_{eff,p}$ = 184.696 mm

$$F_{Rd,8} = b_{eff,t,wb}\, t_{wb}\ f_{ywb}\ /\ g_{Mo} = 184.696 \times 5.9 \times 235 / 1.1 = 226504\ \ \text{N} = 232.8\ \ \text{kN}$$

b) Stiffness:

$$k_8 = \infty$$

Component 10: Bolts in tension

a) Resistance:

$F_{Rd,10} = 2\ B_{t,Rd}$ = 2 × 90.4 = 180.8 kN

b) Stiffness:

$$k_{10} = 1.6 \frac{A_{s,b}}{L_b} = 1.6 \times \frac{157}{38.5} = 6.525\ \ \text{mm}$$

Component 13: Longitudinal slab reinforcement in tension

a) Resistance:

$$A_s^{min} = 0.004\ d_{eff}\ (b_{eff.b} - b_c) = 0.004 \times 80 \times (660 - 140) = 166.4\ mm^2$$

$$A_s^{max} = \frac{1.1(0.85\ f_{ck\ s} / g_c) b_c\ d_{eff}}{b\ (f_{sk} / g_s)} = \frac{1.1 \times (0.85 \times 20 / 1.5) \times 140 \times 80}{1 \times (460 / 1.15)} = 349.07\ mm^2$$

with $\mu = 0.7\% \rightarrow A_s = 291\ mm^2 < A_s^{max} = 349.07\ mm^2$: **OK**

and $A_s = 291\ mm^2 > A_s^{min} = 166.40\ mm^2$: **OK**

$$F_{Rd,13} = \frac{A_s\ f_{sk}}{g_s} = \frac{291 \times 460}{1.15} = 116400\ \text{N} = 116.4\ \text{kN}$$

b) Stiffness:

$\beta = 1$

$$K_b = b\ (4.3 b^2 - 8.9 b + 7.2) = 1 \times (4.3 \times 1^2 - 8.9 \times 1 + 7.2) = 2.6$$

$$k_{s,t} = \frac{A_s}{h_c\ (\frac{1 + b}{2} + K_b)} = \frac{291}{140\left(\frac{1+1}{2} + 2.6\right)} = 0.5774\ \text{mm}$$

$$x = \frac{E_a\ I_a}{d_s^2\ E_s\ A_s} = \frac{210 \times 10^3 \times 2772 \times 10^4}{200^2 \times 210 \times 10^3 \times 291} = 2.3814$$

$$u = \sqrt{\frac{(1+x)\ N\ k_{sc}\ l_{eff.b}\ d_s^2}{E_a\ I_a}} = \sqrt{\frac{(1+2.3814) \times 10 \times 100 \times 10^3 \times 880 \times 200^2}{210 \times 10^3 \times 2772 \times 10^4}} = 4.5218$$

$$K_{sc} = \frac{N\ k_{sc}}{u - \frac{u-1}{1+u}\frac{z_1}{d_s}} = \frac{10 \times 100 \times 10^3}{4.5218 - \frac{4.5218 - 1}{1 + 2.3814} \times \frac{305.4}{200}} = 341134.2747$$

$$k_r = \frac{1}{1 + \frac{E_s\ k_{s.t}}{K_{sc}}} = \frac{1}{1 + \frac{210 \times 10^3 \times 0.5774}{341134.2747}} = 0.7378$$

$$k_{13} = k_{s,t}\ k_r = 0.5774 \times 0.7378 = 0.4260\ \text{mm}$$

Component 1: Column web panel in shear

a) Resistance:

- *If unencased column:*

$$V_{a,wp,Rd} = \frac{0.9\ A_{vc}\ f_{ywc}}{\sqrt{3}\ g_{\ Mo}} = \frac{0.9\times1307.6\times235}{\sqrt{3}\times1.1} = 145155\ \text{ N} = 145.155\ \text{ kN}$$

- *If encased column:*

$$u = 0.55\left[1 + 2\frac{N_{Sd}}{N_{pl,Rd}}\right] = 0.626 \qquad (assumption)$$

$$q = \arctan\left(\frac{h_c - 2t_{fc}}{z}\right) = \arctan\left(\frac{140 - 2\times12}{251.5}\right) = 24.76^{o}$$

$$A_c = 0.8\,(h_c - 2t_{fc})(b_c - t_{wc})\cos q = 0.8\times(140 - 2\times12)\times(140 - 7)\times\cos\left(24.76^{\,o}\right) = 11207.7\,\text{mm}^2$$

$$V_{c,wp,Rd} = u\ A_c\ \sin q\ \frac{0.85\ f_{ck\,c}}{g_{\ c}} = 0.626\times11207.7\times\sin\left(24.76^{o}\right)\times\frac{0.85\times20}{1.5} = 33302\ \text{ N} = 33.302\ \text{ kN}$$

- *Total resistance:*

  - *unencased column:*

$$F_{Rd,1} = \frac{V_{wp,Rd}}{b} = \frac{V_{a,wp,Rd}}{b} = \frac{145.155}{1} = 145.155\ kN$$

  - *encased column:*

$$F_{Rd,1} = \frac{V_{wp,Rd}}{b} = \frac{V_{a,wp,Rd} + V_{c,wp,Rd}}{b} = \frac{145.155 + 33.302}{1} = 178.457\ \text{ kN}$$

b) Lever arm for resistance calculation

- *If unencased column:*

$F_{c,Rd}$ *= min [* $F_{Rd,2}$ *;* $F_{Rd,7}$*]* = min [163.421 ; 289.246] = 163.421 kN

$F_{t,Rd}$ *= min [* $F_{Rd,i\quad i = 3,4,5,8,10}$*]* = min [162.439 ; 134.550 ; 154.965 ; 232.8 ; 180.8] = 134.550 kN

As $F_{c,Rd}$ *= 163.421 kN >* $F_{Rd,13}$ *= 116.4 kN ® upper bolt row subjected to tension forces*

F_{Rdo} *= min [* $F_{c,Rd}$ *;* $F_{Rd,13}$ *+* $F_{t,Rd}$ *]* = min [163.421 ; 116.4 + 134.550] = 163.421 kN

$$F_o = \frac{F_{Rdo}}{F_{Rd,13}} - 1 = \frac{163.421}{116.4} - 1 = 0.403$$

$$z = \frac{1+F_o\,c_o^2}{1+F_o\,c_o}\,z_1 = \frac{1+0.403\times 0.5416^2}{1+0.403\times 0.5416}\times 305.4 = 280.318 \text{ mm}$$

- *If encased column:*

$F_{c,Rd}$ = *min [* $F_{Rd,2}$ *;* $F_{Rd,7}$*]* = min [425.363 ; 289.246] = 289.246 kN
$F_{t,\,Rd}$ = *min [*$F_{Rd,i \quad i=3,4,5,8,10}$*]* = min [162.439 ; 134.550 ; 154.965 ; 232.8 ; 180.8] = 134.550 kN

As $F_{c,Rd}$ *= 289.246 kN >* $F_{Rd,13}$ *= 116.4 kN ® upper bolt row subjected to tension forces*

F_{Rdo} = *min [* $F_{c,Rd}$ *;* $F_{Rd,13}$ + $F_{t,\,Rd}$ *]* = min [289.246 ; 116.4 + 134.550]

F_{Rdo} = min [289.246 ; 250.95] = 250.95 kN

$$F_o = \frac{F_{Rdo}}{F_{Rd,13}} - 1 = \frac{250.95}{116.4} - 1 = 1.1559$$

$$z = \frac{1+F_o\,c_o^2}{1+F_o\,c_o}\,z_1 = \frac{1+1.1559\times 0.5416^2}{1+1.1599\times 0.5416}\times 305.4 = 251.5 \text{ mm}$$

c) Lever arm for stiffness calculation:

$$k_t = \frac{1}{\sum \frac{1}{k_{br,i \quad i=3,4,5,8,10}}} = \frac{1}{\frac{1}{7.728}+\frac{1}{10.949}+\frac{1}{13.897}+\frac{1}{\infty}+\frac{1}{6.525}} = 2.242 \text{ mm}$$

$$k_{eq} = \frac{k_{13}z_1 + k_t z_2}{z_{eq}} = \frac{0.426\times(305.4) + 2.242\times(165.4)}{201.8} = 2.483 \text{ mm}$$

$$z_{eq} = \frac{k_{13}z_1^2 + k_t\,z_2^2}{k_{13}z_1 + k_t\,z_2} = \frac{0.426\times(305.4)^2 + 2.242\times(165.4)^2}{0.426\times(305.4) + 2.242\times(165.4)} = 201.8 \text{ mm}$$

d) Stiffness

- *If unencased column:*

$$k_{a,wp.s} = \frac{0.38\,A_{vc}}{b\;z_{eq}} = \frac{0.38\times 1307.6}{1\times 201.8} = 2.463 \text{mm}$$

- *If encased column:*

$$k_{c,wp.s} = \frac{0.06\;b_c\;h_c}{b\;z_{eq}}\frac{E_{cm.c}}{E_a} = \frac{0.06\times 140\times 140}{1\times 201.8}\;\frac{29}{210} = 0.805 \text{mm}$$

- Total stiffness:

- *unencased column:*

$k_1 = k_{a,wp.s} = 2.463$ mm

- *encased column:*

$k_1 = k_{a,wp.s} + k_{c,wp.s} = 2.463 + 0.805 = 3.268$ mm

D- EVALUATION OF THE MECHANICAL PROPERTIES OF THE JOINT

a) Resistance:

- *If unencased column:*

F_{Rd} = *min [* $F_{1,Rd}$; $F_{c,Rd}$; $F_{Rd,13}$ + $F_{t,Rd}$ *]* = min [145.155 ; 163.421 ; 116.4 + 134.55]

F_{Rd} = min [145.155 ; 163.421 ; 250.95] = 145.155 kN

(Column web panel in shear → **ductile** ® **O.K.**)

→ Plastic design moment resistance:

$$M_{Rd} = F_{Rd}\, z = 145.155 \times 280.318 = 40689.559 \text{ kN.mm} = 40.7 \text{ kN.m}$$

® Elastic moment resistance:

$$M_{e,Rd} = \frac{2}{3} M_{Rd} = \frac{2}{3} \times 40.7 = 27.13 \text{ kN.m}$$

- *If encased column:*

F_{Rd} = *min [* $F_{1,Rd}$; $F_{c,Rd}$; $F_{Rd,13}$ + $F_{t,Rd}$ *]* = min [178.457 ; 289.246 ; 116.4 + 134.55]

F_{Rd} = min [178.457 ; 289.246 ; 250.95] = 178.457 kN

(Column web panel in shear → **ductile** ® **O.K.**)

→ Plastic design moment resistance:

$$M_{Rd} = F_{Rd}\, z = 178.457 \times 251.5 = 44882 \text{ kN.mm} = 44.9 \text{ kN.m}$$

® Elastic moment resistance:

$$M_{e,Rd} = \frac{2}{3} M_{Rd} = \frac{2}{3} \times 44.9 = 29.9 \text{ kN.m}$$

b) Stiffness:

- *If unencased column:*

→ Initial stiffness :

$$S_{j,ini} = \frac{E_a\, z_{eq}^2}{\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_{eq}}} = \frac{210 \times 10^3 \times 201.8^2}{\frac{1}{2.463} + \frac{1}{7.812} + \frac{1}{2.483}} \cong 9126 \text{ kN.m}$$

→ Nominal stiffness:

$S_j = S_{j,ini} / 2 = 9126/2 \cong 4563\, kN.m$

- *If encased column:*

$\rightarrow$ Initial stiffness :

$$S_{j,ini} = \frac{E_a\, z_{eq}^2}{\dfrac{1}{k_1} + \dfrac{1}{k_2} + \dfrac{1}{k_{eq}}} = \frac{210 \times 10^3 \times 201.8^2}{\dfrac{1}{3.268} + \dfrac{1}{11.986} + \dfrac{1}{2.483}} \cong 10792 \text{ kN.m}$$

$\rightarrow$ Nominal stiffness:

$S_j = S_{j,ini} / 2 = 10792/2 = 5396\, kN.m$

SUMMARY

Unencased Column		Encased Column	
M_{Rd} (kNm)	$S_{j,ini}$ (kNm/rad)	M_{Rd} (kNm)	$S_{j,ini}$ (kNm/rad)
40.7	9126	44.9	10792

Course: Eurocode 4

Lecture 9 : Composite joints
Annex B

References:

- COST C1: Composite steel-concrete joints in frames for buildings: Design provisions
Brussels, Luxembourg 1999

Annex B: Calculation procedures and worked examples

1 Introduction

Calculation procedures and design examples are presented for the following joint configurations and connections types:

- **ANNEX B1:** Joints with contact plate connection in single- or double-sided configurations:
  - with an unencased column;
  - with an encased column.

- **ANNEX B2:** Joints with partial-depth end-plate connection in single- or double-sided configurations:
  - with an unencased column;
  - with an encased column.

- **ANNEX B3:** Joints with flush end-plate connection in single- or double-sided configurations:
  - with an unencased column;
  - with an encased column.

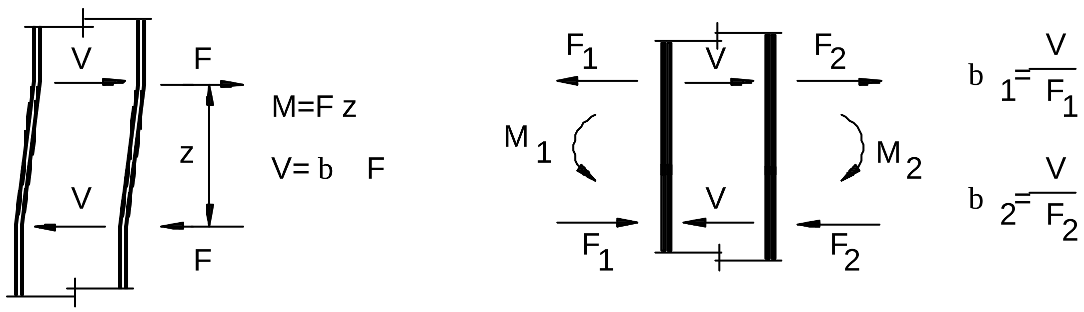
For each of these joints, the calculation procedure is organised as follows:

- a first design sheet is devoted to relevant mechanical and geometrical characteristics of the joint;
- in the next sheets, the calculation procedure gives the expressions of both stiffness and resistance for all the components of the joint;
- finally, the global properties of the joint, i.e. its initial or nominal stiffness and its design moment resistance, are derived and summarized at the end of the design sheets.

2 Additional design considerations

2.1 Transformation parameter b

The web panel deformation is due to the shear force. The shear force (*V*) in the web panel is the combination of the column shear force resulting from the global frame analysis and the local shear force due to the load introduction (*F*) (Figure B 1). For simplicity, this shear force is obtained by magnifying the force *F* by means of a factor *b* (Table B 1).



a. Definition **b.** b **values**

Figure B 1 Shear force in a column web panel

In order to prevent any iterative process in the determination of *b*, some values are proposed in the calculation procedures for three typical cases (see Table B 1).

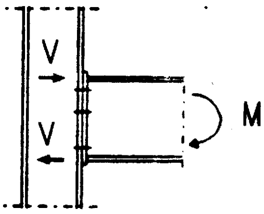
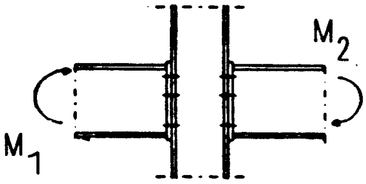
<table>
<tr><th>Configuration and loading</th><th>Proposed value for b</th></tr>
<tr><td>
</td><td>$b = 1$</td></tr>
<tr><td>
</td><td>if $|M_1| \cong |M_2|$
$b_1 = b_2 = 2$
if M_1 or $M_2 \cong 0$
$b_1 = b_2 = 1$</td></tr>
</table>

Table B 1 Approximate values of *b*

For common buildings and loading patterns, the value $b = 1$ can be used in a first step as a safe value. Of course in case of double-sided joint configurations with balanced moments, it could be of value to evaluate more precisely the value of b; in these cases, $0 < b < 1$ and a value of b smaller than 1 results in an increase of the joint stiffness and possibly of the bending resistance.

When the internal forces acting on the joint configuration are available from the global frame analysis, b can be assessed in a more accurate way. Of course, for a preliminary design, a value of b has to be chosen a priori.

2.2 Factor $k_{wc,a}$

The factor $k_{wc,a}$ accounts for the detrimental effect of the longitudinal web stresses (due to the normal force and the bending moment in the steel column) on the local design resistance of the steel column web in compression.

The factor $k_{wc,a}$ is as follows :

$$k_{wc,a} = (1{,}25 - 0{,}5 \frac{s_{com,a,Ed}}{f_{ywc}}) \leq 1{,}0 \qquad \text{(B1)}$$

and is plotted in figure B2 where f_{ywc} is the yield stress of the column web and $s_{com,a,Ed}$ is the maximum direct stress in the column web at the root of the radius.

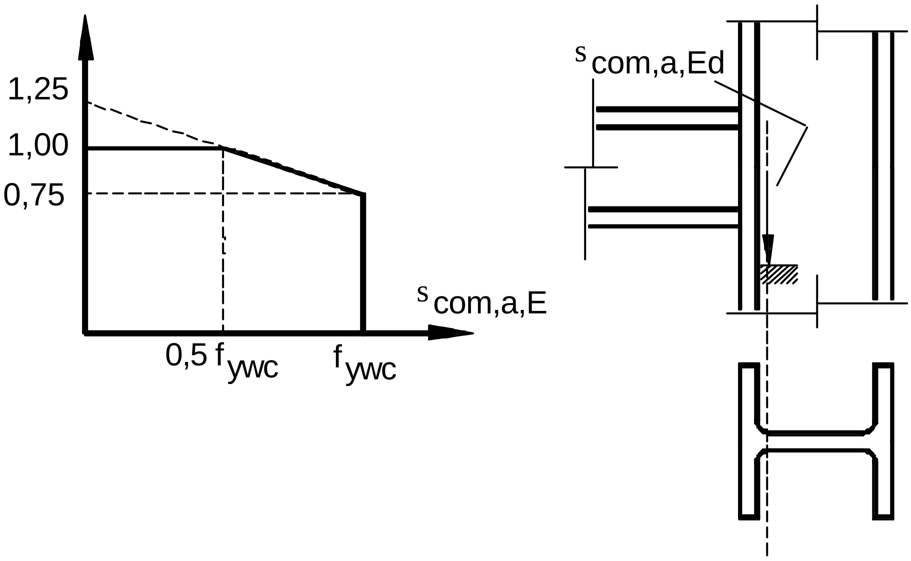


Figure B 2 Evolution of the factor $k_{wc,a}$

In most situations $s_{com,a,Rd} < 0{,}5\,f_{ywc}$ and therefore to adopt an upper bound of 1 for the factor $k_{\mathrm{wc,a}}$ is safe.

The evaluation of $k_{\mathrm{wc,a}}$ is based on an assessment of $s_{com,a,Ed}$. It is up to the designer, once the global frame analysis is completed, to check whether the actual value of $k_{\mathrm{wc,a}}$ is in compliance with the one adopted as a first approximate. This check is of primary importance.

2.3 Options taken in worked examples

Size and grade of the welds connecting the beam flanges to the end-plates

Welds are used to assemble the beam and the end-plates. Their grade should be higher than that of the weaker connected material.

For joints between small and medium size shapes (say up to 400 mm depth), a very simple rule is to use a weld throat size a_f (Figure B 3) of 50 % of the thickness t_{fb} of the beam flanges, i.e. :

$$a_f \cong 0{,}5\, t_{fb} \qquad \text{(B2)}$$

This value has been rounded up in the examples.

For larger joints, it could however be more economical to proportion the welds with regards to the joint resistance rather than to the beam resistance.

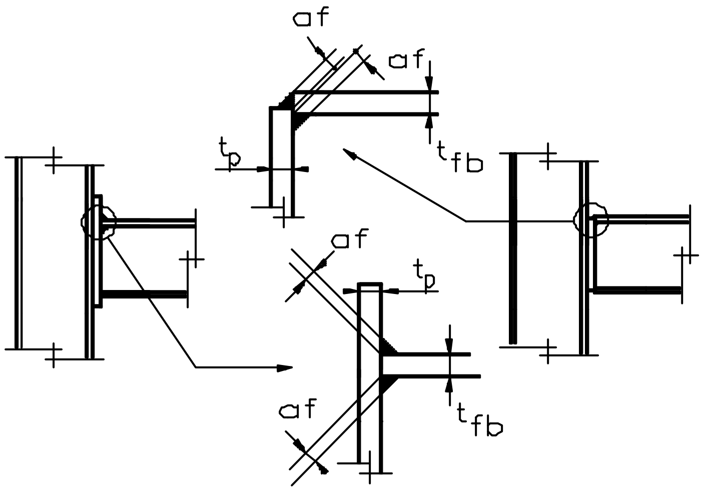


Figure B 3 Beam-flange-to-end-plate welds

Contact plate size and steel grade

No premature failure of the contact plate is likely to occur if:

- its steel grade is equal to or higher than that of the beam;
- its width is equal to or larger than the minimum corresponding dimension of the beam and of the column;
- its height is equal to or larger than that of the beam flange thickness.

In these circumstances it is deemed that the resistance will not be limited by the contact plate.

Structural Steelwork Eurocodes

Development of a Trans-National Approach

Course: Eurocode 4

Lecture 10 : Advanced composite floor systems

Summary:

- Traditional composite construction in buildings results in relatively large structural depth. This may present problems where the beams are required to span a long distance or the building height is restricted to comply with planning regulations. This lecture discusses alternative composite structural systems which are appropriate for long span applications or where a particularly shallow structural floor is required. The impact of the choice of structural system on installation and refitting of services in a building is explained.

Pre-requisites:

- Lecture 1 – Introduction to composite construction

Notes for Tutors:

This material comprises one 30 minute lecture.

Objectives:

- To introduce alternative composite systems for applications requiring either long spans or shallow floors.

References:

- Eurocode 4
- Chien, E.Y.L. and Ritchie, J.K. (1984) Design and Construction of Composite Floor Systems, Canadian Institute of Steel Construction.
- Chien, E.Y.L. and Ritchie, J.K. (1992) Composite Floor Systems, Constructional Steel Design – An International Guide, ed. Dowling, P.J., Harding, J.E. and Bjorhovde, R., Elsevier Applied Science, , pp. 471-500
- Lawson, R.M. and Rackham, J.W. (1989) Design of Haunched Composite Beams in Buildings, Steel Construction Institute
- Brett, P.R., Nethercot, D.A and Owens, G.W. (1987) Continuous Construction in Steel for Roofs and Composite Floors, The Structural Engineer, Vol. 65A, No. 10, October.
- Lawson, R.M., Mullet, D.L. and Rackham, J.W. (1998) Design of Asymmetric Slimflor Beams, Journal of Constructional Steel research, Vol 46: Nos. 1-3, paper No. 123.

Contents:

1. Introduction

Traditional composite construction with one way spanning slabs supported by a grillage of secondary and primary beams is a cost effective and structurally efficient solution for multi-storey buildings. Its advantages over other methods of construction have been explained in lecture 1. The success of this form of building construction is manifest throughout Europe and it has become the most popular form of multi-storey construction in a number of countries. However, a number of trends in modern construction have led to the development of more advanced forms of composite construction – these will be briefly discussed in this lecture.

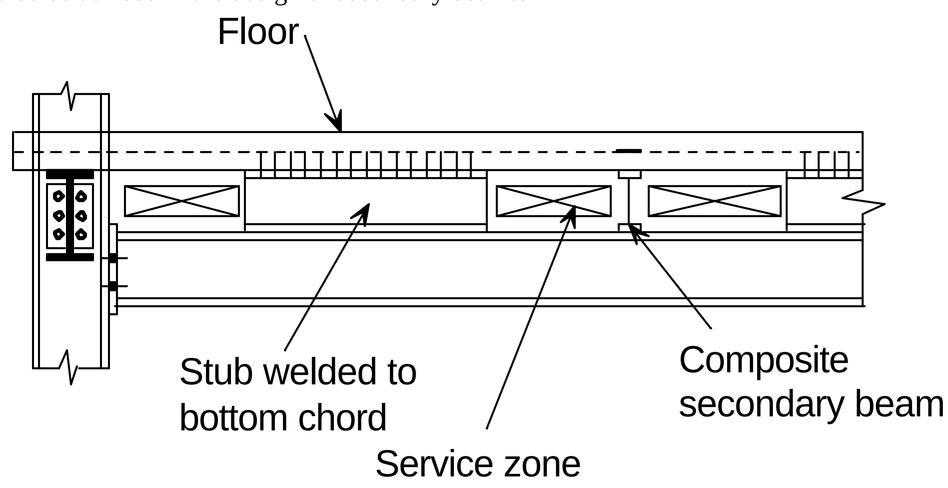
Many clients require large open space for maximum flexibility of use in their buildings. This requires that column spacings be increased from the typical 9m x 6m, which is ideal for traditional composite construction, to perhaps 12 to 20m in one direction. Spans of this size require very large beams, a number of possible long-span alternatives have been developed to improve structural efficiency in these situations. A further complication in modern buildings is the accommodation of services. In traditional composite construction, services were either routed below the deepest beam, in a separate 'service zone' or threaded through holes cut into the webs of the beams. The former approach increases the overall depth of the construction, the latter makes upgrading of services (which is likely to occur within 10-15 years of construction) problematic. This issue is exacerbated when long-spans are required and is an important consideration when considering structural options. Planning laws and regulations frequently restrict the maximum height of new buildings. A reduction in the overall depth of the structural and service zone can considerably affect the economic feasibility of a project, either by reducing the floor to floor height such that the requisite number of floors can be accommodated with in the regulations or by reducing the overall building height leading to economies in cladding costs. To address this problem, a number of structural systems have been developed using shallow floor beams supporting slabs on their lower flanges thus significantly reducing the overall structural depth. A number of long-span alternatives integrate the services within the structural zone in order to minimise the overall construction depth. These will be explained in the following sections.

It is important to realise that a building frame accounts for perhaps only 10-15% of the project cost. Therefore, decisions on structural form should be informed by the impact they have on the overall project cost. In some cases, a more expensive structural alternative (perhaps a long-span or a shallow floor) may prove cost-effective when considered in the context of overall project costs.

2. Long span solutions

2.1 Stub girders

Figure 1 shows an innovative structural solution which is particularly suitable for buildings requiring a column free space of 11.5 – 13.5m between the building exterior and a central core. The system was devised in North America in the 1970s and is described in detail by Chien and Ritchie (1984, 1992). Stub girders are basically a Vierendeel-type truss, the bottom boom of which is usually a column section, the web is formed from short lengths of a beam section (the ‘stubs’) and the top flange is provided by a concrete slab. Composite action is assured by the provision of connectors along the stub beams. Secondary beams may frame into the stubs or sit on the bottom boom, in which case they may be designed based on cantilever and suspended span construction with beam to beam connections located at inflection points. Composite action may also be utilised in the design of secondary beams.



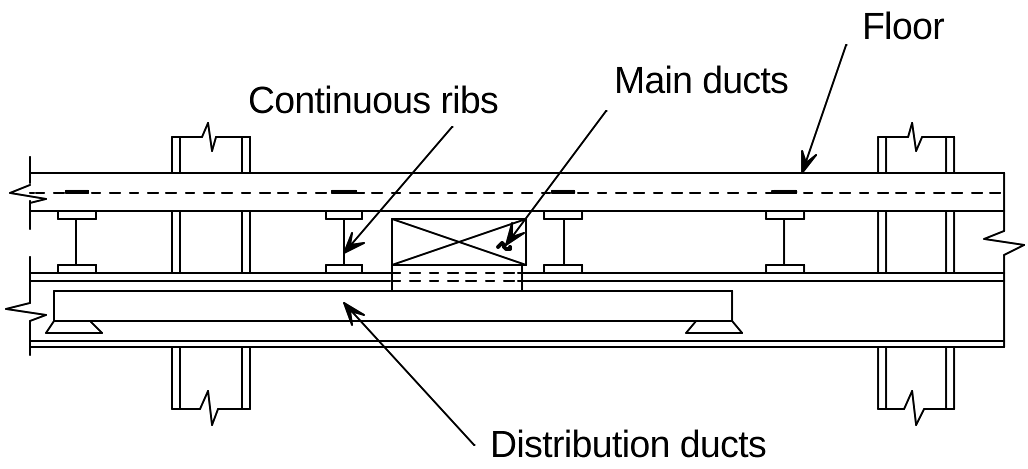


Figure 1 Stub girder system

The openings formed between the shear panels (the stubs) provide room for mechanical services thus reducing the storey-height. As the concrete slab forms the compression boom of the truss, temporary propping is necessary during construction unless an additional member (often a T-section) is provided to enable the truss to resist construction loading. Particular attention should be given to the design of the shear connectors, slab reinforcement and combined bending and shear at critical locations. Chien and Ritchie (1984, 1992) provide detailed recommendations. Although most stub-girders are designed as simply supported and used in braced frames, they may also be designed as part of a sway frame but in such cases a length of stub beam is required adjacent to the supporting column to effect a moment resisting connection.

2.2 Haunched beams

For spans in the region of 15 to 20m a haunched beam as shown in figure 2 may be appropriate. Continuity at the beam column connection is developed by provision of a full-strength moment resisting joint, requiring the use of a haunch (hence the name) as frequently used in portal frame construction. Continuity reduces beam moments and deflections thus permitting the use of shallower and lighter beam sections but at the expense of increased column moments. Overall economy can be achieved, particularly when total project costs are considered. The depth of the haunch sets the level for the suspended ceiling and creates a service zone of uniform depth over the majority of the span. This is particularly attractive to service engineers as it affords them freedom to install services in any location and also allows complete flexibility in subsequent refits.

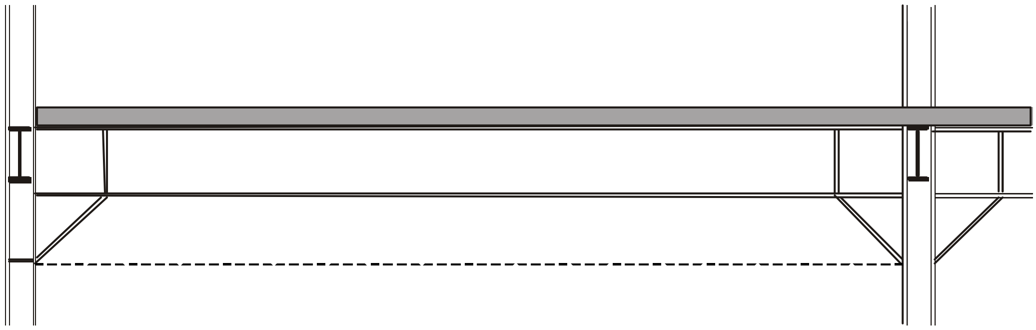
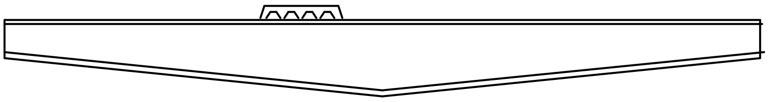


Figure 2 – based on SCI pub 60 page 5

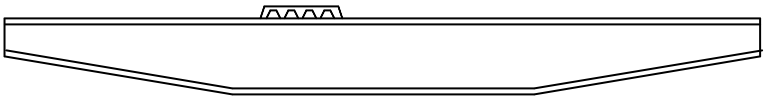
Haunch lengths are typically 5 to 7% of the span of the beam, although 7 to 15% may be required if the frame is a sway frame as a greater length of the beam will be subjected to negative moment. The size of the beam is normally determined by provision of sufficient moment capacity at the tip of the haunch to resist the negative moment at this location (the contribution of any slab reinforcement is ignored here). Elastic or plastic analysis methods may be used. As the *composite* beam capacity is likely to be considerably greater than the mid-span moment, taking account of moment redistribution derives some benefit. Particular attention should be given to the design of the connections and to the stability of the haunch region under negative moments. Detailed guidance on design to the UK code is provided by Lawson and Rackham (1989), the principles of which are equally applicable to design to EC4.

2.3 Tapered fabricated beams

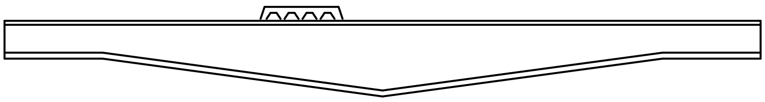
Plate girders are commonly used in bridges but use in buildings has been restricted to applications where no suitable rolled section can be found. Application of plate girder fabrication expertise to beams for long spans in buildings provides the opportunity to shape the profile to suit both the shape of the bending moments and the integration of services. Figure 3 shows a number of possible profiles resulting in structural efficiency and, more importantly, creation of a service zone within the structural depth of the floor. Tapered beams spanning 15 to 20m across a building may be placed at centres to suit the span of the composite decking or be combined with secondary beams thus permitting the use of fewer fabricated beams placed at increased centres. Span to depth ratios range between 15 to 25.



Fabricated beam with straight taper



Fabricated beam with semi-taper



Fabricated beam with cranked taper

Figure 3 Tapered beams

Tapered beams allow for some flexibilty in positioning services but large ducts can only readily be accommodated near supports. Holes may be cut in the beam web but doing so is likely to require costly stiffening as the web is generally quite slender. For economic fabrication, tapered beams require specialist automatic or semi-automatic equipment as used by fabricators in plate girder construction.

The design of composite tapered sections requires particular attention to the identification of the critical sections for bending and combined shear and bending. The non-prismatic shape means that in many cases the critical section for bending is not at the centre. Superposition of the design bending moment diagram and the tapered profile of the beam should reveal where the critical sections probably occur. Care should be taken when considering the distribution of shear connectors placing more near the ends of the beams.

2.4 Composite trusses

Composite trusses were developed in North America and are popular for spans of 10 to 20m. The Warren truss arrangement occupies the full ceiling space allowing services to be threaded through the openings formed between the web members. Large ducts may require the use of a Vierendeel panel at some point, usually the mid-span. The open nature of the truss at first sight appears ideal for integration of services but the size of the available openings quickly reduces when fire protection is added to the web members. Insulation around many services significantly increases the opening size required to accommodate ducts which can lead to problems if either the structural designer and services engineer overlook these considerations. A typical composite truss is shown in figure 4.

Figure 4 Composite truss

A composite truss must be capable of supporting the construction loading in an unpropped condition and this will determine the size of the top chord. The metal decking may be assumed to provide lateral restraint during construction. Member forces are normally found by elastic analysis of a pin-jointed truss with the effective depth based on the distance between the centroids of the top and bottom chords. Once the concrete has hardened, the slab becomes an effective compression chord and analysis of the truss under live superimposed dead and live loads should take account of the composite properties of the truss. Design procedures are recommended by Chien and Ritchie (1984,1992).

2.5 Beams with web openings

In order to accommodate large service ducts within the structural depth, thus reducing the ceiling to floor zone, large holes are sometimes required through the webs of beams. Long span beams of uniform depth generally have reserve capacity along much of their length permitting the formation of holes through the web. If these are not too large (say, not greater than 0.6D* nor longer than 1.5D), holes without horizontal stiffening may be made (as the web contribution to the moment capacity of the composite cross-section is small), provided of course that the remaining web has adequate shear resistance. If larger holes are necessary, stiffeners in the form of horizontal plates welded above and below the hole should be provided. Although quite large holes can be provided, as shown in figure 5, the problem with this approach is inflexibility. Even

* D is the depth of the steel beam

minor changes during the design or construction stage can be very difficult to accommodate and future changes of the building services layout may be impossible.

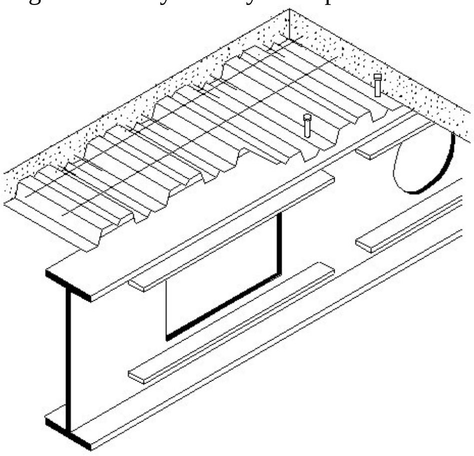


Figure 5 – beam with large opening in web for services.

An alternative to providing customised holes at required locations in a rolled beam is to use a castellated or cellular beam. Cutting a castellated line or a series of semi-circles through the web of a standard beam section and then reassembling the two halves of the beam by welding creates a deeper beam with a series of hexagonal or circular holes, as shown in figure 6. The resulting sections are structurally efficient, due to their greater depth, and have numerous holes through which services may be threaded. Although the size of the holes may prove a limitation, the use of these beams assures good flexibility for future upgrades in servicing.

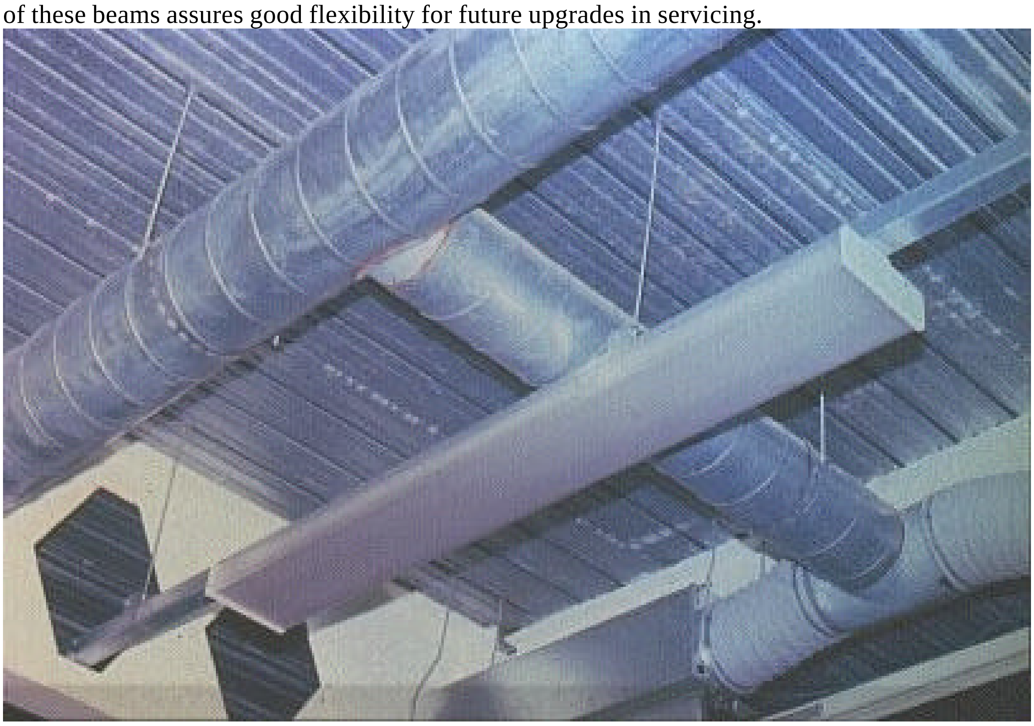


Figure 6 Cellular and castellated beams

2.6 Parallel beams

Brett et al (1987) describe a novel approach to composite floor construction in which the secondary beams are supported on the top flange of a pair of primary beams located either side of a column rather than on the column centreline. Figure 7 illustrates the arrangement. The principal attractions of this framing arrangement are the use of continuity, which reduces the weight of the beams, and a reduction in the number of members in a frame and the complexity of the connections, which reduces erection and fabrication costs. Although the combined structural depth can be quite large, services can run in parallel with the secondary and primary beams at two levels resulting in efficient service integration.

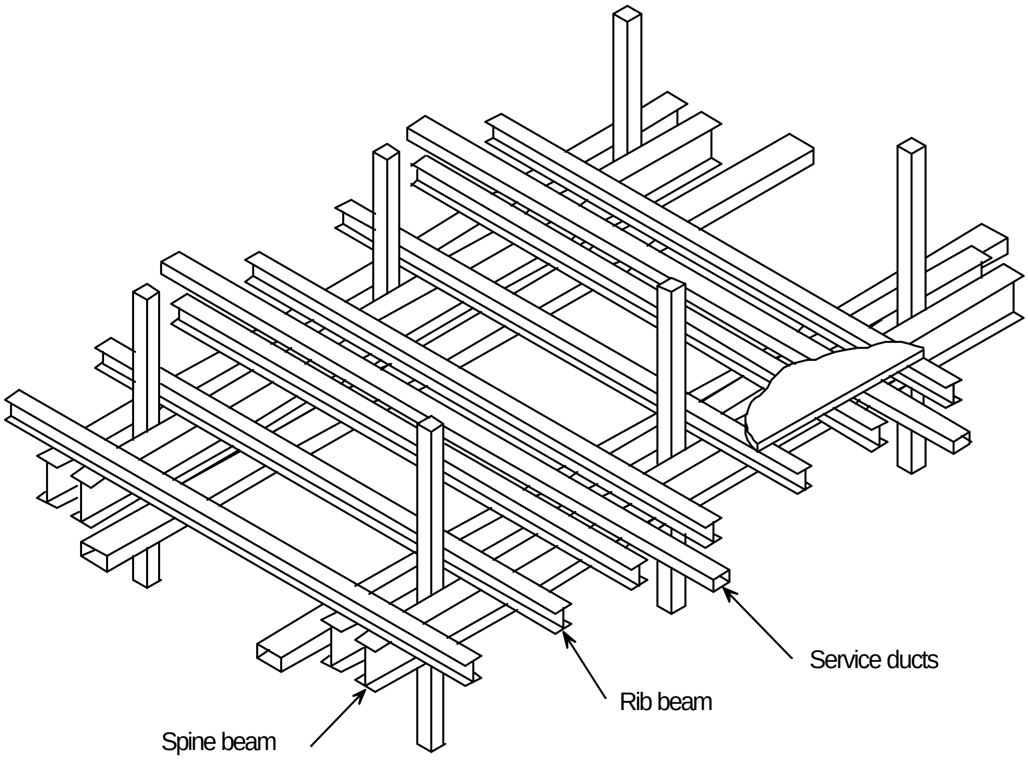


Figure 7 Parallel beam system

3. Shallow floor systems

Traditional multi-storey construction is efficient but the structural depth can prevent its use in countries with very restrictive controls on building height. To address this problem, a number of systems have been developed in Scandinavia and the UK in which the steel beams are integrated in the slabs. These systems are illustrated in Figure 8. In each case precast concrete slabs are supported on the bottom flange of a, usually fabricated, section. In addition to the advantage gained by a shallow floor depth and the usual attractions of steel construction, these systems have a flat soffit for ease of servicing and excellent inherent fire resistance as the majority of the steel cross-section is encased in concrete. These systems do not however incorporate composite action.

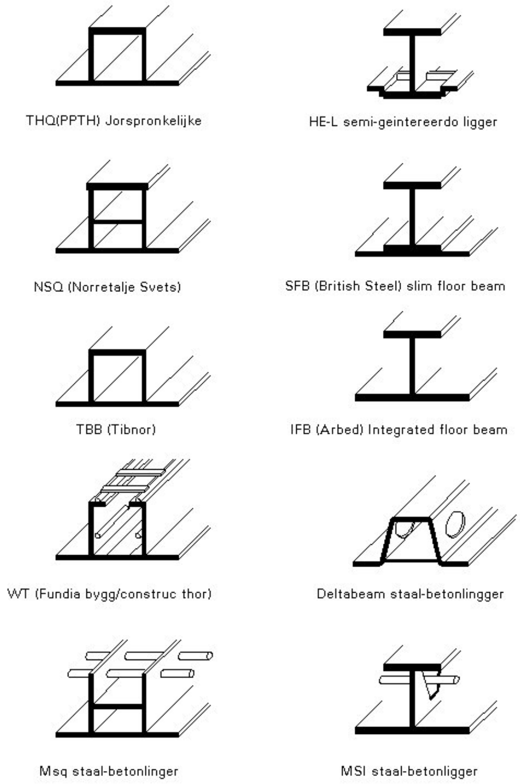


Figure 8 Slim floor construction

A universal column section with a plate welded beneath provides a good balance between fabrication and ease of erection. This section may be used compositely with precast units provided sufficient depth of concrete is placed over the precast units to cover shear connectors.

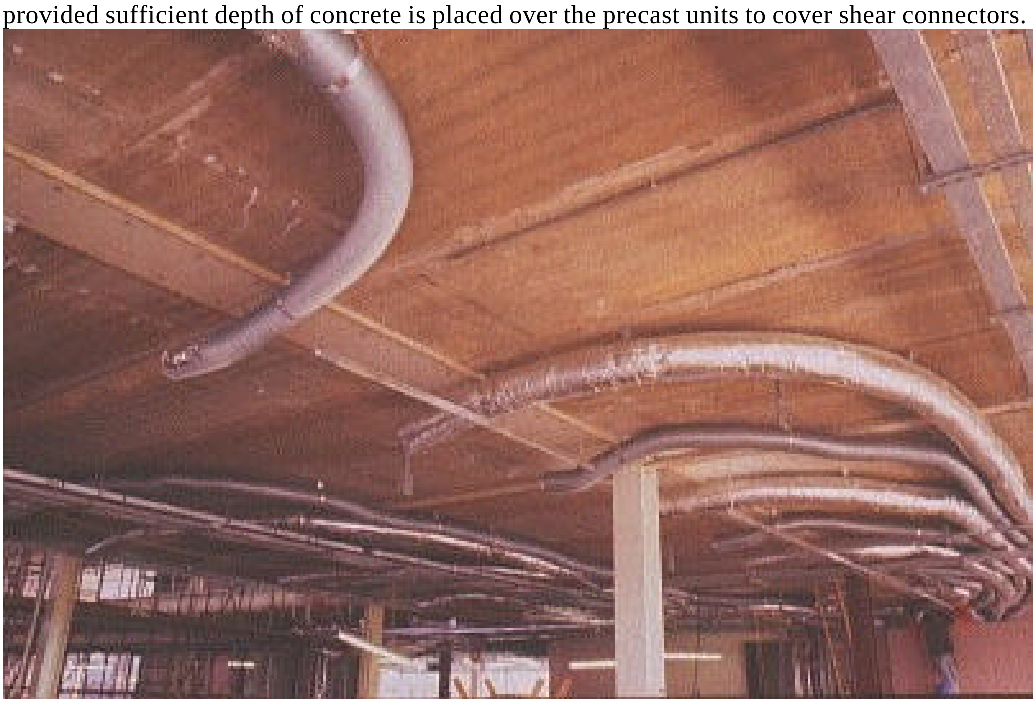


Figure 9 Slim floor beam composite with pc units

A recent development in the design and construction of shallow floor systems has been the replacement of precast floor units, which are heavy and difficult to manoeuvre on site, with composite floor slabs cast on deep metal decking. The metal decks are about 200mm deep and can support a total slab depth of about 300mm. Unsupported decking in construction spans around 6m, increasing to 8m if propped. Design recommendations for long span composite slabs with deep profile steel sheets were presented by Brekelmans et al (1997) as part of an ECSC research project on steel intensive shallow floor construction. Figure 10 shows details of a shallow floor arrangement with deep decking.

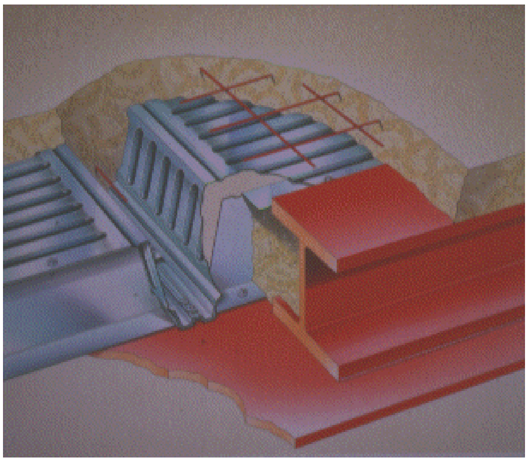


Figure 10 Deep deck shallow floor

Use of a composite metal deep deck allows composite action between the slab and supporting beam to be developed. Shear connection may be achieved by studs connectors, reinforcement bars passed through holes in the beam web or by some form of embossed pattern on the beam. Recently Corus introduced a series of rolled asymmetric beams, which avoids the need to weld a plate onto a column section thereby reducing fabrication costs, with a pattern rolled onto the top surface to develop composite action without shear connectors. Details of tests to verify the system and formulate design guidance in accordance with Eurocodes 3 and 4 are provided by Lawson et al (1998). The rolled beam has the added advantage of a thicker web than a column section to enhance fire resistance. Figure 11 shows the system (registered as Slimdek by Corus)– notice that services can be run routed through the voids created beneath the deep deck and passed through holes cut in the beam web.

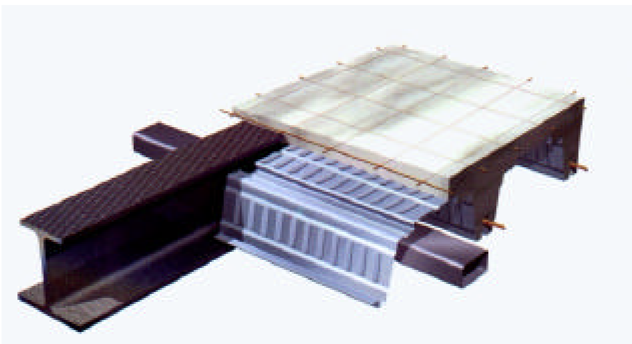


Figure 11 Asymmetric rolled beam with deep deck composite slab

4. Conclusions

The use of composite construction in buildings may often be based on an orthogonal grid of primary and secondary beams with an overlaying composite deck. This arrangement gives excellent structural efficiency but may result in a structural depth which is too large. In such cases shallow floor systems, which combine the floor and slab in the same vertical space, offer a competitive alternative to concrete flat slab construction.

Where large column free spaces are required in multi-storey buildings, large span simple beams can become prohibitively deep. Various alternatives have been described, many of which provide the opportunity to integrate mechanical services within the the structural zone thereby reducing overall construction depth. In seeking an ecomonic design, consideration should be given to overall project costs and flexibility for future changes in building use and services.

Structural Steelwork Eurocodes

Development of
a Trans-National Approach

Course: Eurocode 4

Lecture 11a: Introduction to Structural Fire Engineering

Summary:

- Both steel and concrete suffer a progressive reduction in both strength and stiffness as their temperature increases in fire conditions. EC3 and EC4 provide material models of stress-strain curves for both materials over an extensive range of temperatures.
- Fire resistance of structural elements is quoted as the time at which they reach a defined deflection criterion when tested in a furnace heated according to a standard ISO834 time-temperature curve.
- It is possible to assess the severity of a natural fire as a time-equivalent between its peak temperature and the same temperature on the ISO834 standard curve.
- The behaviour of elements in furnace tests is very different from that in a building frame, and the only practical way of assessing whole-structure behaviour is to use advanced calculation models.
- EC3 or EC4 calculation of fire resistance takes account of the loading level on the element. However the safety factors applied are lower than in those used in strength design.
- Critical temperature is calculated for all types of member of classes 1, 2 or 3 from a single equation in terms of the load level in fire. Class 4 sections are universally assumed to have a critical temperature of 350°C.
- It is possible to calculate the temperature growth of protected or unprotected members in small time increments, in a way which can easily be implemented on a spreadsheet.

Pre-requisites: an appreciation of

- Simple element design for strength and serviceability according to EC3 and EC4.
- Framing systems currently used in steel-framed construction, including composite systems.

Notes for Tutors:

- This material is a general introduction to structural fire engineering, and may be used as the initial stage of a course which leads into EC4 design of composite structures for fire conditions.
- The lecturer can break up the session with formative exercises at appropriate stages (calculation of member capacities in fire, and fire resistance times are suggested).

Objectives:

After completing the module the student should:

- Understand that both steel and concrete progressively lose strength and stiffness at elevated temperatures.
- Understand that fire resistance is quoted in relation to furnace testing using a standard time-temperature curve in which the temperature never reduces, and does not refer to actual survival in a real fire.
- Know that EC1 specifies three such standard fire curves, of which two refer only to hydrocarbon and external fires, but also provides a method of modelling parametric natural fires if sufficient detail of fire loads, ventilation etc are known.
- Understand the concept of time-equivalence in rating the severity of a natural fire in terms of the standard fire curve.
- Know about traditional methods of passive fire protection of steel members.
- Understand that other strategies may be used in fire engineering design to provide the required fire resistance, including overdesign, selection of framing systems, and use of sprinklers.
- Understand the principles of the simple design calculations of resistance in fire conditions of beams and columns, and the concept of critical temperature.
- Understand the methods of calculating the thermal response of protected and unprotected members to increase of atmosphere temperature in a fire.

References:

- ENV 1991-1: *Eurocode 1: Basis of Design and Actions on Structures. Part 1: Basis of Design.*
- prEN 1991-1-2: *Eurocode 1: Basis of Design and Actions on Structures. Part 1.2: Actions on Structures Exposed to Fire.*
- ENV 1992-1-1: *Eurocode 2: Design of Concrete Structures. Part 1.1: General Rules: General Rules and Rules for Buildings.*
- ENV 1992-1-2: *Eurocode 2: Design of Concrete Structures. Part 1.2: General Rules: Structural Fire Design.*
- prEN 1993-1-1: *Eurocode 3: Design of Steel Structures. Part 1.1: General Rules: General Rules and Rules for Buildings.*
- prEN 1993-1-2: *Eurocode 3: Design of Steel Structures. Part 1.2: General Rules: Structural Fire Design.*
- prEN 1994-1-1: *Eurocode 4: Design of Composite Steel and Concrete Structures. Part 1.1: General Rules: General Rules and Rules for Buildings.*
- ENV 1994-1-2: *Eurocode 4: Design of Composite Steel and Concrete Structures. Part 1.2: General Rules: Structural Fire Design.*
- EN yyy5: *Method of Test for the Determination of the Contribution to Fire Resistance of Structural Members.*

Contents:

1 Introduction

Any structure must be designed and constructed so that, in the case of fire, it satisfies the following requirements:

EC4 Part 1.2

- The load-bearing function of the structure must be maintained during the required time,
- The development and spread of fire and smoke within the building is restricted,
- The spread of the fire to adjacent buildings is restricted,
- People within the building must be able to leave the area safely or to be protected by other means such as refuge areas,
- The safety of fire fighters is assured.

2 Temperatures in fires

A real fire in a building grows and decays in accordance with the mass and energy balance within the compartment in which it occurs (Fig. 1). The energy released depends upon the quantity and type of fuel available, and upon the prevailing ventilation conditions.

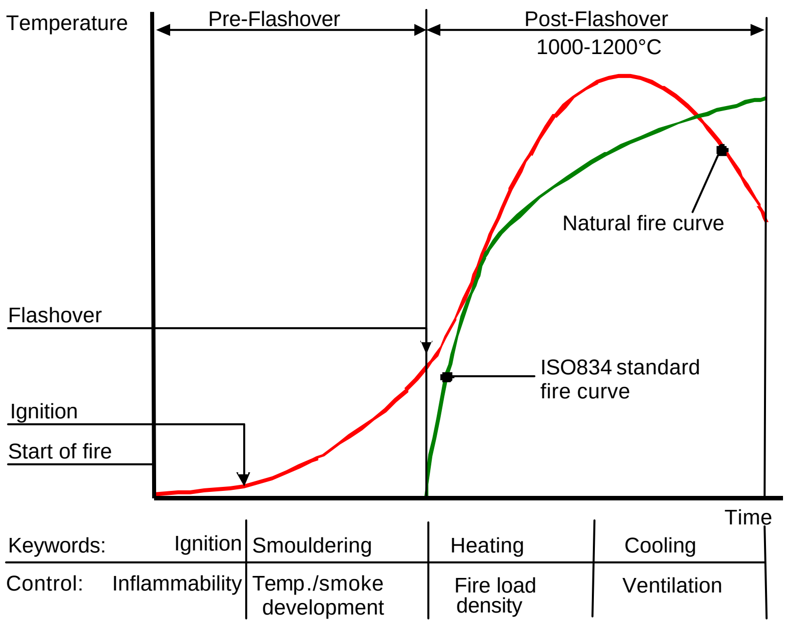


Figure 1 Phases of a natural fire, comparing atmosphere temperatures with the ISO834 standard fire curve

It is possible to consider a real fire as consisting of three phases, which may be defined as growth, full development and decay. The most rapid temperature rise occurs in the period following flashover, which is the point at which all organic materials in the compartment spontaneously combust.

EC1 Part 1.2 4.2.2

Fire resistance times specified in most national building regulations relate to test performance when heated according to an internationally agreed atmosphere time-temperature curve defined in ISO834 (or Eurocode 1 Part 1.2), which does not represent any type of natural building fire. It is characterised by an atmosphere temperature which rises continuously with time, but at a diminishing rate (Fig. 2). This has become the standard design curve which is used in furnace testing of components. The quoted value of fire resistance time does not therefore indicate the actual time for which a component will survive in a building fire, but is a like-against-like comparison indicating the severity of a fire which the component will survive.

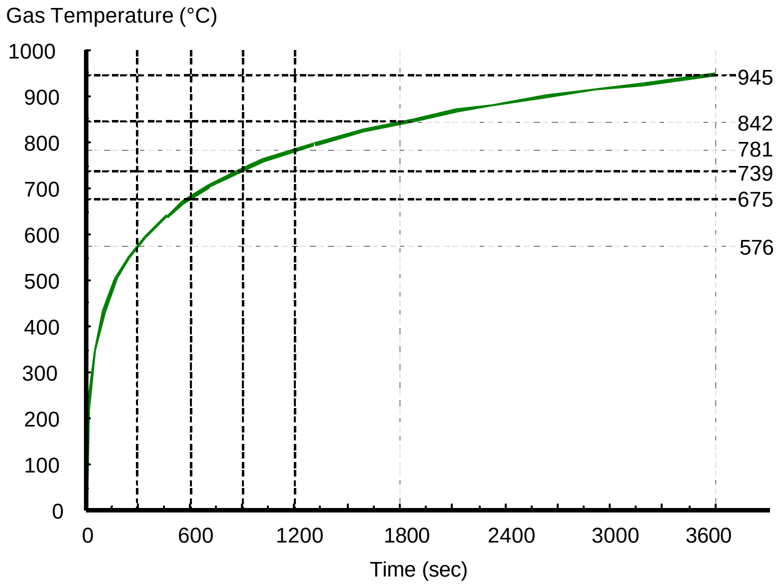


Figure 2 Atmosphere temperature for ISO834 standard fire

Where the structure for which the fire resistance is being considered is external, and the atmosphere temperatures are therefore likely to be lower at any given time (which means that the temperatures of the building materials will be closer to the corresponding fire temperatures), a similar "External Fire" curve may be used. *4.2.3*

In cases where storage of hydrocarbon materials makes fires extremely severe a "Hydrocarbon Fire" curve is also given. These three "Nominal" fire curves are shown in Fig. 3. *4.2.4*

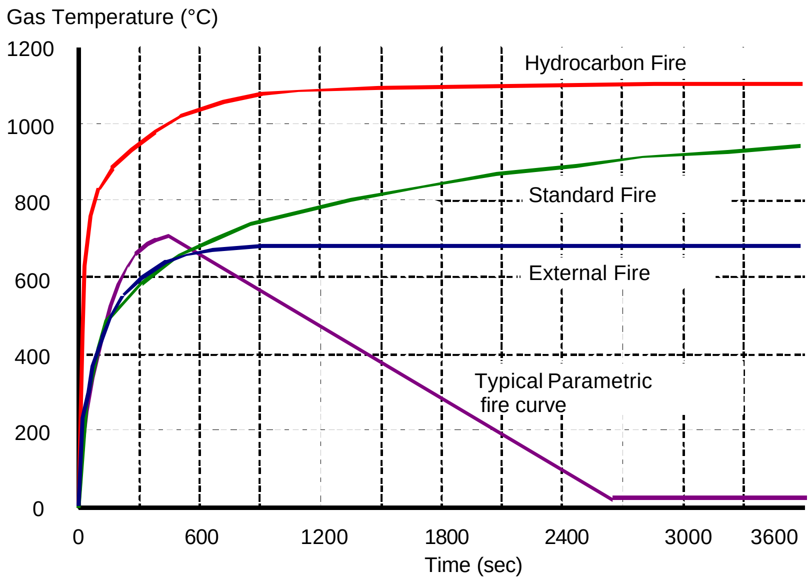


Figure 3 EC1 Part 1.2 nominal fire curves compared with a parametric fire

Any of the normal means of establishing fire resistance times (prescriptive rules, tabulated data or calculation models) may be used against these curves.

An alternative method to the use of fire resistance times related to nominal fire curves, which may only be used directly with fire resistance calculation models, is to attempt to model a natural fire using a "parametric" fire curve for which equations are provided in EC1 Part 1.2. This enables fairly simple modelling of fire temperatures in the heating and cooling phases of the post-flashover fire (the initial growth phase is not addressed), and the time at which the maximum temperature is attained. It is necessary to have data on the properties (density, specific heat, thermal conductivity) of the materials enclosing a compartment, the fire load (fuel) density and ventilation areas when using these equations. They are limited in application to compartments of up to $100m^2$ with mainly cellulosic (paper, wood etc ...) fire loads. *EC1 Part 1.2 Annex A*

It may be advantageous to the designer to use parametric curves in cases where the density of combustible materials is low, where using the nominal fire curves is unnecessarily conservative.

In using a parametric curve the concept known as 'equivalent time' can be used both to compare the severity of the fire in consistent terms and to relate the resistance times of structural elements in a real fire to their resistance in the standard fire. The principle is shown in Fig. 4. *EC1 Part 1.2 Annex D*

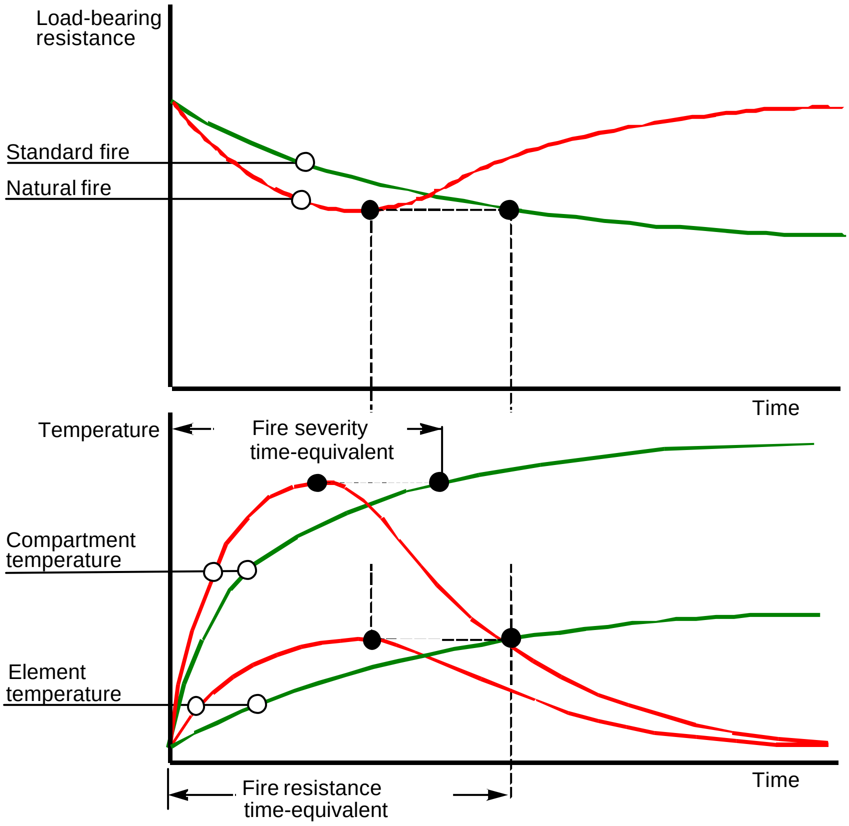


Figure 4 Time-equivalent severity of natural fires

This is useful in applying calculation models which are based on the standard fire heating curve, but the important aspect of using parametric fire curves and the calculated structure temperatures which come from these is that they represent an *absolute* test of structural fire resistance. The comparison is between the maximum temperature reached by the structure and its critical temperature, rather than an assessment of the way it would perform if it were possible to subject it to a standard fire time-temperature curve based on furnace testing.

3 Behaviour of beams and columns in furnace tests

Furnace testing using the standard time-temperature atmosphere curve is the traditional means of assessing the behaviour of frame elements in fire, but the difficulties of conducting furnace tests of representative full-scale structural members under load are obvious. The size of furnaces limits the size of the members tested, usually to less than 5m, and if a range of load levels is required then a separate specimen is required for each of these. Tests on small members may be unrepresentative of the behaviour of larger members. *EN yyy5*

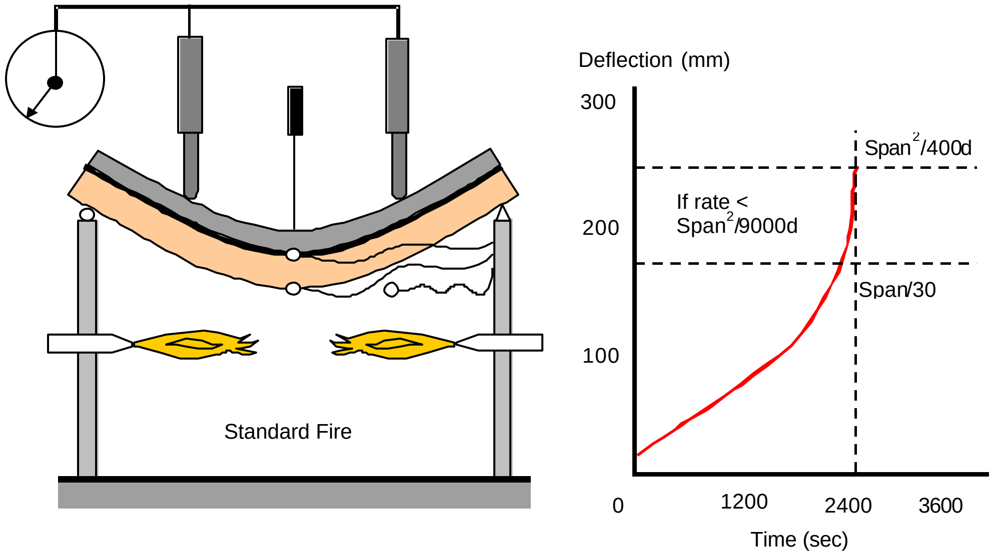


Figure 5 Typical beam fire test

A further serious problem with the use of furnace tests in relation to the behaviour of similar elements in structural frames is that the only reliable support condition for a member in a furnace test is simply supported, with the member free to expand axially. When a member forms part of a fire compartment surrounded by adjacent structure which is unaffected by the fire its thermal expansion is resisted by restraint from this surrounding structure.

This is a problem which is unique to the fire state, because at ambient temperatures structural deflections are so small that axial restraint is very rarely an issue of significance. Axial restraint can in fact work in different ways at different stages of a fire; in the early stages the restrained thermal expansion dominates, and very high compressive stresses are generated. However, in the later stages when the weakening of the material is very high the restraint may begin to support the member by resisting pull-in. Furnace tests which allow axial movement cannot reproduce these restraint conditions at all; in particular, in the later stages a complete collapse would be observed unless a safety cut-off criterion is applied. In fact a beam furnace test is always terminated at a deflection of not more than *span/20* for exactly this reason.

Only recently has any significant number of fire tests been performed on fire compartments within whole structures. Some years may pass before these full-scale tests are seen to have a real impact on design codes. In fact full-scale testing is so expensive that there will probably never be a large volume of documented results from such tests, and those that exist will have the major function of being used to validate numerical models on which future developments of design rules will be based. At present, Eurocodes 3 and 4 allow for the use of advanced calculation models, but their basic design procedures for use in routine fire engineering design are still in terms of isolated members and fire resistance is considered mainly in terms of a real or simulated furnace test.

4 Fire protection methods

In general for composite steel and concrete structures can be used the same fire protection methods as for steel structures.

This may be in alternative forms:

- Boarding (plasterboard or more specialised systems based on mineral fibre or vermiculite) fixed around the exposed parts of the steel members. This is fairly easy to apply and creates an external profile which is aesthetically acceptable, but is inflexible in use around complex details such as connections. Ceramic fibre blanket may be used as a more flexible insulating barrier in some cases.

- Sprays which build up a coating of prescribed thickness around the members. These tend to use vermiculite or mineral fibre in a cement or gypsum binder. Application on site is fairly rapid, and does not suffer the problems of rigid boarding around complex structural details. It can, however, be extremely messy, and the clean-up costs may be high. Since the finish produced tends to be unacceptable in public areas of buildings these systems tend to be used in areas which are normally hidden from view, such as beams and connections above suspended ceilings. They are sometimes susceptible to cracking and shrinkage.
- Intumescent paints, which provide a decorative finish under normal conditions, but which foam and swell when heated, producing an insulating char layer which is up to 50 times as thick as the original paint film. They are applied by brush, spray or roller, and must achieve a specified thickness which may require several coats of paint and measurement of the film thickness.

All of these methods are normally applied as a site operation after the main structural elements are erected. This can introduce a significant delay into the construction process, which increases the cost of construction to the client. The only exception to this is that some systems have recently been developed in which intumescents are applied to steelwork at the fabrication stage, so that much of the site-work is avoided. However, in such systems there is clearly a need for a much higher degree than usual of resistance to impact or abrasion.

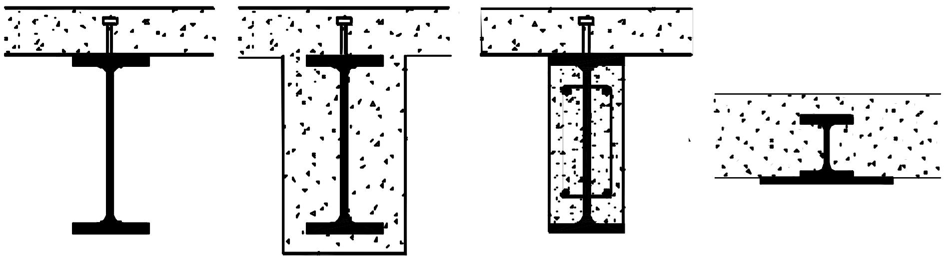
These methods can provide any required degree of protection against fire heating of steelwork, and can be used as part of a fire engineering approach. However traditionally thicknesses of the protection layers have been based on manufacturers' data aimed at the relatively simplistic criterion of limiting the steel temperature to less than 550°C at the required time of fire resistance in the ISO834 standard fire. Fire protection materials are routinely tested for insulation, integrity and load-carrying capacity in ISO834 furnace test. Material properties for design are determined from the results by semi-empirical means.

Open steel sections fully or partially encased in concrete, and hollow steel sections filled with concrete, generally do not need additional fire protection. In fire the concrete acts to some extent as a heat-sink as well as an insulator, which slows the heating process in the steel section.

The most recent design codes are explicit about the fact that the structural fire resistance of a member is dependent to a large extent on its loading level in fire, and also that loading in the fire situation has a very high probability of being considerably less than the factored loads for which strength design is performed. This presents designers with another option which may be used alone or in combination with other measures. A reduction in load level by selecting composite steel and concrete members which are stronger individually than are needed for ambient temperature strength, possibly as part of a strategy of standardising sections, can enhance the fire resistance times, particularly for beams. This can allow unprotected or partially protected beams to be used.

The effect of loading level reduction is particularly useful when combined with a reduction in exposed perimeter by making use of the heat-sink effects of the supported concrete slab and concrete full or partial encasement. The traditional downstand beam (Fig. 6a)) gains some advantage over complete exposure by having its top flange upper face totally shielded by the slab; beams with concrete encasement (Fig. 6b)) provide high fire resistance (up to 180´), but their big disadvantage are complicated constructions of joints and the need of sheeting. Better solution is to use steel beams with partial concrete encasement (Fig. 6c)). Concrete between flanges reduces the speed of heating of the profile's web and upper flange, contributes to the load bearing resistance, when lower part of the steel beam loses its strength very quickly during the fire. The big advantage is that the partial encasement of the beam can be realised in the shop without the use of sheeting, the beam is concreted in stages in the side way position. The construction of joints is always very simple, typical joint types in common use in steel structures can be adopted also here.

The recent innovation of "Slimflor" beams (Fig. 6d)), in which an unusually shallow beam section is used and the slab is supported on the lower flange, either by pre-welding a plate across this flange or by using an asymmetric steel section, leaves only the lower face of the bottom flange exposed.



(a) (b) (c) (d)

Figure 6 Inherent fire protection to steel beams

Alternative fire engineering strategies are beyond the scope of this lecture, but there is an active encouragement to designers in the Eurocodes to use agreed and validated advanced calculation models to analyse the behaviour of the whole structure or sub-assemblies. The clear implication of this is that designs which can be shown to gain fire resistance overall by providing alternative load paths when members in a fire compartment have individually lost all effective load resistance are perfectly valid under the provisions of these codes. This is a major departure from the traditional approach based on the fire resistance in standard tests of each component. The preambles to Parts 1-2 of both Eurocodes 3 and 4 also encourage the use of integrated fire strategies, including the use of combinations of active (sprinklers) and passive protection. However it is acknowledged that allowances for sprinkler systems in fire resistant design are at present a matter for national Building Regulations.

5 Basic structural fire resistant design of members

Details of Eurocode structural fire resistance calculations are given in the appropriate articles on EC3 and EC4, together with an example design calculation using the simple calculation models, and so this section concentrates on the principles of these methods rather than their detail.

5.1 Notation

Eurocodes use a very systematic notation in which different symbols are used for general and particular versions of parameters. For example an “effect of an action” is denoted in general terms as E in establishing a principle; in particular members this might become the axial force N or the internal bending moment M. Subscripts denoting different attributes of a parameter may be grouped, using dots as separators, as in $E_{fi.d.t}$ which denotes the ***d**esign* value of the ***e**ffect* of an action in ***fi**re*, at the required ***t**ime* of resistance. Commonly used notations in the fire engineering parts of Eurocodes 1, 3 and 4 are:

EC4 Part 1.2
1.4

E = *effect of actions*
G = *permanent action*
Q = *variable action*
R_{fi} = *load-bearing resistance*
t_{fi} = *standard fire resistance time of a member*
$t_{fi.requ}$ = *standard fire resistance time nominal requirement*
q = *temperature*
q_{cr} = *critical temperature of a member*
g = *partial safety factor*
y = *load combination factor*

and the following subscript indices may be used alone or in combination:

A = *accidental design situation*
cr = *critical value*
fi = *relevant to fire design*

d = *design value*
q = *associated with certain temperature (may be replaced by value)*
k = *characteristic value*
t = *at certain fire exposure time (may be replaced by value)*
1, 2 .. = *ranking order for frequency of variable actions*

5.2 Loadings

Eurocode 1 Part 1.2 presents rules for calculating design actions (loadings) in fire, which recognise the low probability of a major fire occurring simultaneously with high load intensities. The normal Eurocode classification of loads is as permanent and variable; in fire the characteristic permanent actions (dead loading) are used unfactored (g_{GA}=1,0) while the principal characteristic variable action (imposed loading) is factored down by a combination factor $y_{1.1}$ whose value is between 0,5 and 0,9 depending on the building usage. The “reduction factor” or “load level for fire design” can be defined either as

$$h_{fi,t} = \frac{E_{fi.d.t}}{R_d}$$ (loading in fire as a proportion of ambient-temperature design resistance),

which is relevant when global structural analysis is used, or

EC4 Part 1.2
Fig. 2.1

$$h_{fi,t} = \frac{E_{fi.d.t}}{E_d}$$ (loading in fire as a proportion of ambient-temperature factored design load),

which is the more conservative, and is used in simplified design of individual members, when only the principal variable action is used together with the permanent action. This may be expressed in terms of the characteristic loads and their factors as

$$h_{fi} = \frac{g_{GA}G_k + y_{1.1}Q_{k.1}}{g_G G_k + g_{Q.1}Q_{k.1}} \qquad (1)$$

Typical values of the safety factors specified in Eurocode 1 are:

g_{GA}	= 1,0	(Permanent loads: accidental design situations)
$y_{1.1}$	= 0,5	(Combination factor: variable loads, office buildings)
g_G	= 1,35	(Permanent loads: strength design)
$g_{Q.1}$	= 1,5	(Variable loads: strength design)

5.3 Basic principles of fire resistant design

Structural fire-resistant design of a member is concerned with establishing that it satisfies the requirements of national building regulations over the designated time period when subjected to the appropriate nominal fire curve. This can be expressed in three alternative ways:

EC1 Part 1.2
Section 2

- The *fire resistance time* should exceed the requirement for the building usage and type when loaded to the design load level and subjected to a nominal fire temperature curve:

  $$t_{fi,d} \geq t_{fi,requ}$$

- The *load-bearing resistance* of the element should exceed the design loading when it has been heated for the required time in the nominal fire:

  $$R_{fi,d,t} \geq E_{fi,d,t}$$

- The *critical temperature* of an element loaded to the design level should exceed the design temperature associated with the required exposure to the nominal fire:

  $$q_{cr,d} \geq q_d$$

6 Material properties

6.1 Mechanical properties

6.1.1 Steel strengths

EC3 Part 1.2 3.2 Table 3.1 Fig. 3.1 EC4 Part 1.2 3.2.1; Annex A

Most construction materials suffer a progressive loss of strength and stiffness as their temperature increases. For steel the change can be seen in EC3/4 stress-strain curves (Fig. 7) at temperatures as low as 300°C. Although melting does not happen until about 1500°C, only 23% of the ambient-temperature strength remains at 700°C. At 800°C this has reduced to 11% and at 900°C to 6%.

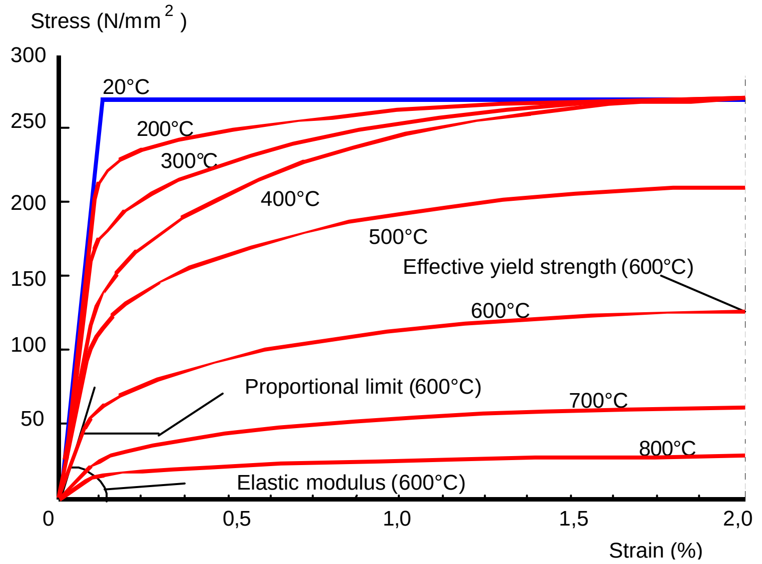


Figure 7 Reduction of stress-strain properties with temperature for S275 steel (EC4 curves)

EC3 Part 1.2 Fig. 3.2 EC4 Part 1.2

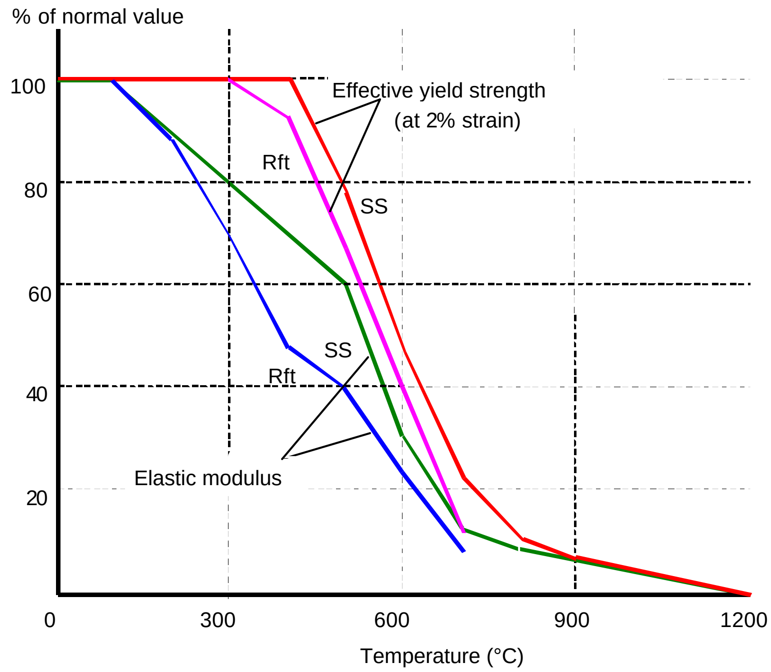


Figure 8 EC3 Strength reduction for structural steel (SS) and cold-worked reinforcement (Rft) at high temperatures

These are based on an extensive series of tests, which have been modelled by equations representing an initial linear elastic portion, changing tangentially to a part-ellipse whose gradient is zero at 2% strain. When curves such as these are presented in normalised fashion, with stresses shown as a proportion of ambient-temperature yield strength, the curves at the same temperatures for S235, S275 and S355 steels are extremely close to one another. It is therefore possible to use a single set of strength reduction factors (Fig. 8) for all three grades, at given temperatures and strain levels. In Eurocodes 3 and 4 strengths corresponding to 2% strain are used in the fire engineering design of all types of structural members.

Hot-rolled reinforcing bars are treated in Eurocode 4 in similar fashion to structural steels, but cold-worked reinforcing steel, whose standard grade is S500, deteriorates more rapidly at elevated temperatures than do the standard grades. Its strength reduction factors for effective yield and elastic modulus are shown on Fig. 8. It is unlikely that reinforcing bars or mesh will reach very high temperatures in a fire, given the insulation provided by the concrete if normal cover specifications are maintained. The very low ductility of S500 steel (it is only guaranteed at 5%) may be of more significance where high strains of mesh in slabs are caused by the progressive weakening of supporting steel sections.

6.1.2 Concrete strengths

Concrete also loses strength properties as its temperature increases (Fig. 9), although a variety of parameters contribute to the relevant characteristics of any given concrete element in the structure.

The stress-strain curves at different temperatures for concrete have a significant difference in form from those for steel. The curves all have a maximum compressive strength, rather than an effective yield strength, which occurs at strains which progressively increase with temperature, followed by a descending branch. Tensile strength for all concretes is normally considered to be zero. As is normal in Eurocodes, alternative material constitutive laws may be used provided that they are supported by experimental evidence. *EC4 Part 1.2 Fig. B.1*

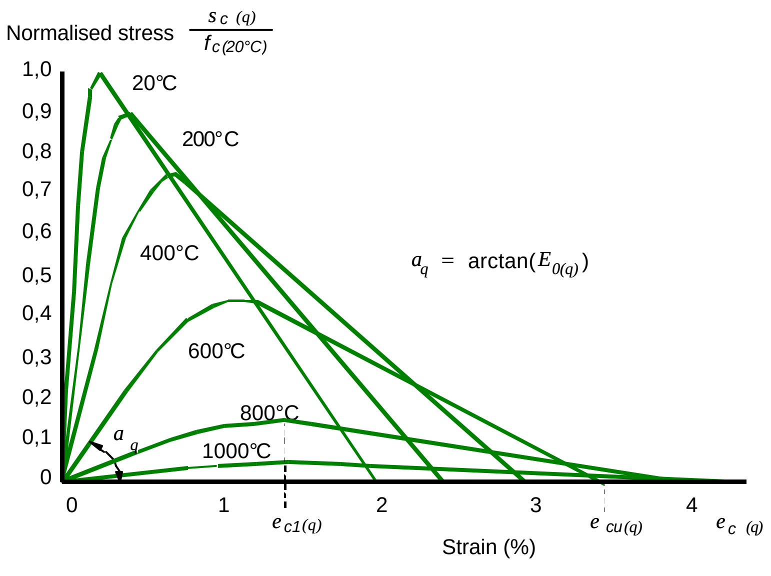


Figure 9 EC4 stress-strain- temperature curves for normal-weight and lightweight concrete

EC4 Part 1.2 3.2.2 Annex B

For normal-weight concretes (density around 2400 kg/m^3) only the lower range of strength values, corresponding to the siliceous type which is shown in Fig. 10, are tabulated in Eurocode 4 Part 1.2. For calcareous-aggregate concrete these are also used, being inherently conservative values. Where more detail is required designers are referred to Eurocode 2 Part 1.2.

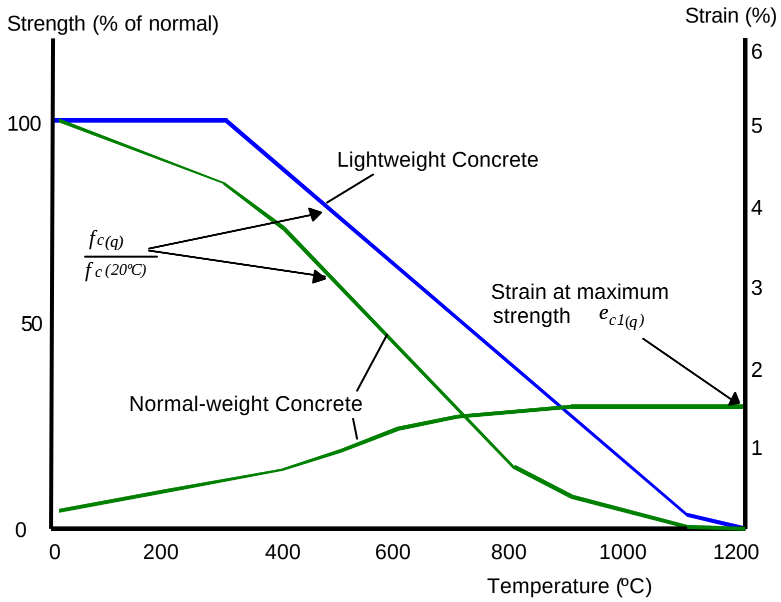
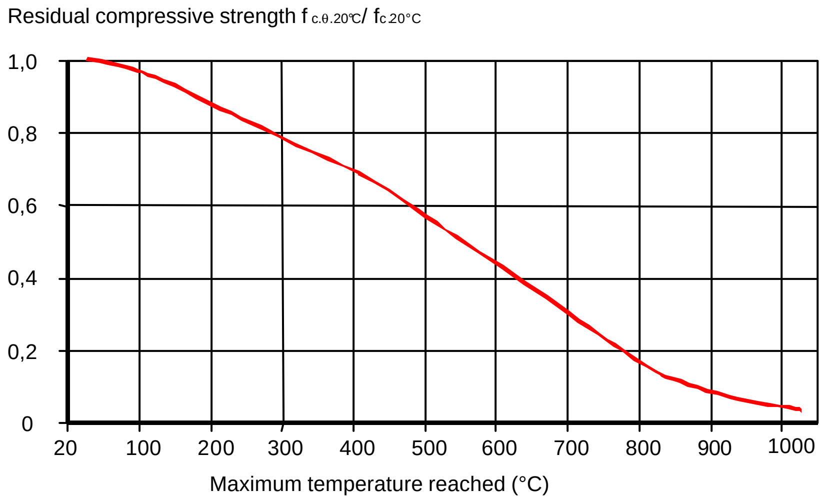


Figure 10 EC4 Strength reduction for normal-weight siliceous concrete and lightweight concrete at elevated temperatures

Lightweight concretes are defined as those within the density range 1600-2000 kg/m^3. Although in practice they may be created using different forms of aggregate, they are treated in EC4 Part 1.2 as if they degrade similarly with temperature. Hence the single set of strength reduction factors (Fig. 10) for lightweight concrete is again necessarily on the conservative side.

It is important to notice that concrete, after cooling down to ambient temperature does not regain its initial compressive strength. Its residual strength $f_{c,q,20°C}$ depends on the maximum temperature which was reached during the heating phase (Fig. 11).



Annex C

Figure 11 Proportional loss of residual compressive strength $f_{c,q,20°C}$ after heating to different maximum temperatures

During the cooling phase it is possible to define the corresponding compressive cylinder strength for a certain temperature q ($q_{max} > q > 20$ºC) by linear interpolation between $f_{c.q.max}$ and $f_{c.q.20°C}$ in the way illustrated in Fig. 12.

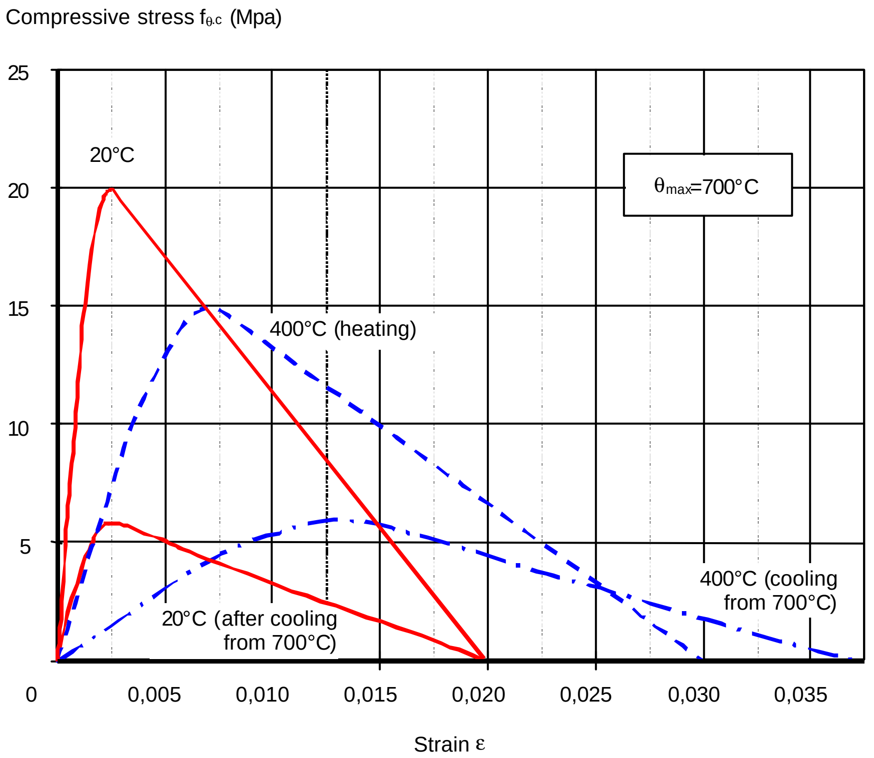


Figure 12 Stress-strain relationships of concrete C20/25 at 400ºC during the heating and cooling phases, after reaching maximum temperature 700ºC

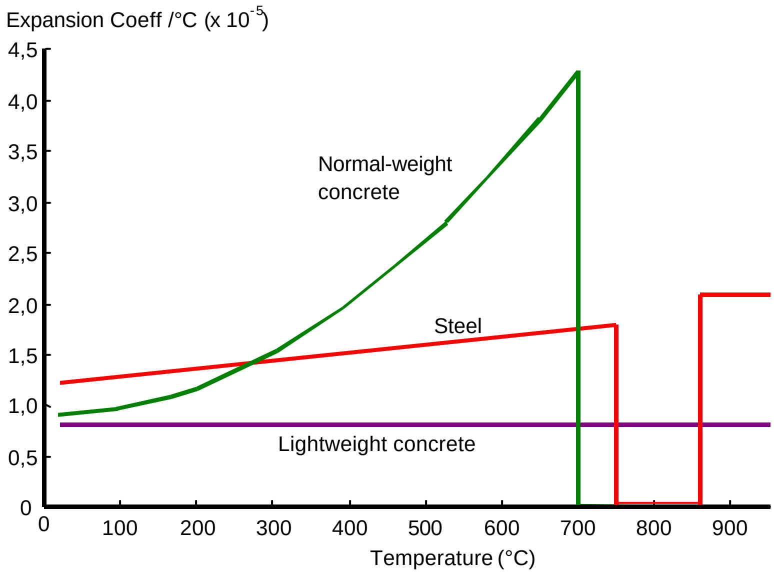
Concrete has a lower thermal conductivity than steel and is therefore a relatively good insulator to reinforcement or embedded parts of sections. Fire resistance of reinforced concrete members tends to be based on the strength reduction of reinforcement, which is largely controlled by the cover specification. However, concrete is affected by spalling, which is a progressive breaking away of concrete from the fire-exposed surface where temperature variation is high, and this can lead to the exposure of reinforcement as a fire progresses. Its behaviour at elevated temperatures depends largely on the aggregate type used; siliceous (gravel, granite) aggregates tends to cause concrete to spall more readily than calcareous (limestone) aggregates. Lightweight concrete possesses greater insulating properties than normal-weight concrete.

6.2 Thermal properties

6.2.1 Thermal expansion of steel and concrete

In most simple fire engineering calculations the thermal expansion of materials is neglected, but for steel members which support a concrete slab on the upper flange the differential thermal expansion caused by shielding of the top flange, and the heat-sink function of the concrete slab, cause a "thermal bowing" towards the fire in the lower range of temperatures. When more advanced calculation models are used, it is also necessary to recognise that thermal expansion of the structural elements in the fire compartment is resisted by the cool structure outside this zone, and that this causes behaviour which is considerably different from that experienced by similar members in unrestrained furnace tests. It is therefore necessary at least to appreciate the way in which the thermal expansion coefficients of steel and concrete vary with respect to one another and with temperature. They are shown in Fig. 13; perhaps the most significant aspect to note is that the thermal expansion coefficients of steel and concrete are of comparable magnitudes in the practical range of fire temperatures.

EC3 Part 1.2 3.3.1.1
EC4 Part 1.2 3.3



EC4 Part 1.2 3.2.2;

Figure 13 Variation of Eurocode 3/4 thermal expansion coefficients of steel and concrete with temperature

Concrete is unlikely to reach the 700°C range at which its thermal expansion ceases altogether, whereas exposed steel sections will almost certainly reach the slightly higher temperature range within which a crystal-structure change takes place and the thermal expansion temporarily stops.

6.2.2 Other relevant thermal properties of steel

Two additional thermal properties of steel affect its heating rate in fire. Thermal conductivity is the coefficient which dictates the rate at which heat arriving at the steel's surface is conducted through the metal. A simplified version of the change of conductivity with temperature, defined in EC3, is shown in Fig. 14. For use with simple design calculations the constant conservative value of 45W/m°K is allowed.

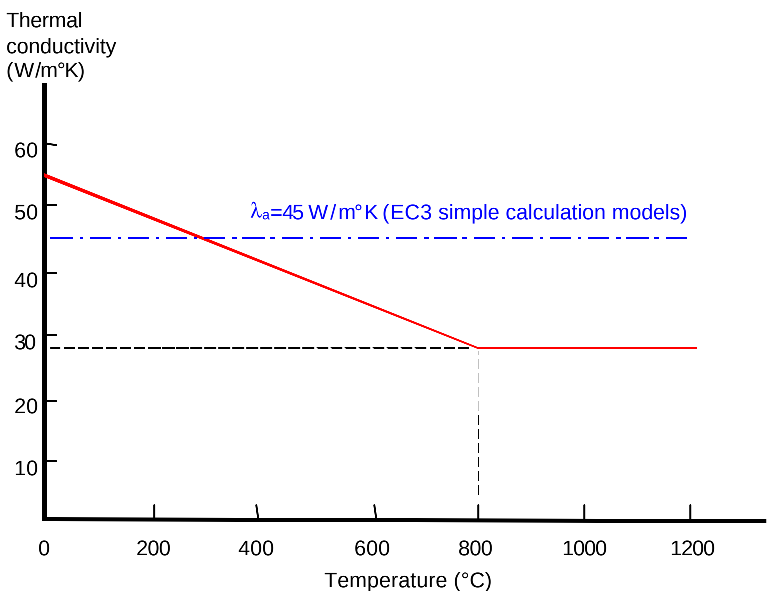


Figure 14 Eurocode 3 representations of the variation of thermal conductivity of steel with temperature

The specific heat of steel is the amount of heat which is necessary to raise the steel temperature by 1°C. This varies to some extent with temperature across most of the range, as is shown in Fig. 15, but its value undergoes a very dramatic change in the range 700-800°C. The apparent sharp rise to an "infinite" value at about 735°C is actually an indication of the latent heat input needed to allow the crystal-structure phase change to take place. Once again, when simple calculation models are being used a single value of 600J/kg°K is allowed, which is quite accurate for most of the temperature range but does not allow for the endothermic nature of the phase change.

EC3 Part 1.2 3.3.1.3
EC4 Part 1.2 3.3.1; 3.3.2; 3.3.3

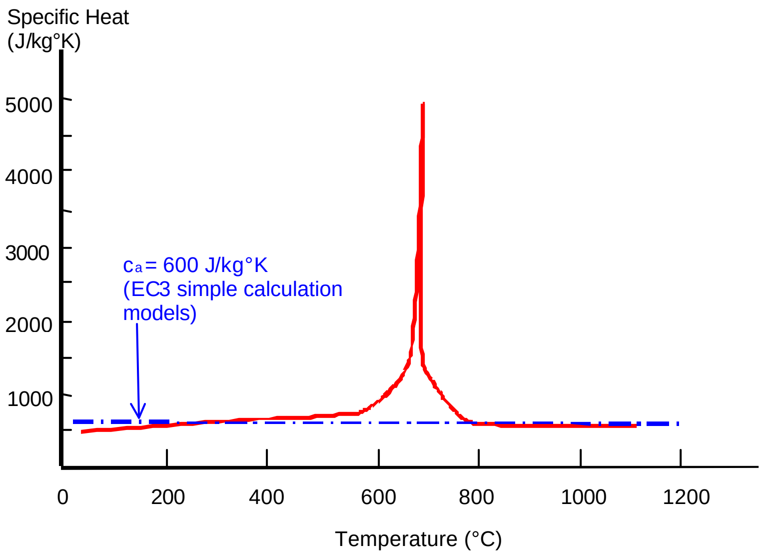


Figure 15 Variation of the specific heat of steel with temperature

6.2.3 Other relevant thermal properties of concrete

The thermal conductivity l_c of concrete depends on the thermal conductivity of its individual components and also on moisture content, aggregate type, mixture proportion and cement type. The aggregate type has the most significant influence on the conductivity of dry concrete. However, as the concrete's moisture content increases its thermal conductivity increases.

EC4 Part 1.2 3.3.2, 3.3.3

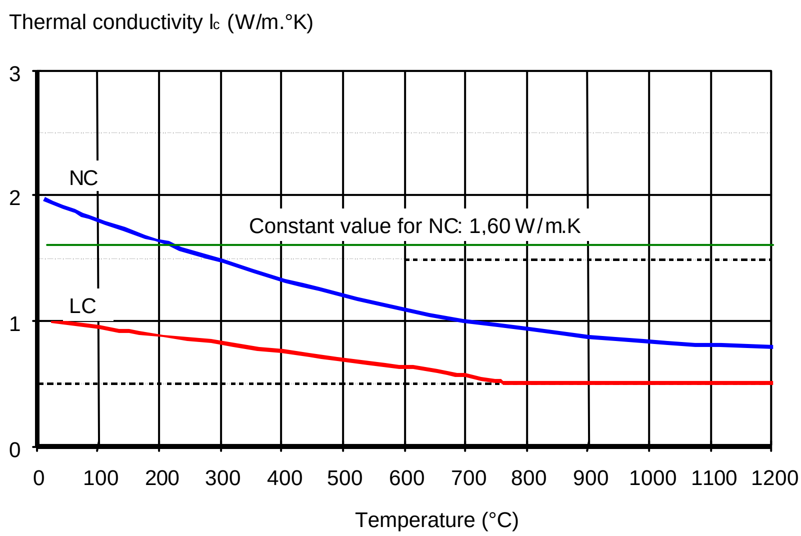


Figure 16 Thermal conductivity of normal-weight (NC) and light-weight concrete (LC) as a function of temperature

EC4 gives the thermal conductivity development with temperature for both normal and light-weight concrete (Fig. 16). In simple calculation models for normal-weight concrete a constant value of thermal conductivity may be used.

The specific heat of concrete c_c is also influenced by gravel type, mixture rate and moisture content. Gravel type is significant particularly in the case of concrete with calcareous aggregate, for which the specific heat increases suddenly because of chemical changes at a temperature of about 800°C. The moisture content is significant at temperatures up to 200°C, because the specific heat of wet concrete is twice that for dry concrete.

EC4 gives simple equations for the variation of the specific heat with temperature (Fig. 17). However in simple calculation models a constant value may be used. Values of specific heat c_c^* for wet concrete are given for several values of moisture content in Table 1.

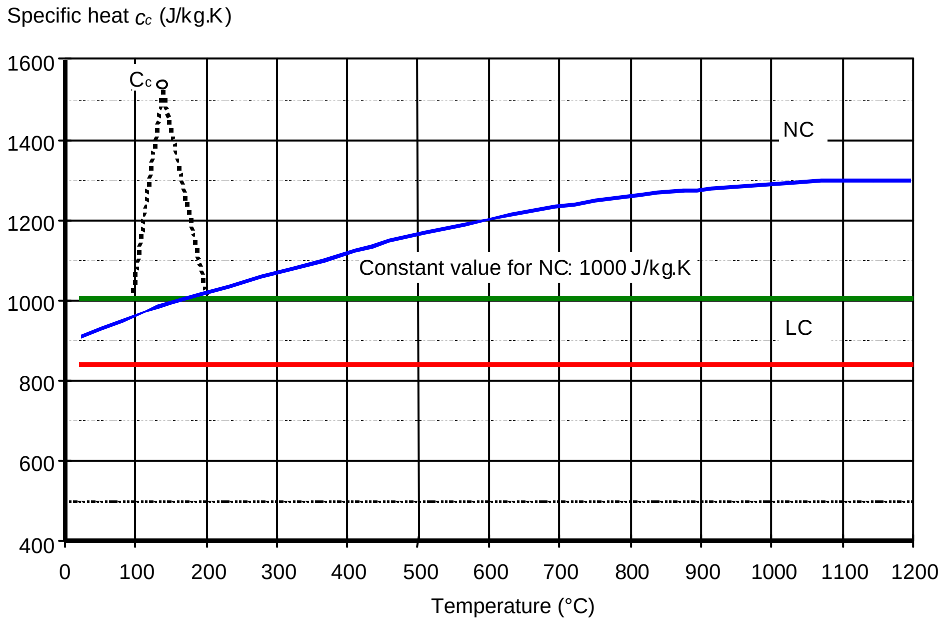


Figure 17 Specific heat of normal-weight (NC) and light-weight concrete (LC) as a function of temperature

Water content [%]	c_c^* [J/kg°K]
2	1875
4	2750
10	5600

Table 1. Variation of concrete specific heat with moisture content.

7 Thermal analysis

The same calculation rules are given in both EC3 Part 1.2 and EC4 Part 1.2 for calculating the temperatures of unprotected and protected steel beams. The temperatures of the lower and upper flanges may differ considerably, so it is very important that they should be calculated properly in order to obtain an accurate estimate of the bending moment resistance of the composite section.

The heat transfer to the member is predominantly by two mechanisms; radiation and convection. Since the rate of heating by both mechanisms is dependent at any time on the temperatures of both the fire atmosphere and the member, the member temperature is related to time via a fairly complex differential equation. This is dealt with in Eurocode 3 by linearising the temperature increments over small time steps, which is impractical for hand calculation but is ideal for setting up in spreadsheet software.

7.1 Unprotected steel sections

EC4 Part 1.2 4.3.3.2

For an unprotected steel section the temperature increase $Dq_{a.t}$ in a small time interval Dt (up to 5 seconds) is given by the net amount of heat which the section acquires during this time:

$$\Delta q_{a.t} = \frac{1}{c_a r_a} \frac{A_m}{V} h_{net.d} Dt \tag{2}$$

in which

c_a = *specific heat of steel*

r_a = *density of steel*

$\frac{A_m}{V}$ = *a "Section Factor" composed of*

- A_m = *exposed surface area of member per unit length (Fig. 22)*
- V = *volume of member per unit length*

$h_{net.d}$ = *design value of net heat flux per unit area*

EC1 Part 1.2 4.1

The net heat flux has radiative and convective components, of which the radiative is:

$$h_{net.r} = 5{,}67 * 10^{-8} \Phi e_{res} \left[(q_r + 273)^4 - (q_m + 273)^4 \right] \tag{3}$$

in which, apart from the Stefan-Boltzmann constant of $5{,}67x10^{-8}$,

F = configuration factor (can be set to 1.0 in the absence of data)

$e_{res} = e_f e_m$ = resultant emissivity = emissivity of fire compartment x emissivity of member surface (0,8 . 0,625=0,5 if no specific data)

q_r, q_m = environment and member surface temperatures

4.1

and the convective heat flux is:

$$h_{net.c} = a_c (q_g - q_m) \tag{4}$$

in which a_c = convective heat transfer coefficient (subject to NAD values, but 25W/m^2°K used for Standard or External Fire curves, 50W/m^2°K for Hydrocarbon Fire)

q_g, q_m = environment (gas) and member surface temperatures

When forming the net heat flux from these, each may be factored in order to account for differences in national practice in fire testing, but usually they are simply added together.
The "Section Factor" A_m/V uses the *exposed* perimeter in calculating an appropriate value of A_m and this means the actual surface which is exposed to radiation and convection. In determination of the Section Factor for the case of a composite slab with profiled steel sheets the three-sided exposure case can be considered, provided that at least 90% of the upper flange is covered by the steel sheet. If this is not the case then void-fillers made from appropriate insulating material must be used.

7.2 Steel sections with passive protection

For members with passive protection the basic mechanisms of heat transfer are identical to those for unprotected steelwork, but the surface covering of material of very low conductivity induces a considerable reduction in the heating rate of the steel section. Also, the insulating layer itself has the capacity to store a certain, if small, amount of heat. It is acceptable to assume that the exposed insulation surface is at the fire atmosphere temperature (since the conduction away from the surface is low and very little of the incident heat is used in raising the temperature of the surface layer of insulation material).

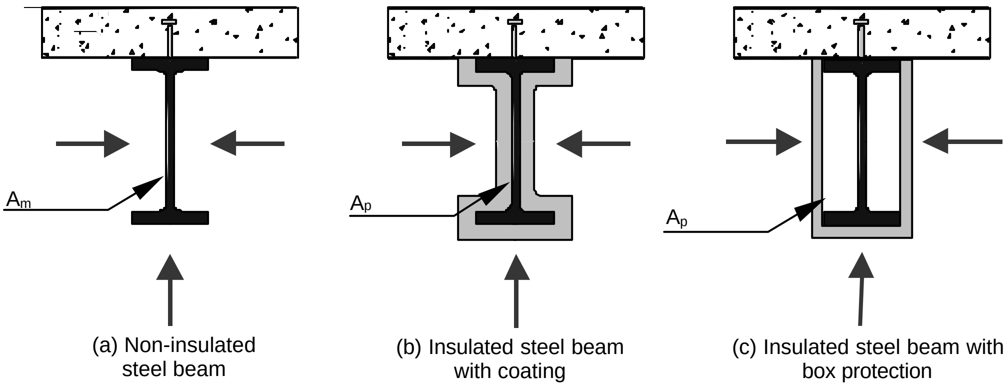


Figure 18 Estimation of the section factors of unprotected and protected steel beams

The calculation of steel temperature rise $Dq_{a.t}$ in a time increment Dt (up to 30 s) is now concerned with balancing the heat conduction from the exposed surface with the heat stored in the insulation layer and the steel section: *EC4 Part 1.2 4.3.3.2*

$$Dq_{a.t} = \frac{l_p / d_p}{c_a r_a} \frac{A_p}{V} \left(\frac{1}{1 + f/3} \right) (q_{g.t} - q_{a.t}) Dt - \left(e^{f/10} - 1 \right) Dq_{g.t} \text{ but } Dq_{a.t} \geq 0 \quad (5)$$

in which the relative heat storage in the protection material is given by the term

$$f = \frac{c_p r_p}{c_a r_a} d_p \frac{A_p}{V} \quad (6)$$

in which

$\frac{A_p}{V} =$	*section factor for protected steel member, where A_p is generally the inner perimeter of the protection material (Fig. 18)*
$c_a, c_p =$	*specific heats of steel and protection material*
$d_p =$	*thickness of fire protection material*
$q_{a.t}, q_{g.t} =$	*temperatures of steel and furnace gas at time t*
$Dq_{g.t} =$	*increase of gas temperature during the time step Dt*
$l_p =$	*thermal conductivity of the fire protection material*
$r_a, r_p =$	*densities of steel and fire protection material*

Fire protection materials often contain a certain percentage of moisture which evaporates at about 100°C, with considerable absorption of latent heat. This causes a "dwell" in the heating curve for a protected steel member at about this temperature while the water content is expelled from the protection layer. The incremental time-temperature relationship above does not model this effect, but this is at least a conservative approach. A method of calculating the dwell time is given, if required, in the European pre-standard for fire testing. *EN yyy5*

8 Concluding Summary

- Both steel and concrete suffer a progressive reduction in both strength and stiffness as their temperature increases in fire conditions. EC3 and EC4 provide material models of stress-strain curves for both materials over an extensive range of temperatures.
- Fire resistance of structural elements is quoted as the time at which they reach a defined deflection criterion when tested in a furnace heated according to a standard ISO834 time-temperature curve.
- It is possible to assess the severity of a natural fire as a time-equivalent between its peak temperature and the same temperature on the ISO834 standard curve.
- The behaviour of elements in furnace tests is very different from that in a building frame, and the only practical way of assessing whole-structure behaviour is to use advanced calculation models.
- EC3 or EC4 calculation of fire resistance takes account of the loading level on the element. However the safety factors applied are lower than in those used in strength design.
- Critical temperature is calculated for all types of member of classes 1, 2 or 3 from a single equation in terms of the load level in fire. Class 4 sections are universally assumed to have a critical temperature of 350°C.
- It is possible to calculate the temperature growth of protected or unprotected members in small time increments, in a way which can easily be implemented on a spreadsheet.

Structural Steelwork Eurocodes

Development of a Trans-National Approach

Course: Eurocode 4

Lecture 11b: Fire Engineering Design of Composite Structures

Summary:

- Traditional fire protection of steelwork is usually achieved by covering it with an insulating material during construction. However it may be possible under EC4 to use a combination of strategies to ensure fire resistance.
- EC4 calculation of fire resistance takes account of the loading level on the element. However the safety factors applied are lower than in those used in strength design.
- EC4 provides simple calculations for the load resistance in fire of common types of elements. In case of composite beams lateral-torsional buckling is neglected, and for columns the buckling fire resistance can be estimated according to code rules only for the case of braced frames.
- Fire resistance of composite beams comprising steel beam and concrete or composite slab may be calculated in terms of time, as a load-bearing resistance at a certain time, or as a critical element temperature appropriate to the load level and required time of exposure. Other members (composite slabs, composite beams comprising steel beams with partial concrete encasement, composite columns with partially encased steel sections and concrete-filled hollow sections) are examined in terms of the required fire resistance time.
- EC4 provides tabular design data for some structural types which are not easily addressed by simplified calculation methods.
- To assure the composite action during the fire exposure and the transmission of the applied forces and moments in the beam to column connections some constructional requirements must be fulfilled.

Pre-requisites: an appreciation of

- Simple element design for strength and serviceability according to EC3 and EC4.
- Framing systems currently used in steel-framed construction, including composite systems.
- The effects of temperature on the properties of steel and concrete.

Notes for Tutors:

- This module should follow a general introduction to the effects of fire on structures, and the basic principles of fire engineering design, such as that provided by the companion Lecture 11a.
- The lecturer can break up the session with formative exercises at appropriate stages (calculation of member capacities in fire, and fire resistance times are suggested).
- The lecturer may apply a summative assessment at the end of the session requiring that students consider the consequences of adopting different fire protection strategies at the design stage for part of an example building.

Objectives:

The student should:

- Understand that some types of composite members provide a considerable degree of inherent fire resistance which may either reduce or eliminate the need for additional passive protection materials.
- Understand that a range of strategies may be used in fire engineering design to provide the required fire resistance, including over-design, selection of framing systems, and use of sprinklers.
- Understand the principles of the simple design calculations of resistance in fire conditions of composite slabs, beams and columns, and the concept of critical temperature.
- Know how to calculate the sagging and hogging moment resistance of composite slabs.
- Know how to use the bending moment resistance method for calculation of the fire resistance of composite beams.
- Know how to calculate the capacity in fire of composite columns of different types, including the use of EC4 tabular data.

References:

- **ENV 1991-1:** *Eurocode 1: Basis of Design and Actions on Structures. Part 1: Basis of Design.*
- **prEN 1991-1-2:** *Eurocode 1: Basis of Design and Actions on Structures. Part 1.2: Actions on Structures Exposed to Fire.*
- **ENV 1992-1-1:** *Eurocode 2: Design of Concrete Structures. Part 1.1: General Rules: General Rules and Rules for Buildings.*
- **ENV 1992-1-2:** *Eurocode 2: Design of Concrete Structures. Part 1.2: General Rules: Structural Fire Design.*
- **prEN 1993-1-1:** *Eurocode 3: Design of Steel Structures. Part 1.1: General Rules: General Rules and Rules for Buildings.*
- **prEN 1993-1-2:** *Eurocode 3: Design of Steel Structures. Part 1.2: General Rules: Structural Fire Design.*
- **prEN 1994-1-1:** *Eurocode 4: Design of Composite Steel and Concrete Structures. Part 1.1: General Rules: General Rules and Rules for Buildings.*
- **ENV 1994-1-2:** *Eurocode 4: Design of Composite Steel and Concrete Structures. Part 1.2: General Rules: Structural Fire Design.*

Contents:

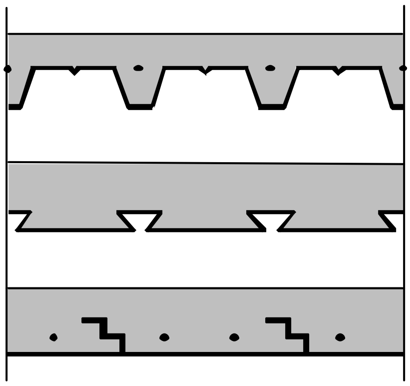
1 Introduction

EC 4 *Part 1.2* deals with the passive fire safety concept applied to composite steel and concrete structures.

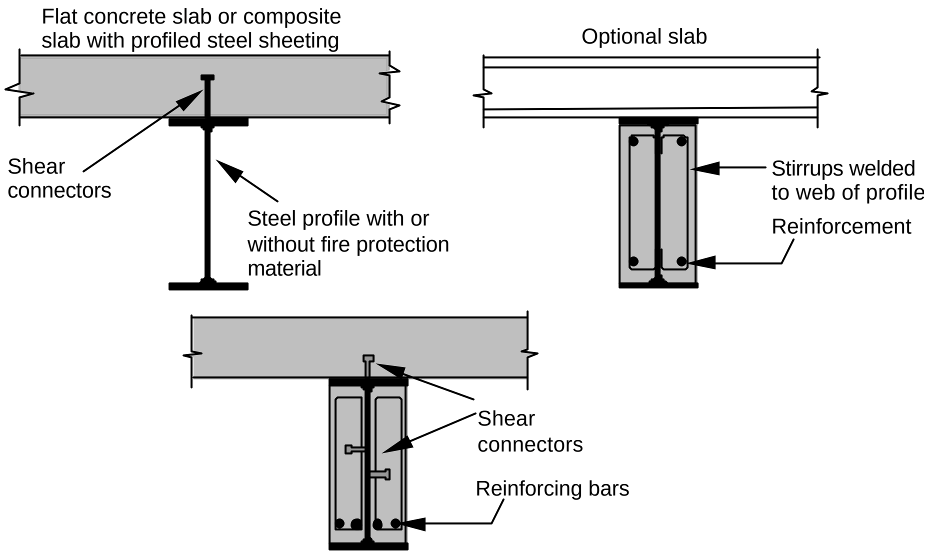
EC4 Part 1.2

The design methods presented in this document are valid only for structures or parts of structures within the scope of EC4 *Part 1.1* (Fig. 1).

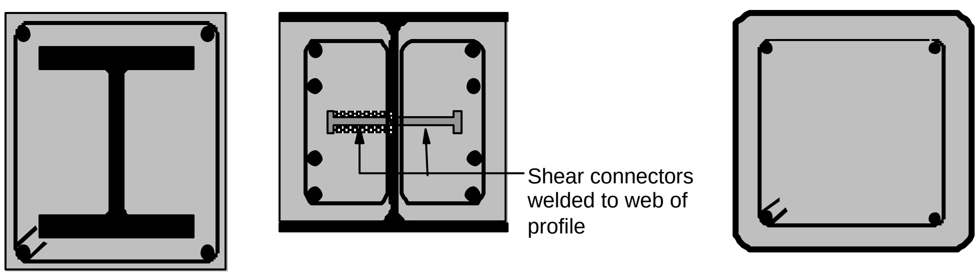
EC4 Part 1.2
1.1



(a) Composite Slabs



(b) Composite beams



(c) Composite columns

Figure 1 Typical examples of various types of composite steel and concrete sections

1.1 Structural fire design

It is necessary to recall that elements of structure must comply with three criteria in the event of fire:

EC4 Part 1.2 2.1

- *Integrity criterion ("E")* – cracks or openings, which can cause fire penetration by hot gases or flames, must not occur,
- *Insulation criterion ("I")* – the temperatures on the non-exposed surface of separating elements must not exceed ignition temperatures,
- *Load-bearing criterion ("R")* – structural members must maintain their load-bearing function during the whole required fire resistance time.

ENV 1994-1-2 covers principally the load-bearing criterion "R", although at a simpler level it also covers the integrity of compartments "E" and insulation "I". It allows three approaches to the assessment of structural behaviour in a fire design situation:

EC4 Part 1.2 4.1

- <u>*Simple Calculation Models*</u> for specific types of structural members,
- Established solutions, presented as <u>*Tabular Data*</u> for specific types of structural members,
- <u>*Advanced Calculation Models*</u> which simulate the behaviour of the global structure, of parts of the structure, or of isolated structural members.

Tabular data and simple calculation models can only be used for particular types of structural members under prescribed conditions. It is assumed that structural members are directly exposed to fire over their full length, so that the temperature distribution is the same over the whole length. Both methods give conservative results.

2 Simple calculation models

EC4 Part 1.2 4.3

It is sensible to consider fire engineering design of the main structural elements in composite construction in the order of their positions in the structural load-path. The design of slabs affects the beams more directly than is the case in non-composite steel construction.

2.1 Unprotected composite slabs

EC4 Part 1.2 4.3.1, 4.3.1.1

Composite slabs are commonly used, particularly in unpropped construction when cast onto ribbed steel decking, a system which is advantageous in terms of speed and simplicity of construction. It is both a structural element and in general has also the function of separating individual fire compartments, and so it must comply with all three criteria.

All the rules given in EC4 Part 1.2 for slabs are valid for both simply supported and continuous slabs. It is assumed that steel decking is not insulated but is heated directly, and that there is also no insulation between the structural concrete slab and surface screeds.

2.1.1 Criterion "E"

For composite slabs designed according to EC4 *Part 1.1* it is assumed that the integrity criterion is satisfied automatically.

2.1.2 Criterion "I"

EC4 Part 1.2 4.3.1.2

The effectiveness of the insulation function of the composite slab depends on its effective thickness.

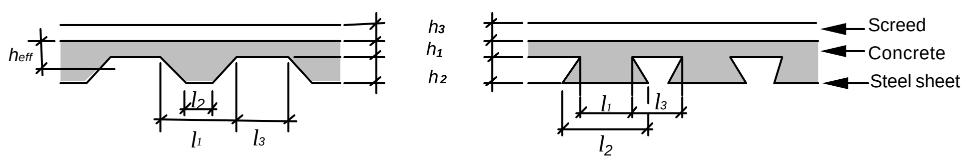


Figure 2 Slab dimensions for estimation of effective thickness

The effective slab thickness is calculated using the formula:

$$h_{eff} = h_1 + 0{,}5h_2\left(\frac{l_1 + l_2}{l_1 + l_3}\right) \qquad \text{for } h_2 / h_1 \le 1{,}5 \text{ and } h_1 > 40 \text{ mm,} \qquad (1)$$

$$h_{eff} = h_1\left[1 + 0{,}75\left(\frac{l_1 + l_2}{l_1 + l_3}\right)\right] \qquad \text{for } h_2 / h_1 > 1{,}5 \text{ and } h_1 > 40 \text{ mm.} \qquad (2)$$

The effective thickness value obtained is then compared with the minimum values (Table 1) necessary to achieve the required fire resistance time.

EC4 Part 1.2 Table 4.8

Standard Fire Resistance	Minimum effective thickness
R30	$60 - h_3$
R90	$100 - h_3$
R180	$150 - h_3$

Table 1 Effective slab thicknesses related to slab fire resistance.

For lightweight concrete values 10 % lower than these may be used.

In calculation of effective thickness it is permissible also to take into account screed, up to a maximum thickness of 20 mm.

2.1.3 Criterion "R"

EC4 Part 1.2 4.3.1.3

In a fire the mechanical properties of all structural materials degrade due to the high temperatures, which causes a decrease of both strength and flexural stiffness of the slab. When the design load-bearing resistance has decreased to the level of the design effect of the actions in the fire limit state then an ultimate condition has been reached.

The steel decking is not taken into account in calculation of fire resistance. In fact, due to the high heat capacity of the concrete the concrete slab and the release of steam from the concrete surface, the temperature of the sheeting is much lower than the gas temperature at early stages of a fire. Considering this fact, a slab which has been designed for ambient temperatures according to the rules of EC4 *Part 1.1* is assumed to have a fire resistance of 30 minutes without additional calculations.

In many cases, when the steel sheet is fixed to the supports (e.g. by stud connectors to the beams), or the parts of slabs near supports are cooler (in the case of a large plan), then axial deformations are prevented, so the slab is restrained in-plane. In such cases membrane forces can develop, and this may lead to an increase of the load-bearing resistance of the slab. This effect is the subject of current research at the present and is not yet included in the EC4 Part 1.2 rules.

Rules given in EC4 Part 1.2 for evaluation of load-bearing capacity are based on plastic global analysis. In the case of continuous slabs a redistribution of moments occurs as a result of changing stiffness, strength and thermal curvature due to high temperatures, so sufficient rotational capacity is required. This entails the provision of tensile reinforcement with sufficient deformation capacity and an adequate reinforcement ratio. This can be assured if the ambient-temperature slab design conforms to the rules of EC2 Part 1.2.

2.1.3.1 Sagging moment resistance

In the calculation of sagging moment resistance not only the steel sheeting but also concrete in tension is neglected. As the insulation criterion must be fulfilled the temperature on the *4.3.1.4* unexposed side will be low, and due to this fact the concrete in compression can be considered to have no reduction of strength. From this it is dear that the sagging moment resistance depends on the amount of tensile reinforcement (the reinforcement ratio) and its temperature. The temperature of the reinforcement depends on its distance from the heated surfaces. These are the distances shown as u_1, u_2 and u_3 in Fig. 3, and the reinforcement temperature is expressed as the function of its position "z":

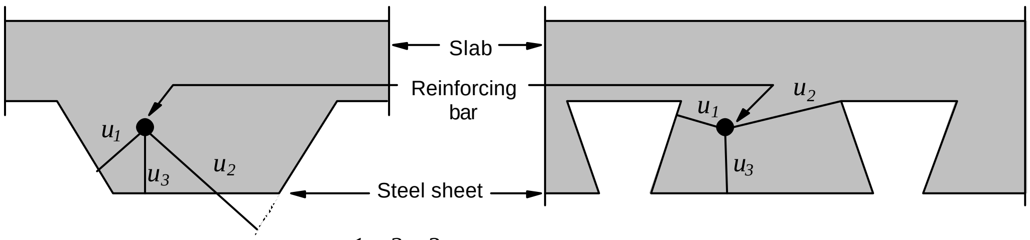
$$\frac{1}{z} = \frac{1}{\sqrt{u_1}} + \frac{1}{\sqrt{u_2}} + \frac{1}{\sqrt{u_3}} \tag{3}$$

Limitations on the edge distances of the reinforcement are

u_1 and u_2 = 50 mm,

u_3 = 35 mm.

EC4 Part 1.2 Fig. 4.2



($u1$, $u2$, $u3$ are perpendicular distances)

Figure 3 Geometrical position of the rebar

An example of temperature functions of reinforcement for some fire duration times is shown in Table 2.

Standard Fire Resistance	Temperature of the reinforcement [°C]
R60	$\theta_s = 1175 - 350\,z \le 810°C$ for $(z \le 3{,}3)$
R120	$\theta_s = 1370 - 350\,z \le 930°C$ for $(z \le 3{,}8)$
R240	$\theta_s = 1575 - 350\,z \le 1050°C$ for $(z \le 4{,}2)$

Table 2 Reinforcement temperature functions for some Standard fire resistance times

2.1.3.2 Hogging moment resistance

The concrete in compression is on the exposed side of the slab, so a reduced strength must be considered. This can be done in two ways; by integration over the depth of the ribs or by *4.3.1.5* replacing the ribbed slab by an equivalent slab of uniform thickness h_{eff} according to 2.1.2, which is a more conservative method. Temperatures of uniform-thickness slabs are given in EC4 Part 1.2.

The temperature of the tensile reinforcement can be taken as equal to the concrete temperature at the position of the bars. As this is usually placed at a minimum cover distance from the exposed surface the temperature influence is negligible in most cases.

In Fig. 4 the heating of a slab with effective thickness 100 mm at a standard fire duration of 60 minutes is shown.

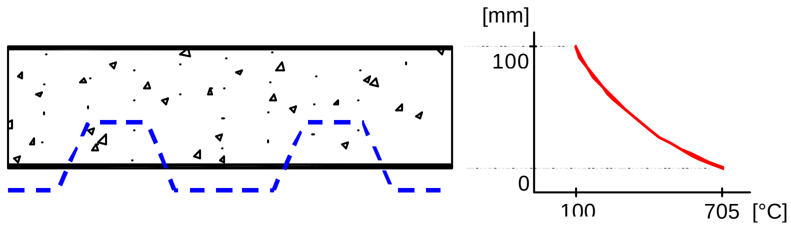


Figure 4 Heating of a 100mm flat composite slab at 60 minutes

2.2 Protected composite slabs

Composite slabs can be protected (Fig. 5) by the use of fire protection material or a suspended ceiling.

4.3.2

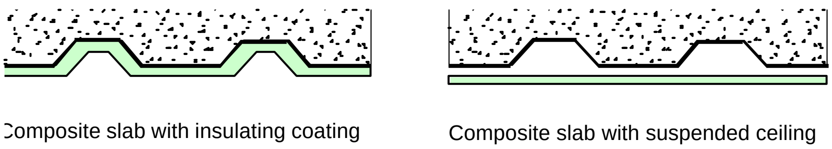


Figure 5 Fire protection of the slab

The relevant data for proprietary fire protection materials is given in manufacturers' literature, and for generic materials typical figures are provided in relevant codes. The fulfilment of the insulation criterion “I” is assured by the use of the EC4 rules for the load-bearing criterion "R", if the fire protection material is taken into account in the equivalent concrete thickness according to the appropriate codes.

It is assumed that the load-bearing criterion “R” is automatically fulfilled before the temperature of the steel sheeting reaches 350 °C.

2.3 Composite beams including steel sections with no concrete encasement

The analysis of a composite beam including a "downstand" steel beam with no concrete encasement is divided into two steps:

EC4 Part 1.2
4.3.3, 4.3.3.1

- ***thermal analysis*** for estimation of the temperature distribution in the cross-section,
- ***mechanical analysis*** for calculation of the load-bearing resistance of the member under fire conditions.

2.3.1 Thermal analysis

In EC4 Part 1.2 the same rules apply to calculation of the temperatures of unprotected and protected steel beams as are given in EC3 Part 1.2, which are described in *Lecture 11a: Introduction to Structural Fire Engineering* of this package. There may be considerable differences in the temperatures of the lower and upper flanges, so it is very important that these should be calculated properly in order to obtain an accurate value of the bending moment resistance of a composite section.

4.3.3.2

2.3.2 Mechanical analysis

In EC4 Part 1.2 two methods are given for calculation of sagging bending moment resistance of beams without concrete encasement.

2.3.2.1 The Critical Temperature Method

EC4 Part 1.2
4.3.3.3

The Critical Temperature Method is a simplified method, which can be used for the case of simply supported composite beams composed of hot-rolled downstand steel sections of up to 500mm depth and concrete slabs with a thickness of not less than 120mm. For such configurations it is assumed that the temperature over the depth of the steel section is uniform.

The advantage of this method is that it is not necessary to calculate the bending moment resistance in fire directly. The critical temperature is a function of the load level for the fire limit state, $h_{fi,t}$:

$$h_{fi,t} = \frac{E_{fi,d,t}}{R_d} = \frac{h_{fi} E_d}{R_d} \tag{4}$$

where $E_{fi,d,t}$ is the design effect of the actions in the fire situation, R_d is the design load-bearing resistance for normal temperature design, E_d is the design effect of actions for normal temperature design and

$$h_{fi} = (g_{GA} + y_{1,1}\, x) \,/\, (g_G + g_Q\, x). \tag{5}$$

In the fire situation the ultimate limit state is reached when the load-bearing resistance $R_{fi,d,t}$ decreases to the level of the design effect of the actions in fire $E_{fi,d,t}$ so that the load level can be written as

$$h_{fi,t} = \frac{R_{fi,d,t}}{R_d} \tag{6}$$

It has been shown experimentally that the compressive strength of concrete has not a significant influence on the bending moment resistance of composite beams in fire. The reason for this is that the resultant tension in the steel section is rather small due to its high temperature. The neutral axis position is therefore high in the concrete slab, and only a small part of the slab is in compression (see Fig. 6). Considering this fact, it is clear that the bending moment resistance in the fire situation is influenced mainly by the steel strength, so

$$h_{fi,t} = \frac{R_{fi,d,t}}{R_d} = \frac{f_{a\,max,qcr}}{f_{ay,20°C}} \tag{7}$$

The critical temperature of the steel part is determined from the formula

$$0{,}9\,h_{fi,t} = \frac{f_{a\,max,qcr}}{f_{ay,20°C}} \tag{8}$$

and the value of the critical temperature obtained is then compared with the temperature of the steel section after the required fire duration, calculated from the formulas for unprotected or protected sections, as given in Section 2.1.3.1. The term $0{,}9\,h_{fi,t}$ is almost completely equivalent to the "Utilisation Factor" which is used in the same way in EC3 Part 1.2 for non-composite steel construction. *EC3 Part 1.2 Table 4.1*

2.3.2.2 Bending moment resistance method

4.3.3.4

If the steel section is deeper than 500 mm or the slab thickness is less than 120 mm, the Bending Moment Resistance Method must be used.

The bending moment resistance is calculated using simple plastic theory, so the steel section must be Class 1 or 2. The concrete slab must have sufficient rotational capacity, which is assured by the fulfilment of EC2 Part 1.2 requirements.

At the required fire resistance time the neutral axis position is obtained as usual from equilibrium of the tensile force T in the lower part and the compressive force F in the upper part (Fig. 6). *Annex D*

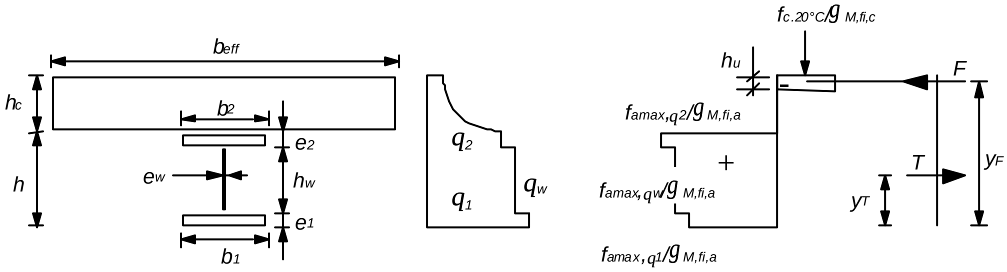


Figure 6 Temperature and stress distribution for composite beam comprising concrete slab and downstand steel section

Assuming that the neutral axis position is in the concrete slab, the tensile force in the steel section is given by:

$$T = \left[f_{amax,q\,1}(b_1 e_1) + f_{amax,\,qw}(h_w e_w) + f_{amax,q\,2}(b_2 e_2) \right] / g_{M,\,fi,a} \tag{9}$$

and the depth of concrete in compression results from the equation:

$$F = T \Rightarrow h_u \;=\; T \,/\, (b_{eff} \;\; f_{c,20°C} \;/ g_{?\;,fi,c}) \tag{10}$$

The sagging moment resistance is then obtained from

$$M_{fi,Rd^+} = T\,(y_F - y_T) \tag{11}$$

This process can also be used for a composite slab with profiled steel sheeting, if the slab depth is replaced by h_{eff} (Section 2.1.1.2). It is also important to check whether the temperature of the compressed concrete zone h_u is less than 250°C (using the process shown in Section 2.1.1.3), otherwise the following more complicated formula for the estimation of h_u should be used:

$$F = T = \left[\int_0^{h_u} b_{eff}\, 0{,}85\, f_{c,qi}\, d_i \right] \Big/ g_{M,fi,c} \tag{12}$$

which can be solved by iteration, assuming a stepped temperature profile using the average temperatures at 10mm steps:

To define in fig6 h_{cr}, $h_{u,n}$; $f_{c,?n}$

$$T = F = \left[(h_c - h_{cr})(b_{eff}) f_{c,20°C} + \sum_{i=2}^{n-1} (10 b_{eff}) f_{c,qi} + (h_{u,n}\, b_{eff}) f_{c,qn} \right] \Big/ g_{M,fi,c} \tag{13}$$

2.3.2.3 Shear resistance

4.3.3.5

For composite beams it is also necessary to verify the shear resistance of shear connectors to assure that the slab and the steel section act as a single structural member. They must have sufficient strength and stiffness to resist the shear force acting at the interface between the steel and the concrete slab, which is increased in the fire situation as a result of different thermal elongations of the slab and the steel section.

The shear resistance is calculated according to the rules of EC4 Part 1.1 (g_v is replaced by $g_{M,fi,v}$) and is equal to the lower of:

EC4 Part 1.1 6.13

$$P_{fi,Rd} = P_{Rd} \;\; k_{max,q} = k_{max,q} \frac{0{,}8 \dfrac{pd^2}{4} f_u}{g_{M,fi,v}} \tag{14}$$

6.14

$$P_{fi,Rd} = P_{Rd} \;\; k_{c,q} \;=\; k_{c,q} \frac{0{,}29 a d^2}{g_{M,fi,v}} \sqrt{f_{ck} E_{cm}} \tag{15}$$

where

q is the temperature of connectors or the adjacent concrete,

f_{ck} and E_{cm} are the characteristic values of cylinder strength and secant modulus of concrete,

f_u is the value of the specified ultimate tensile strength of the stud material, but not more than 500 N/mm^2,

$k_{max,q}$ and $k_{c,q}$ are reduction factors for the stud connector and concrete strengths.

The formulas are valid for studs of diameter up to 25mm. For greater diameters they should be verified by testing.

The temperature of the stud connectors (q_v) and of the concrete (q_c) may be taken as 80% and 40% respectively of the steel section's upper flange temperature.

To mobilise the full plastic bending moment resistance, the shear resistance must be greater than the tensile resultant, otherwise the value of $N.P_{fi,Rd}$ (where N is the number of shear connectors in half of the span of the simply supported beam) should be used instead of T for the calculation of the bending moment resistance.

2.4 Composite beams comprising steel beams with partial concrete encasement

EC4 Part 1.2 4.3.4

This type of composite beam consists of a concrete slab (either flat or ribbed), a steel section and concrete placed between the flanges of the steel section. The rules given in EC4 Part 1.2 are valid for either simply supported or continuous beams including cantilevers. This contrasts with beams without concrete encasement to the steel section, which can only be considered as simply supported because of the possibility of local buckling at the connections.

4.3.4.1

For calculation purposes plastic theory is used and three-sided exposure is assumed. To ensure the validity of this assumption in the case of ribbed slabs with trapezoidal steel sheeting at least 90% of the upper flange must be covered.

The validity of the calculation procedures given in the Code is restricted by required minimum slab thickness and steel profile dimensions, both of which depend on the required fire safety class of the building. Examples of these dimensional restrictions are shown in Table 3.

Tables 4.11; 4.12

Restricted dimensions	**Fire Resistance Class**	
	R30	**R90**
Minimum slab thickness h_c [mm]	60	100
Minimum profile height h and width b_c [mm]	120	170
Minimum area h .b_c [mm^2]	17500	35000

Table 3 Limits on validity of EC4 calculation for partially encased slabs.

Additional restrictions on the calculation are:

- $e_w < \dfrac{b_c}{10}$ e
- $e_f < \dfrac{h}{8}$ (e_w, b_c, e_f and h are as defined in Fig. 7)

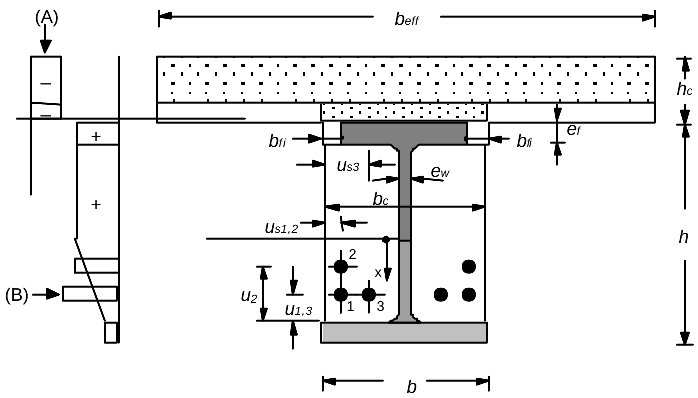


Figure 7 Reduced section for calculation of sagging moment resistance

2.4.1 Thermal analysis

4.3.4.2

The heating of the cross-section is more complicated for partially encased beams than for simple downstand steel beams (Section 2.1.3). The lower flange of the steel beam is heated directly, while other parts are protected by the concrete placed between the flanges. This concrete encasement, as well as the reinforcement placed between the flanges, also contributes to the resistance. Due to these facts it is not possible to estimate the temperatures of the individual parts of the section by simple calculation and to compare them with a general critical temperature. The Code gives rules for calculation of bending moment resistance for different fire resistance classes. For the purpose of calculation of the bending moment resistance, individual parts of the cross-section (lower steel flange and web, rebars between flanges), over which the temperature distribution is uniform or linearly varying, are assumed to have their full section but reduced strength. Horizontal areas heated non-uniformly are assumed to have full strength, but the parts affected by heat are excluded from the calculation (concrete infill, the lower parts $h_{c,fi}$ of the concrete slab, the ends b_{fi} of the upper steel flange) (see Figs. 7 and 10).

2.4.2 Continuity

For simply supported beams the sagging moment resistance is compared with the maximum sagging moment of the beam (Fig. 8(a)), but for continuous beams the sagging moment resistance is compared with the maximum sagging moment in fire and the hogging moment resistance is compared with the maximum support moment in fire (Fig. 8(b)).

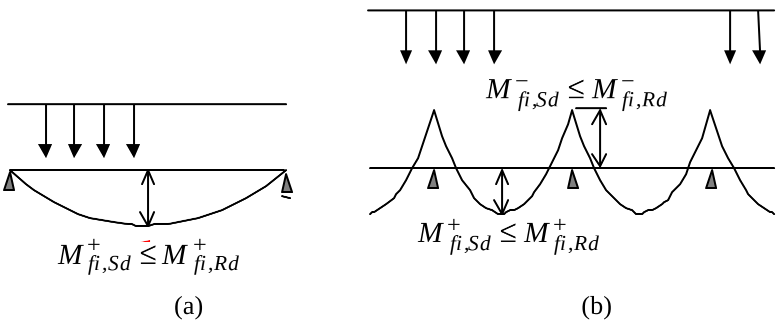


Figure 8 Conditions for maximum sagging and hogging moments

5.4.1

In some cases beams which behave in normal temperature design as a simply supported may be considered as continuous in the fire case. This may occur when the concrete slab is reinforced adequately at its supports to guarantee its continuity, and provided that there can be effective transmission of the compression force through the steel connection (Fig. 9).

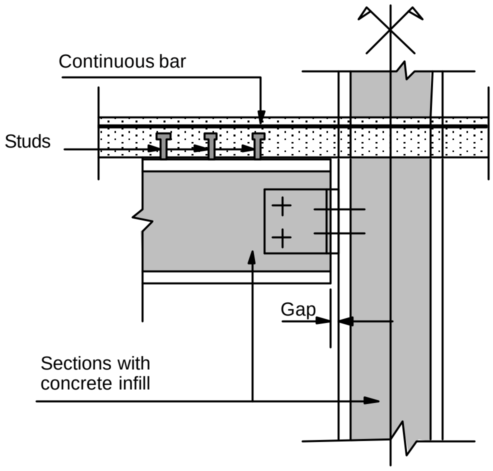


Figure 9 Detail of beam-to-column connection to ensure continuity under fire conditions.

To develop the hogging moment at the support the gap should be in the range between 10mm and 15mm for fire ratings R30 to R180 and a beam span over 5m; in all other cases the gap should be less than 10 mm.

2.4.3 Mechanical analysis

EC4 Part 1.2
4.3.4.4
Annex E, E.1

2.4.3.1 Sagging moment resistance $M_{fi,Rd}$+

The procedure for calculation of the sagging moment resistance is as follows:

2.4.3.1.1 Estimation of the reduced section (Fig. 10)

Concrete slab

Only the part in compression which is not influenced by temperature is taken into account. The design value of the compressive concrete strength is taken as $f_{c,20°C}/\gamma_{M,fi,c}$. The effective width of the concrete slab b_{eff} is the normal effective width, since it is assumed to be at ambient temperature. The reduced thickness $h_{c,fi}$ varies with the fire resistance class. Values of the reduction are given in the tables of the code (Table 4). For composite slabs with steel sheeting the reduced thickness $h_{c,fi} = h_2$ (the height of the rib; see Fig. 2).

Table E.1

Upper flange of steel section

The upper flange is considered to have its full strength $f_{ay,20°C}/\gamma_{M,fi,a}$, but it is assumed that there are directly heated edges, each of width b_{fi} which are not taken into account. The effective width is then $(b-2b_{fi})$. The heated edge value b_{fi} is related to the fire resistance class (Table 4).

Table E.2

Web of steel section

The web is divided into two parts. The upper part h_h is assumed to remain at 20°C, so its full strength is used. In the lower part h_ℓ the temperature is assumed to change linearly from 20°C at its top edge to the temperature of the lower flange at its bottom edge. The value of h_ℓ is calculated as follows (see Table 4):

Table E.3

- For $h/b_c \leq 1$ or $h/b_c \geq 2$: $$h_l = a_1 / b_c + a_2 e_w /(b_c h) \geq h_{l.min} \qquad (16)$$
- for $1 < h/b_c < 2$ the formula varies with the fire resistance time (Table 4).

The values of a_1 and a_2 are given in the Table E.3 of the Code.

Lower flange of steel section

Table E.4

The lower flange is assumed to have uniform temperature distribution, because it is heated directly. Therefore its area is not modified, but its yield point is reduced by the factor k_a depending on the fire resistance class (Table 4).

Reinforcing bars

EC4 Part 1.2
Annex E2 (9)
Table E.5

The temperature of the reinforcing bars depends on their distance from the lower flange u_i and on their concrete cover u_s. The reduction factor k_r is given not only as a function of fire resistance class but also of the position u of the reinforcement, which is estimated as

$$u = 1/\left[(1/u_i) + (1/u_{si}) + (1/(b_c - e_w - u_{si}))\right] \tag{17}$$

The reduction factor k_r is calculated by the empirical formula:

$$k_r = (ua_3 + a_4)a_5 / \sqrt{A_m / V} \tag{18}$$

(in which u, A_m and V are all in mm units) subject to the limits $k_{r,min} = 0,1 = k_r = k_{r,max} = 1$.

Concrete between flanges

Concrete between the section flanges is not included in the calculation of sagging moment resistance, but is assumed to resist the vertical shear by itself, so its shear resistance must be verified.

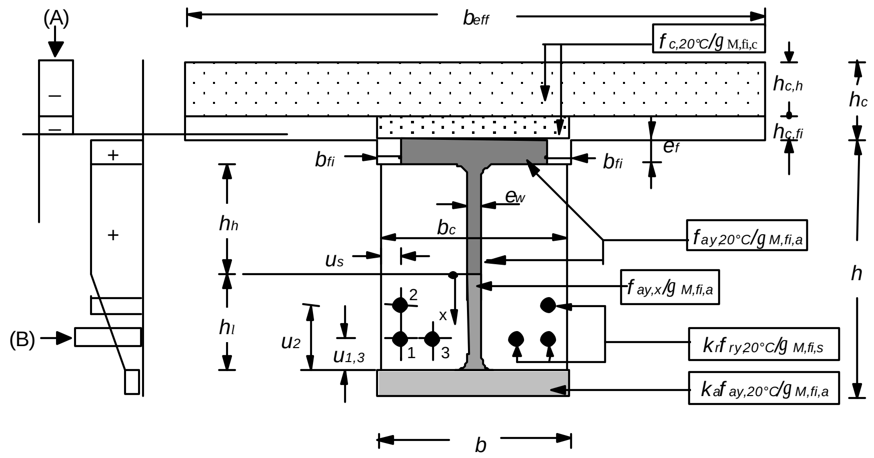


Figure 10 Stress distributions in concrete (A) and steel (B) for sagging moment

Fig. E.1

	Fire resistance class R30	Fire resistance class R90
Thickness reduction of the concrete slab $h_{c,fi}$ [mm]	10	30
Width reduction of the upper flange b_{fi} [mm]	$(e_f / 2) + (b - b_c) / 2$	$(e_f / 2) + 30 + (b - b_c) / 2$
Bottom part of the web h_ℓ [mm] for $1 < h / b_c < 2$	$h_\ell = 3\,600 / b_c$	$h_\ell = 14\,000 / b_c + 75\,000 (e_w / b_c h) + 85\,000 (e_w / b_c h)(2 - h / b_c)$
$h_{\ell,min}$ [mm]	20	40
Reduction factor for the strength of lower flange k_a	$[(1,12) - (84 / b_c) + (h / 22b_c)]a_0$	$[(0,12) - (17 / b_c) + (h / 38b_c)]a_0$

Table 4 Reduced section parameters for sagging.

Definition of the neutral axis for bending

The position of the neutral axis should be defined on the basis of a plastic distribution of stresses and from the equilibrium of tensile and compressive resultants.

2.4.3.1.2 Calculation of the sagging moment resistance $M_{fi,Rd}+$

Assuming that no net axial force is taken into account the moment resistance is simply calculated by summation of the contributions of each of the stress blocks shown in Fig. 10. A detailed example is given in Section 6. The moment resistance must exceed the design moment in the fire limit state:

$$M^{+}_{fi,Sd} = \eta_{fi} . M_{Sd} \leq M^{+}_{fi,Rd} \qquad (19)$$

2.4.3.2 Hogging moment resistance $M_{fi,Rd}-$

EC4 Part 1.2
4.3.4.5
Annex E, E.2

The calculation method is the same as that for sagging moment resistance, except for some differences in definition of the reduced section.

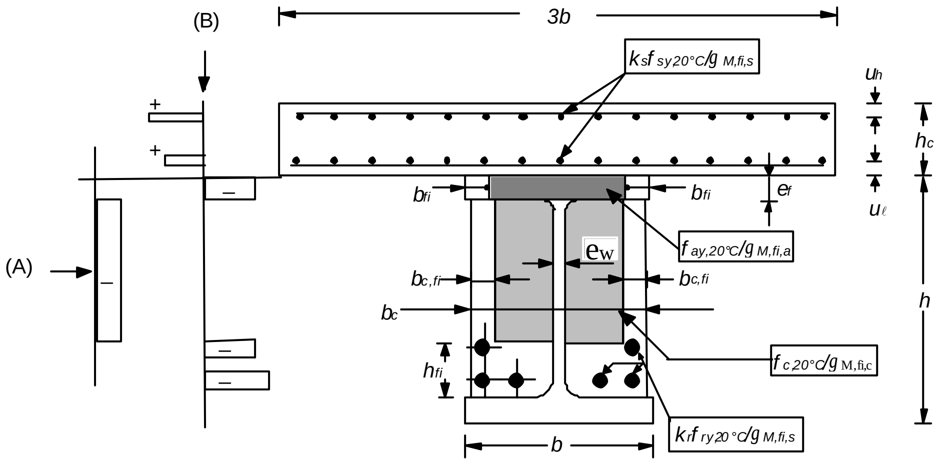


Figure E.2

Figure 11 Stress distribution in concrete (A) and steel (B) for hogging moment

Concrete slab and reinforcement

Concrete in tension is excluded from the calculation, but the tensile reinforcement lying in the effective area is taken into account. The effective width of the concrete slab is reduced to three times the width of the steel profile (Fig. 11). The temperature and the strength reduction depends on the distance u of the reinforcing bars from the lower slab edge. The reduction factor k_s of the yield point of the reinforcing bars in the concrete slab is given as a function of the distance u. Two examples of the factor k_s are shown in Table 5.

Upper flange of steel section

The same rules are adopted as in calculation of sagging moment resistance. In the case of a simply supported beam, which is assumed to be continuous in the fire situation, the upper flange should not be taken into account if it is in tension.

Concrete between the flanges

Concrete infill between flanges is considered to have its full compressive strength but to have a reduced cross-section. The appropriate reductions of height h_{fi} and width b_{fi} are given in EC4 Part 1.2. Some examples, including minimum values, are shown in Table 5.

Reinforcement between flanges

The rules used in calculating the sagging moment resistance should be adopted.

Steel web

In areas of hogging bending moment the shear force is assumed to be transmitted by the steel web, which is neglected when calculating the hogging bending moment resistance.

Lower flange

For the purpose of calculation of hogging moment resistance the compressed lower flange should be ignored.

	Fire Resistance Class	
	R30	**R90**
Reduction factor k_s	1	(0,0275 u) – 0,1
Reduction of the concrete h_{fi} [mm]	25	220 – (0,5b_c) – (h/b_c)
$h_{fi,min}$ [mm]	25	45
Reduction of the concrete $b_{c,fi}$ [mm]	25	70 – (0,1b_c)
$b_{c,fi,min}$ [mm]	25	35

Table 5 Reduced section parameters for hogging

2.5 Slim-floor beams

In recent years slim-floor beams have grown in popularity throughout Europe. The most commonly used types are open or closed sections, combined either with pre-cast slabs or with ribbed slabs cast *in situ* onto deep steel decking (Fig. 12). The advantages of these systems are that the low depth of the floor structure allows a free zone for building services due to the flat soffit, and good inherent fire resistance (up to 60 min.) without additional fire protection because the steel beams are encased by the concrete slabs.

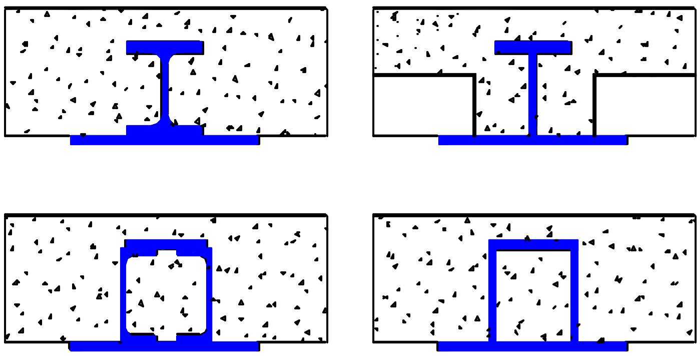


Figure 12 Some types of slim-floor beams

The fire resistance of slim-floor beams is not directly covered in simplified methods within EC4 Part 1.2, so only general principles are discussed here.

Temperature distributions should be estimated for unprotected or protected slim-floor beams using a two-dimensional heat transfer model. Thermal properties of materials and the effect of moisture content can be taken from EC4 Part 1.2, and heat flux should be determined by considering thermal radiation and convection. When the temperature distribution over the cross-section is known, the resistance of the slim-floor beam in the fire limit state can be calculated using the moment capacity method either for composite or non-composite beams, with reduction factors for steel and concrete strength taken from EC4 Part 1.2. In order to calculate the bending resistance, the section is divided to several components: the plate and/or the bottom flange, the lower web, the upper web, the upper flange, reinforcing bars and the concrete slab. Concrete in tension is ignored and, as the neutral axis is in most cases very close to the upper flange, the temperature of the concrete in compression can be assumed to be below 100°C.

2.6 Composite columns

EC4 Part 1.2 4.3.6, 4.3.6.1

The simplified rules given in EC4 Part 1.2 are valid only for braced frames. Provided that a fire is limited to a single storey, and that the fire-affected columns are fully connected to the colder columns below and above, it is possible to assume that their ends are rotationally restrained so that the buckling length in the fire situation is estimated assuming fixed ends. This means that for intermediate storeys the buckling length in fire is $l_{fi,cr} = 0{,}5\ L$ and for the top floor (or for a ground floor with pinned base connection) $l_{fi,cr} = 0{,}7\ L$ (Fig. 13).

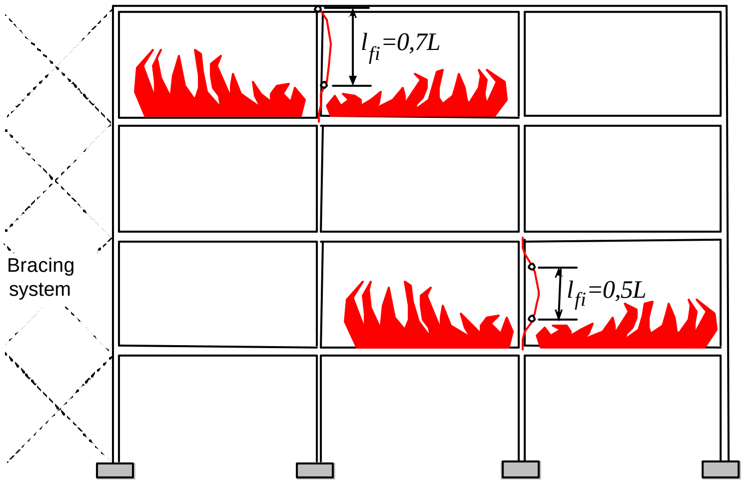


Figure 13 Buckling lengths in fire

In the simple calculation model the buckling resistance in fire is obtained from:

$$N_{fi,Rd,z} = c_z \,.\, N_{fi,pl.Rd}$$

where

c_z is the reduction coefficient for buckling about the minor axis z, evaluated according to rules of EC3 Part 1.1, but using only buckling curve (c) to relate it to the non-dimensional slenderness ratio $\bar{l}_{z,q}$,

$N_{fi,pl.Rd}$ is the design value of the plastic resistance to axial compression in the fire situation.

The non-dimensional slenderness ratio $\bar{l}_{z,q}$ is given by:

$$\bar{l}_{z,q} = \sqrt{\frac{N_{fi,pl.R}}{N_{fi,cr,z}}} \tag{20}$$

where $N_{fi,pl.R}$ is the value of $N_{fi,pl.Rd}$ when the factors $g_{M,fi,a}$, $g_{M,fi,s}$ and $g_{M,fi,c}$ are taken as 1,0 and $N_{fi,cr,z}$ is the Euler critical buckling load for the fire situation, obtained from

$$N_{fi,cr,z} = \frac{p^2 (EI)_{fi,eff,z}}{l_q^2} \tag{21}$$

In this equation the buckling length l_q in the fire situation is obtained according to Figure 13, and

$(EI)_{fi,eff,z}$ is the flexural stiffness of the cross-section in the fire situation.

In the more detailed calculation rules given by EC4 Part 1.2 there are some differences between their application to different types of cross-section. The code gives methods for the analysis of two basic types (see Fig. 1(c)):

- Steel sections with partial concrete encasement (unprotected and protected),
- Concrete-filled circular and square hollow sections (unprotected and protected).

2.6.1 Steel section with partial concrete encasement 4.3.6.2

There are some restrictions to the use of the simple calculation model given in EC4 Part 1.2:

- Buckling length l_θ£ *13,5 b*
- Depth of cross-section *h* is between 230 mm and 1100 mm,
- Width of cross-section *b* is between 230 mm and 500 mm,
- Minimum *h* and *b* for R90 and R120 is 300 mm,
- Percentage of reinforcing steel is between 1% and 6%,
- Standard fire resistance period ≤ 120 min.

To determine the axial plastic resistance $N_{fi,pl.Rd}$ and the flexural stiffness $(EI)_{fi,eff,z}$ in the fire situation, the cross-section is divided (Fig. 14) into: the flanges of the steel section, the web of the steel section, the reinforcing bars and the concrete infill between the flanges. Annex F

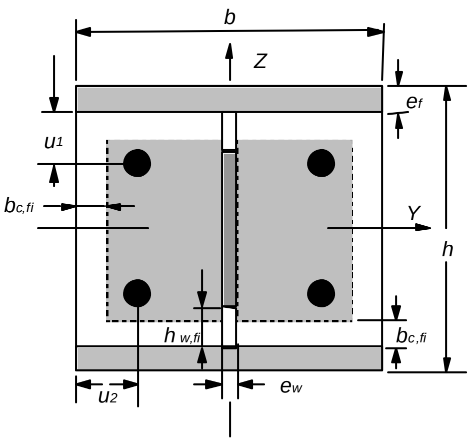


Figure 14 Division of the cross-section into individual components

For each of these components the temperature for the required standard fire resistance (R30, R60, R90 or R120) is estimated. A reduced strength and modulus of elasticity is then determined as a function of temperature. In the simple calculation model a uniform temperature distribution is assumed over certain elements, but in case of the steel web and the concrete infill the outer parts have a considerably higher temperature and thus a high thermal gradient occurs. Because of this the sections of the steel web and the concrete infill are reduced, with the outer parts ($h_{w,fi}$ and $b_{c,fi}$) being ignored.

The process for fire engineering design of partially encased composite steel and concrete columns, in the context of braced frames, may be summarised as follows:

Plastic resistance to axial compression

$$\boxed{\begin{aligned} N_{fi,pl.Rd} &= N_{fi,pl.Rd,a} + N_{fi,pl.Rd,c} + N_{fi,pl.Rd,s} \\ &= \sum_j \left(A_{a,\theta}\ f_{amax,\theta}\right)/\gamma_{M,fi,a} + \sum_m \left(A_{c,\theta}\ f_{c,\theta}\right)/\gamma_{M,fi,c} \\ &\quad + \sum_k \left(A_{s,\theta}\ f_{smax,\theta}\right)/\gamma_{M,fi,s} \end{aligned}}$$

Effective flexural stiffness

$$\boxed{\begin{aligned} (EI)_{fi,eff} &= (E_a I_a)_{fi,eff} + (E_s I_s)_{fi,eff} + (E_c I_c)_{fi,eff} \\ &= \sum_j \left(\varphi_{a,\theta}\ \overline{E}_{a,\theta}\ I_{a,\theta}\right) + \sum_k \left(\varphi_{s,\theta}\ \overline{E}_{s,\theta}\ I_{s,\theta}\right) \\ &\quad + \sum_m \left(\varphi_{c,\theta}\ \overline{E}_{c,\theta}\ I_{c,\theta}\right) \end{aligned}}$$

Determination of critical length:

$$\boxed{l_\theta = 0{,}5 l_{cr} \text{ or } 0{,}7 l_{cr}}$$

Euler critical buckling load:

$$\boxed{N_{fi,cr,z} = \frac{\pi^2 (EI)_{fi,eff,z}}{l_\theta^2}}$$

Non-dimensional slenderness ratio:

$$\boxed{\overline{\lambda}_\theta = \sqrt{\frac{N_{fi,pl.R}}{N_{fi,cr,z}}}}$$

$$\boxed{\chi_z \text{ for buckling curve “c”}}$$

Buckling resistance:

$$\boxed{N_{fi,Rd,z} = \chi_z N_{fi,pl,Rd}}$$

Verification:

$$\boxed{\begin{aligned} &N_{fi,Sd} \le N_{fi,Rd,z} \\ &(\eta_{fi} N_{Sd} \le N_{fi,Rd,z}) \end{aligned}}$$

Eccentricity of loading

In the case of eccentric loading the application point should remain inside the composite section of the column. The design buckling load for eccentricity δ is then obtained from: *Annex F.7*

$$N_{fi,Rd,\delta} = N_{fi,Rd} \frac{N_{Rd,\delta}}{N_{Rd}} \tag{22}$$

where N_{Rd} and $N_{Rd,\delta}$ are the values of the axial buckling resistance and buckling resistance for the case of eccentric loading for normal temperature design, and are calculated according to EC4 Part 1.1 (see Lecture 8).

2.6.2 Unprotected concrete filled hollow sections

Filling the steel hollow sections with concrete has some advantages. It can either be used to increase the load-bearing capacity or reduce of the section size, which increases usable space inside the building, and allows rapid erection without requiring formwork. It also gives high

EC4 Part 1.2
4.3.6.3

inherent fire resistance without additional fire protection. This combination of steel and concrete is very convenient for both materials; the steel hollow section confines the concrete laterally, and the concrete core helps to increase the local buckling resistance of the steel section.

During the first stages of fire exposure the steel part expands more rapidly than the concrete, and so at this stage the steel section carries most of the load. Heat from the steel shell is gradually transferred to the concrete filling, but as the thermal properties of the concrete are very favourable (it has low heat conductivity) the heating of the core is relatively slow. After some time (usually 20 to 30 minutes) the strength of steel begins to degrade rapidly due to its high temperature, and the concrete part progressively takes over the load-carrying rôle. As the temperature of the concrete core increases, its strength decreases and failure eventually occurs either by buckling or compression. The decrease of the mechanical properties of the concrete is slower than in the case of encased steel sections, because the steel section protects it from direct fire exposure and prevents spalling.

For hollow concrete-filled sections it is very important to realise that at high temperatures both the free moisture-content of the concrete and also the chemically bonded water of crystallisation is driven out of the concrete, and it is necessary to avoid any build-up of pressure by allowing it to escape. All hollow sections should therefore have openings of at least 20 mm diameter at both the top and bottom of each storey. The calculation model given in EC4-1-2 is valid only for circular and square hollow sections, and does not include non-square rectangular sections. There are also some restrictions for the use of the model:

- Buckling length $l_\theta \leq 4{,}5\ m$
- Depth b or diameter d of cross-section is between 140 mm and 400 mm,
- Concrete grade is either C20/25 or C40/50,
- Percentage of reinforcing steel is between 0% and 5%,
- Standard fire resistance period ≤ 120 min.

EC4 Part 1.2 4.3.6.3

The whole analysis is divided into two steps; calculation of temperatures over the cross-section, and calculation of the buckling resistance in fire for the temperatures obtained.

2.6.2.1 Temperature across the section

The assumptions for the temperature calculations are:

Annex G.2

- The temperature of the steel wall is homogeneous,
- There is no thermal resistance between the steel wall and the concrete,
- The temperature of the reinforcing bars is equal to the temperature of the concrete surrounding them,
- There is no longitudinal thermal gradient along the column.

The net heat flux transmitted to the concrete core can be obtained from:

$$\dot{h}_{net,d} = \rho_a c_a e \left[\frac{\partial \theta_a}{\partial t} \right] \tag{23}$$

and the heat transfer in the concrete core is calculated according to:

$$c_{c\theta}\, \rho_c \frac{\partial \theta_c}{\partial t} = \frac{\partial}{\partial y}\left(\lambda_{c\theta} \frac{\partial \theta_c}{\partial y} \right) + \frac{\partial}{\partial z}\left(\lambda_{c\theta} \frac{\partial \theta_c}{\partial z} \right) \tag{24}$$

Estimation of the temperature distribution can be made by means of either finite difference or finite element methods. When using the finite difference method the unit dimension of the square mesh "*m*" for square sections, or the distance between two adjacent circular meshes "*n*" for circular sections, should not be greater than 20 mm. The number of nodes n_1 across the width b of the square member or n_2 across the diameter d of a circular member is obtained as follows:

For square members $n_1 = \dfrac{b}{m\sqrt{2}}$ (25)

For circular members $n_2 = \dfrac{d}{n}$ (26)

2.6.2.2 Buckling resistance in fire

The design buckling resistance in fire is calculated in the same way as for concrete-encased sections (Section 2. 6.1):

$$N_{fi,Rd} = c \ \ N_{fi,pl,Rd} \tag{27}$$

Annex G.3

The principles for determination of the non-dimensional slenderness and the strength reduction coefficient for buckling are identical to those used previously. However there are some differences in evaluating the plastic resistance to axial compression and the Euler critical load.

The plastic resistance to axial compression is the sum of the plastic resistances of all components (the wall of the steel section, reinforcing bars and the concrete core), and is determined from

$$N_{fi,pl.Rd} = \sum \left(A_{a,q} \ s_{a,q}\right)/g_{M,fi,a} + \sum \left(A_{c,q} \ s_{c,q}\right)/g_{M,fi,c} + \sum \left(A_{s,q} \ s_{s,q}\right)/g_{M,fi,s} \tag{28}$$

where A_i is the cross-section area of material i,

$s_{i,q}$ is the limiting stress in material i, at the temperature q.

The Euler critical load is given by

$$N_{fi,cr} = \frac{p^2 \left[E_{a,?,s} \ I_a + E_{c,?,s} \ I_c + E_{s,?,s} \ I_s\right]}{l_q^2} \tag{29}$$

where, $E_{i,q,s}$ is the tangent modulus of the stress-strain relationship for the material i at temperature q and stress $s_{i,q}$; l_θ is the buckling length in the fire situation; I_i is the second moment of area of the material i, related to the central axis y or z of the composite cross-section.

$E_{i,q,s} I_i$ and $A_i s_{i,q}$ have to be calculated by summation of elements *(dy, dz)* at the appropriate temperatures.

The stress-strain relationships for the steel section, reinforcing bars and concrete may be modelled for these cases as follows:

Steel section and reinforcement:

$$\frac{s_{a,q}}{f_{ay,q}} = 0{,}06 + 1{,}416\left[\frac{E_{a,?}e_{a,?}}{f_{ay,?}}\right] - 0{,}651\left[\frac{E_{a,?}e_{a,?}}{f_{ay,?}}\right]^2 + 0{,}103\left[\frac{E_{a,?}e_{a,?}}{f_{ay,?}}\right]^3 \tag{30}$$

This gives the tangent modulus relationship

$$\frac{E_{a,?,s}}{E_{a,?}} = 1{,}416 - 1{,}302\left[\frac{E_{a,?}e_{a,?}}{f_{ay,?}}\right] + 0{,}309\left[\frac{E_{a,?}e_{a,?}}{f_{ay,?}}\right]^2 \tag{31}$$

Concrete core:

$$\frac{s_{c,q}}{f_{c,q}} = \frac{E_{c,?}}{f_{c,?}}\left[1 - \left(\frac{E_{c,?} \ e_{c,?}}{4 f_{c,?}}\right)\right] \quad \text{and} \quad \frac{E_{c,?,s}}{E_{c,?}} = 1 - \left[\frac{E_{c,?}}{2 f_{c,?}}\right] \tag{32}$$

The rules for estimation of $f_{ay\theta}$, $f_{sy\theta}$, and $f_{c\theta}$, under the unique conditions which apply to concrete-filled sections, and the tangent moduli $E_{a,?}$, $E_{s,?}$, and $E_{c,?}$ are also given in the form of equations in EC4 Part 1.2. These relationships are shown graphically in Figs. 15(a) and 15(b).

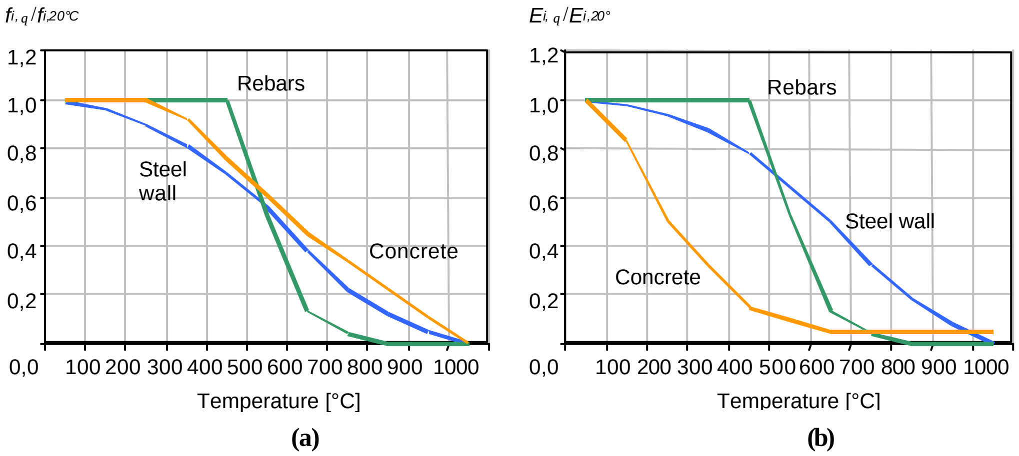
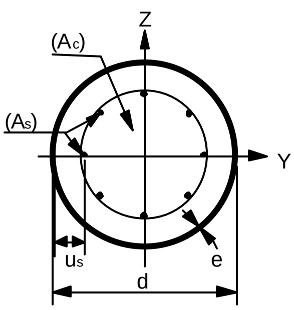


Figure 15 Degradation of strength and tangent stiffness for the constituents of concrete-filled hollow sections

It is possible to present the axial buckling resistance for particular cases in tabular or graphical form. The value of $N_{fi,Rd}$ is a function of the buckling length, concrete grade and percentage of reinforcing steel. An example of such a design graph is shown in Fig. 16.

Design curve	Concrete grade	$A_s/(A_c+A_s)$ %
1	C20/25	1,0
2	C20/25	2,5
3	C20/25	4.0
4	C30/37	1,0
5	C30/37	2,5
6	C30/37	4,0
7	C40/50	1,0
8	C40/50	2,5
9	C40/50	4,0



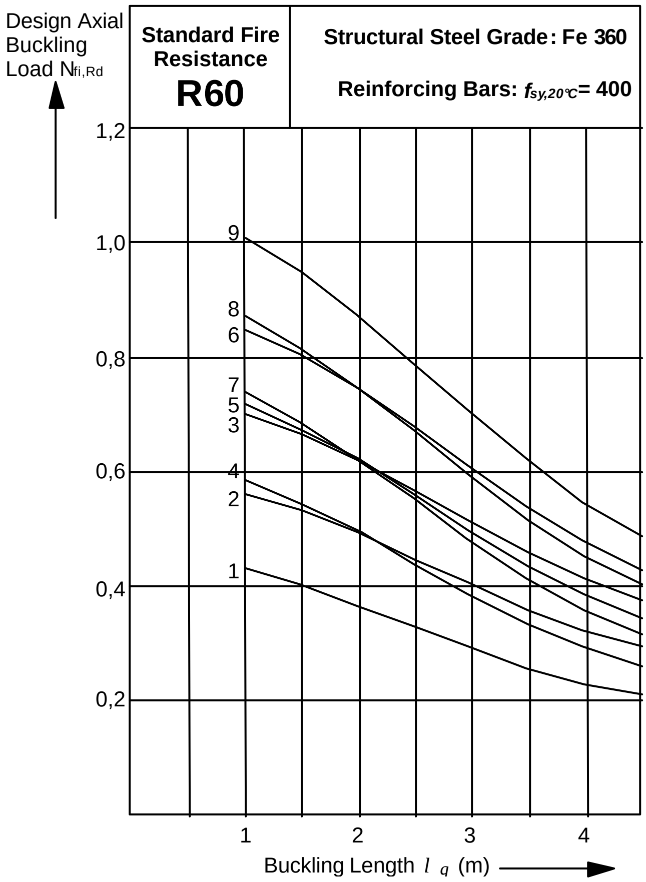


Figure 16 Example design graph for a concrete-filled circular

hollow section 219,1 x 4,5

Fig.G.3

Eccentricity of loading

Unlike partially encased steel sections, in the case of concrete-filled hollow sections any eccentricity of loading is taken into account by artificially increasing the axial loading. The equivalent axial load N_{equ} may be obtained from:

$$N_{equ} = \frac{N_{fi,Sd}}{j_s j_d} \tag{33}$$

where j_s is a correction coefficient for the percentage of reinforcement, and j_d is a coefficient which takes account of the eccentricity of loading and depends also on the buckling length and the section size.

The eccentricity of the load $d = \frac{M}{N}$ at the end of the column should not exceed half the dimension *b* or *d* of the cross-section.

2.6.3 Protected concrete-filled hollow sections

In some cases, such as those which include a high load factor, or a high required fire resistance time, it is necessary to use an additional passive fire protection system around the column. The behaviour of these systems (screens, coatings, sprayed materials) should be assessed according to appropriate codes and manufacturers' data. It is assumed, that the load-bearing criterion is fulfilled provided that the temperature of the steel wall remains below 350°C.

EC4 Part 1.2
Annex G.4

EC4 Part 1.2
4.3.6.4

3 Tabular data

For some special cases under standard fire conditions, and for braced frames, solutions are presented in EC4 Part 1.2 as tabular data. *EC4 Part 1.2 4.2*

It is assumed, that neither the boundary conditions nor the internal forces at the ends of members change during the fire, and that the loading actions are not time-dependent. The only deformations taken into account are those caused by thermal gradient. The fire resistance then depends on the load level $h_{fi,t}$ (see Section 2.1), the cross-section proportions and the reinforcement ratio. *4.2.1*

Structural members for which tabular data is available are as follows:

Simply supported beams *4.2.2*

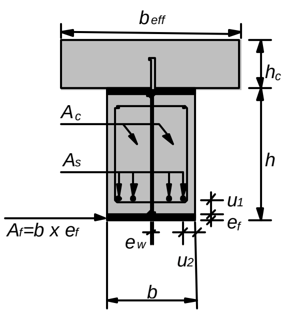
- Composite beams comprising a steel beam with partial concrete encasement,
- Encased steel beams, for which the concrete has only an insulating function.

Columns *4.2.3*

The column at the level under consideration must be fully connected to the columns above and below, and the fire must be limited to only a single storey.

- Composite columns comprising totally encased steel sections, *4.2.3.2*
- Composite columns comprising partially encased steel sections, *4.2.3.3*
- Composite columns comprising concrete-filled hollow sections. *4.2.3.4*

In certain cases the the application of tabular data depends on additional conditions. An example from a design table given in EC4 Part 1.2 is shown in Table 6.



	Condition for application: Slab: h_c ³ 120 mm b_{eff} £ 5 m Steel section: b / e_w ³ 15 e_f / e_w £ 2 Additional reinforcement area, related to total area between the flanges: $A_s/(A_c + A_s)$ £ 5%	**Standard Fire Resistance**				
		R30	**R60**	**R90**	**R120**	**R180**
1	Minimum cross-sectional dimensions for load level $h_{fi,t}$ = **0,3**					
	Min b [mm] and additional reinforcement A_s in relation to the area of flange A_s / A_f					
1.1	h ≥ 0,9 × min b	***70/0,0***	***100/0,0***	***170/0,0***	***200/0,0***	***260/0,0***
1.2	h ≥ 1,5 × min b	***60/0,0***	***100/0,0***	***150/0,0***	***180/0,0***	***240/0,0***
1.3	h ≥ 2,0 × min b	***60/0,0***	***100/0,0***	***150/0,0***	***180/0,0***	***240/0,0***
2	Minimum cross-sectional dimensions for load level $h_{fi,t}$ = **0,5**					
	Min b [mm] and additional reinforcement A_s in relation to the area of flange A_s / A_f					

2.1	h ≥ 0,9 × min b	***80/0,0***	***170/0,0***	***250/0,4***	***270/0,5***	-
2.2	h ≥ 1,5 × min b	***80/0,0***	***150/0,0***	***200/0,2***	***240/0,3***	***300/0,5***
2.3	h ≥ 2,0 × min b	***70/0,0***	***120/0,0***	***180/0,2***	***220/0,3***	***280/0,3***
2.4	h ≥ 3,0 × min b	***60/0,0***	***100/0,0***	***170/0,2***	***200/0,3***	***250/0,3***

Table 6 Tabular design data for composite beams with partial encasement to steel beam

4 Constructional details

The fire resistance of joints must be at least the same as for the connected members. This means that beam-to-column connections should be able to transmit the internal forces during the whole fire resistance time. When passive fire protection is used on the members this requirement is fulfilled if the same thickness of fire protection is applied to the joints. In general the beam-to-column joints do not present a major problem because, due to the concentration of material, the temperature of the joint tends to be lower than that of the connected members. For special cases additional requirements are given in the code. *EC4 Part 1.2* *5.4*

In composite structures it is very important to guarantee the required level of shear connection between the steel and concrete in the fire situation as well as at ambient temperature. Alternatively the steel and concrete parts must be able to fulfil the fire resistance requirements individually. Shear connectors should not be attached to the directly heated parts of the steel sections. *5.1*

In case of fully or partially encased sections the concrete must be reinforced (if the concrete encasement has only an insulating function then nominal steel reinforcement meshes should be sufficient), the concrete cover of the reinforcing bars should be greater than 20 mm and less than 50 mm in order to prevent spalling of the concrete during the fire.

Additional requirements are given in the code for particular types of structure. *5.2, 5.3*

5 Use of advanced calculation models

EC3 Part 1.2 4.3
EC4 Part 1.2 4.4

Both Eurocodes 3 and 4 also permit the use of advanced calculation models based upon fundamental physical behaviour, which give a realistic analysis of the behaviour in fire of the structure. These may be used to represent the behaviour of individual members, the whole structure or sub-assemblies. All computational methods are to some extent approximate, are based on different assumptions, and are not capable of predicting all possible types of behaviour. It is therefore stipulated that the validity of any such model used in design analysis must be agreed by the client, the designer and the competent building control authority.

Computational models may cover the thermal response of the structure to any defined fire, either nominal or parametric, and should not only be based on the established physical principles of heat transfer but should also on known variations of thermal material properties with temperature. The more advanced models may consider non-uniform thermal exposure, and heat transfer to adjacent structure. Since the influence of moisture content in protection materials is inevitably an additional safety feature it is permissible to neglect this in analysis.

When modelling the mechanical response of structures the analysis must be based on acknowledged principles of structural mechanics, given the change of material properties with temperature. Thermally induced strains and their effects due to temperature increase and differentials must be included. Geometric non-linearity is essential when modelling in a domain of very high structural deflections, as is material non-linearity when stress-strain curves are highly curvilinear. It is, however, acknowledged that within the time-scale of accidental fires transient thermal creep does not need to be explicitly included provided that the elevated-temperature stress-strain curves given in the Code are used.

6 Worked Examples

6.1 Composite Beams

In this worked example the design calculations for a simply supported composite beam are examined. Two basic structural types are considered:

- Composite beam comprising a concrete slab and:
  a) An unprotected steel section,
  b) A steel section protected by sprayed material,
  c) A steel section protected by gypsum boarding.
- Composite beam comprising a concrete slab and a partially encased steel section.

6.1.1 Composite beam comprising steel section IPE 400 and concrete slab

The composite beam is a part of floor structure, and the spacing of the beams is 6,0 m. The beam is considered as simply supported with a span of 7,0 m, and consists of an IPE400 steel section and a concrete slab of depth 120 mm (Fig. 17). The effective width of the slab is assumed to be 1,0 m. The beam is checked for a standard fire resistance R 90.

Loads

	Characteristic values		*Load factors*	*Design values*
Dead load	3,6 kN/m^2	3,6 . 6,0 = 21,6 kN/m	$\gamma_G = 1{,}35$	29,16 kN/m
Imposed load	4,5 kN/m^2	4,5 . 6,0 = 27,0 kN/m	$\gamma_{Q1} = 1{,}5$	40,50 kN/m
Total	**8,1 kN/m^2**	**48,6 kN/m**		**69,66 kN/m**

Section characteristics:

Steel grade: S235

Concrete grade: C20/25

Steel area: 8450 mm^2

Concrete area: 120 . 10^3 mm^2

For the ambient-temperature design the plastic bending moment resistance is estimated:

$M_{u,pl}$ = 425,18 kNm

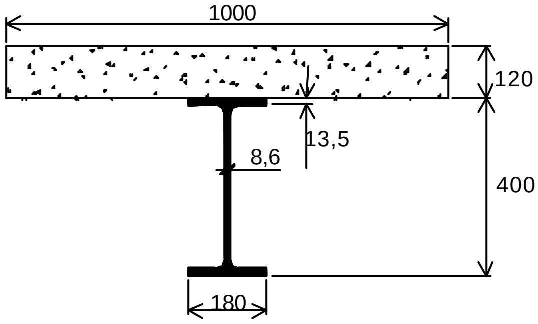


Figure 17 Beam cross-section

The conditions for use of the critical temperature model are satisfied (h < 500 mm [= 400]; h_C = 120 mm [= 120]; simply supported beam). *ENV 1994-1-2* *4.3.3.2*

Loading in the fire situation is obtained from: *4.3.3.3*

$$\sum g_{GA}.G_k + y_{1,1}.Q_{k,1} + \sum y_{2,i}.Q_{k,i} + \sum A_d$$ *ENV 1991-2-2*

where G_k is the permanent action,

$Q_{k,1}$ is the main variable action

$Q_{k,i}$ are other variable actions

A_d are indirect fire actions

$g_{GA}, y_{1,1}$ and $y_{2,i}$ are action and combination factors for accidental loadings.

For the structure analysed $g_{GA} = 1{,}0; y_{1,1} = 0{,}7$, so

$$q_{fi} = 1{,}0.21{,}6 + 0{,}7.27{,}0 = 40{,}5 kN/m$$

The action of loads in fire is then

$$M_{fi,Sd} = \frac{1}{8} 40{,}5.7^2 = 248{,}06 kNm$$

and the load level can be calculated as

$$h_{fi,t} = \frac{E_{fi,d,t}}{R_d} = \frac{M_{fi,Sd}}{M_{u,pl}} = \frac{248{,}06}{425{,}18} = 0{,}583$$

The critical temperature of the steel section is determined from the load level and from the strength of the steel at elevated temperature as follows: *ENV 1994-1-2* *4.1*

$$0{,}9h_{fi,t} = f_{a\,max\,q\,cr} / f_{ay,20°C}$$

$$f_{a\,max\,q\,cr} = 0{,}9h_{fi,t} \cdot f_{ay,20°C} = 0{,}9.0{,}583.235 = 123{,}4 MPa$$

From the value of 123,4 MPa it follows that the strength reduction coefficient at time t is $k_{max\,q} = \frac{123{,}4}{235} = 0{,}525$. *4.3.3.3*

The task is now to calculate the critical temperature (the temperature at which the yield strength decreases to the value of 123,4 MPa) and the time at which the critical temperature will be reached. This will then be compared with the required fire resistance period of 90 min. *3.2.1*

Unprotected steel section

By linear interpolation from Table 3.2 of the Code the value $q_{crit} = 582°C$ is obtained. The required time will be estimated, from the equation for heating of unprotected steel cross-sections, either computationally or using design sheets. In this case the calculation was performed using spreadsheet software.

3.2.1
Table 3.2

$$Dq_{a.t} = \frac{1}{c_a r_a} \frac{A_m}{V} h_{net.d} Dt$$

in which (see Section 2.1.3.1)

c_a =	*specific heat of steel*	[600 J/kgK]	*3.3.1*
r_a =	*density of steel*	[7850kg/m^3]	
$\frac{A_m}{V}$ =	*the "Section Factor"*	[153 m^{-1}]	
$h_{net.d}$ =	*design value of net heat flux per unit area [7.1.3.1]*	[$e_f = 0{,}8$; $e_m = 0{,}625$]	*EC1 Part 2-2 4.1*

Estimation of the section factor:

$$A_m = 1{,}47 m^2 / m$$

$$V = 845 0.10^{-6} m^3 / m$$

$$A_{m,3} = A_m - 0{,}180 = 1{,}470 - 0{,}180 = 1{,}290 m^2 / m$$

$$\frac{A_{m,3}}{V} = \frac{1{,}290}{845 0.10^{-6}} = 153 m^{-1}$$

<u>For Δt = 5s the fire resistance time is 13,4 min < 90 min, so it is necessary to reduce the heating of the steel part by using fire protection material.</u>

Two types of fire protection materials (sprayed protection and gypsum boarding) were applied:.

Sprayed protection

Sprayed protection is applied directly on the surface of the steel section, so the section factor remains the same at 153 m^{-1}.

The heating of the insulated steel beam is obtained from:

4.3.3.2

$$Dq_{a.t} = \frac{l_p / d_p}{c_a r_a} \frac{A_p}{V} \left(\frac{1}{1 + f/3} \right) \left(q_{g.t} - q_{a.t} \right) Dt - \left(e^{f/10} - 1 \right) Dq_{g.t}$$

in which

$\frac{A_p}{V}$ =	*section factor for protected steel member*	[153 m^{-1}]
c_p =	*specific heat of protection material*	[1200 J/kgK]
d_p =	*thickness of fire protection material*	[0,025 mm]
$q_{a.t}, q_{g.t}$ =	*temperatures of steel and furnace gas at time t*	
$Dq_{g.t}$ =	*increase of gas temperature during the time step Dt*	
l_p =	*thermal conductivity of the fire protection material*	[0,174 W/mK]
r_p =	*density of fire protection material*	[430 kg/m^3]
f =		[0,419]

<u>For time interval Δt = 30s, and for a sprayed protection thickness of 25 mm the fire resistance time is by chance exactly 90 min.</u>

Gypsum boarding

This is a boxed protection, so the section factor must be modified:

$$V = 8450.10^{-6}\ m^3\ /\ m$$

$$A_{m,3} = 0{,}180 + 2.0{,}400 = 0{,}980 m^2\ /\ m$$

$$\frac{A_{m,3}}{V} = \frac{0{,}980}{8450.10^{-6}} = 116 m^{-1}$$

The temperature of the steel section is calculated using the same formulas as for sprayed protection. Properties of the fire protection material are as follows:

$c_p =$ [1700 J/kgK]

$d_p =$ [0,02 m]

$l_p =$ [0,200 W/mK]

$r_p =$ [800kg/m^3]

$f =$ [0,7369]

<u>For time interval Δt = 30s and for gypsum boarding with thickness 20 mm the fire resistance time is 90,5 min.</u>

6.1.2 Composite beam comprising partially encased steel section and concrete slab

The composite beam using a partially encased steel section is part of a floor structure, with beams spaced at 6,0 m. The beam considered is simply supported with span 8,0 m, using an IPE 400 steel section infilled between flanges with concrete. The concrete slab has depth 120 mm (Fig. 18) and an assumed effective width of 1,0 m. The beam is checked for R90 standard fire resistance.

Loads

	Characteristic values		*Load factors*	*Design values*
Dead load	3,6 kN/m^2	3,6 . 6,0 = 21,6 kN/m	γ_G = 1,35	29,16 kN/m
Imposed load	4,2 kN/m^2	4,2 . 6,0 = 25,2 kN/m	γ_{Q1} = 1,5	37,80 kN/m
Total	**7,8 kN/m^2**	**46,8 kN/m**		**66,96 kN/m**

Section characteristics:

Steel grade: S235

Concrete grade: C20/25

Steel area: 8450 mm^2

Concrete area: 120 . 10^3 mm^2

Additional reinforcement area: 1256,6 mm^2

For ambient-temperature design the plastic bending moment resistance is estimated as

$M_{u,pl}$ = 625,38 kNm

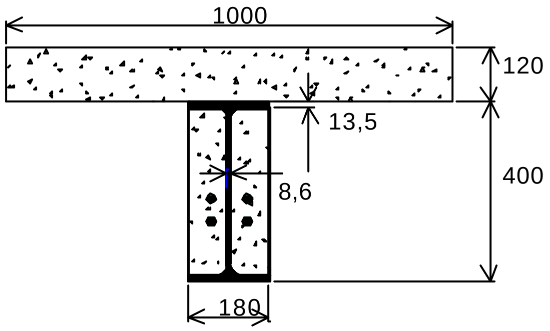


Figure 18 Cross-section of the beam

Simple calculation model

Loading in the fire situation is obtained from:

ENV 1991-2-2

$$\sum g_{GA}.G_k + y_{1,1}.Q_{k,1} + \sum y_{2,i}.Q_{k,i} + \sum A_d$$

so

$$q_{fi} = 1{,}0.21{,}6 + 0{,}7.25{,}2 = 39{,}24 kN/m$$

The bending moment action of the loads in fire is then:

$$M_{fi,Sd} = \frac{1}{8} 39{,}24.8^2 = 313{,}92 kNm$$

This must be less than the resistance in fire:

$$M_{fi,Sd} \le M_{fi,Rd} \rightarrow 313{,}92 kNm \le M_{fi,Rd}$$

The task is then to calculate the bending moment resistance at the fire duration time of 90 min. First the reduced section and the reduced strength of individual elements for R90 must be estimated (Fig. 19).

Partial safety factors:

ENV 1994-1-2
2.3

$g_{M,fi,c} = 1{,}0$; $f_{c,20°C} = 20$ MPa (C 20/25)

$g_{M,fi,r} = 1{,}0$; $f_{ry,20°C} = 325$ MPa (1 335 J)

$g_{M,fi,a} = 1{,}0$; $f_{ay,20°C} = 235$ MPa (S 235)

ENV 1994-1-2
4.3.4, Annex E

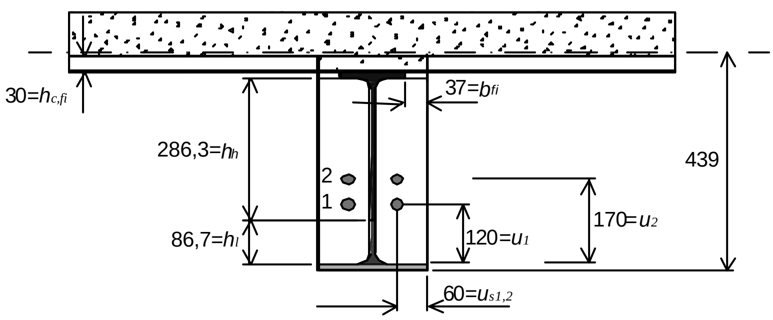


Figure 19 Dimensions of the reduced section

Reduction of the concrete thickness for fire resistance R90

$h_{c,fi} = 30$ mm

$h_{c,h} = 90$ mm

Reduction of the width of the upper flange

Annex E.1
Table E.1

$$b_{fi} = (e_f / 2) + 30 + (b - b_c)/2$$
$$b_{fi} = (13{,}5/2) + 30 + (180 - 180)/2$$
$$b_{fi} = 36{,}75 mm$$

Upper and lower parts of web

$$h_l = a_1 / b_c + a_2 e_w / (b_c h)$$

Table E.2

$$\frac{h}{b_c} = \frac{400}{180} = 2{,}2 > 2 \Rightarrow \quad a_1 = 14000 mm^2$$
$$a_2 = 75000 mm^2$$

$$h_{l,min} = 40mm$$

$$h_l = 14000/180 + 75000 \cdot 8{,}6/(180 \cdot 400)$$
$$h_l = 86{,}74mm$$
$$h_h = 286{,}26mm$$

Table E.3

Strength Reduction Coefficient for the lower flange

$$k_a = [0{,}12 - 17/b_c + h/(38b_c)] \cdot a_0 \qquad a_0 = 0{,}018e_f + 0{,}7$$
$$k_a = [0{,}12 - 17/180 + 400/(38 \cdot 180)] \cdot 0{,}943 \qquad a_0 = 0{,}018 \cdot 13{,}5 + 0{,}7$$
$$0{,}06 < k_a = 0{,}079 < 0{,}12 \qquad a_0 = 0{,}943$$

Strength Reduction Coefficient for the reinforcement

Positions of the reinforcing bars (Fig. 19): $u_1 = 120\ mm$, $u_2 = 170\ mm$, $u_{s1,2} = 60\ mm$

Table E.4

$$k_r = \frac{(ua_3 + a_4)a_5}{\sqrt{\frac{A_m}{V}}} \qquad A_m = 2h + b_c = 2 \cdot 400 + 180$$
$$A_m = 980mm$$
$$V = h \cdot b_c = 72000mm^2$$

$$u = \frac{1}{\frac{1}{u_i} + \frac{1}{u_{si}} + \frac{1}{b_c - e_w - u_{si}}}$$

$$u(1) = \frac{1}{\frac{1}{120} + \frac{1}{60} + \frac{1}{180 - 8{,}6 - 60}} = 29{,}43mm \qquad u(2) = \frac{1}{\frac{1}{170} + \frac{1}{60} + \frac{1}{180 - 8{,}6 - 60}} = 31{,}72mm$$

Where u_i is the distance from the reinforcing bar to the inside of the flange
u_{si} is the distance from the reinforcing bar to the outside of the concrete

Table E.5

$a_3 = 0{,}026$ $\quad a_4 = -0{,}154$ $\quad a_5 = 0{,}090$

The reduction factor for the yield point of the reinforcement is calculated from:

$$k_r(1) = \frac{(29{,}43 \cdot 0{,}026 - 0{,}154) \cdot 0{,}090}{\sqrt{\frac{980}{72000}}} = 0{,}471$$

$$k_r(2) = \frac{(31{,}72 \cdot 0{,}026 - 0{,}154) \cdot 0{,}090}{\sqrt{\frac{980}{72000}}} = 0{,}517$$

Calculation of position of the neutral axis

Initially it is assumed that the neutral axis position is in the concrete slab ($h_{c,h}$). Equilibrium of horizontal forces must apply:

$$\sum F_H = 0: \qquad F_H^+ = F_H^-$$

<u>Negative forces in concrete slab</u>:

$$N_{C,h_{c,h}} = 0{,}85 f_{c,20°C} \cdot b_{eff} \cdot x / g_{M,fi,c} = 0{,}85 \cdot 20 \cdot 1000 \cdot x / 1{,}0$$

<u>Tensile forces in steel section and reinforcement</u>:

Upper flange:

$$N_{f1} = f_{ay,20°C} \cdot (b_f - 2b_{fi}) \cdot e_f / g_{M,fi,a} = 235 \cdot (180 - 2 \cdot 36{,}75) \cdot 13{,}5 / 1{,}0 = 375{,}41kN$$

Upper part of the web:

$$N_{w,h_h} = f_{ay,20} \cdot h_h \cdot e_w / g_{M,fi,a} = 235 \cdot 286{,}26 \cdot 8{,}6 / 1{,}0 = 642{,}81kN$$

Lower part of the web:

The yield strength has a trapezoidal profile, so the tensile force is calculated separately for the constant part and the linear part.

$$N_{w,h_l}^{const.} = k_a \cdot f_{ay,20} \cdot h_l \cdot e_w / g_{M,fi,a} = 0{,}079 \cdot 235 \cdot 86{,}74 \cdot 8{,}6 / 1{,}0 = 15{,}39kN$$

$$N_{w,h_l}^{lin.} = (f_{ay,20} - k_a \cdot f_{ay,20}) \cdot \frac{h_l}{2} \cdot e_w / g_{M,fi,a}$$

$$= (235 - 0{,}079 \cdot 235) \cdot \frac{86{,}74}{2} \cdot 8{,}6 / 0{,}9 = 89{,}70kN$$

Lower flange:

$$N_{f2} = k_a \cdot f_{ay,20} \cdot b_f \cdot e_f / g_{M,fi,a} = 0{,}079 \cdot 235 \cdot 180 \cdot 13{,}5 / 0{,}9 = 50{,}12kN$$

Reinforcement:

Bar 1: $N_{r1} = 2 \cdot A_{s1} \cdot k_{r1} \cdot f_{ry,20} / g_{M,fi,r} = 628{,}5 \cdot 0{,}471 \cdot 325 / 1 = 96{,}21kN$

Bar 2: $N_{r2} = 2 \cdot A_{s1} \cdot k_{r2} \cdot f_{ry,20} / g_{M,fi,r} = 628{,}5 \cdot 0{,}517 \cdot 325 / 1 = 105{,}6kN$

The position x of the neutral axis can be now calculated:

$$\sum F_H = 0 : \qquad F_H^+ = F_H^-$$

$$F_H^+ = N_{f1} + N_{w,h_h} + N_{w,h_l}^{const.} + N_{w,h_l}^{lin.} + N_{f2} + N_{r1} + N_{r2}$$

$$F_H^+ = 375{,}41 + 642{,}81 + 15{,}39 + 89{,}70 + 50{,}12 + 96{,}21 + 105{,}6 = 1375{,}24kN$$

$$F_H^- = 0{,}85 \cdot f_{c,20} \cdot b_{eff} \cdot x = 0{,}85 \cdot 20 \cdot 1000 \cdot x = 17000x$$

$$x = \frac{1375{,}24 \cdot 10^3}{17000} = 80{,}9mm < 90mm$$

$$F_H^- = 1377kN$$

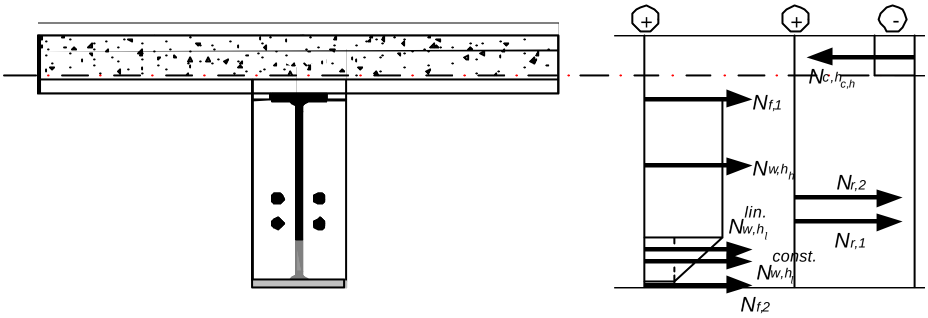


Figure 20 Force components of the bending moment resistance

Bending moment resistance:

The bending moment resistance in the fire situation is calculated by summation of the moments of the individual stress resultants (Fig. 20).

$$M_{u,fi,qcr} = \sum M_y$$

$$\sum M_y^- = 1377.40,5 = 55,768 kNm$$

$$\sum M_y^+ = 375,41.45,75 + 642,81.195,63 + 15,39.382,13 + 89,70.367,70$$
$$+ 50,12.432.25 + 96,21.305,5 + 105,6.255,5 = 259,83 kNm$$

$M_{u,fi,qcr}$ = 315,59kNm > 313,92 kNm

<u>So the beam has a standard fire resistance in excess of 90 min.</u>

6.2 Composite column

6.2.1 Composite column composed of a partially encased HEB600 steel section

EC4 Part 1.2
4.3.4
Annex F

The composite column is a part of a seven-storey building frame structure. The spacing of frames is 6,0 m and the span of the main beams is 8,0 m. The column is an HEB600 section infilled with concrete between its flanges (Fig. 21) and is checked for the standard fire resistance R90.

Steel grade: S235

Concrete grade: C20/25

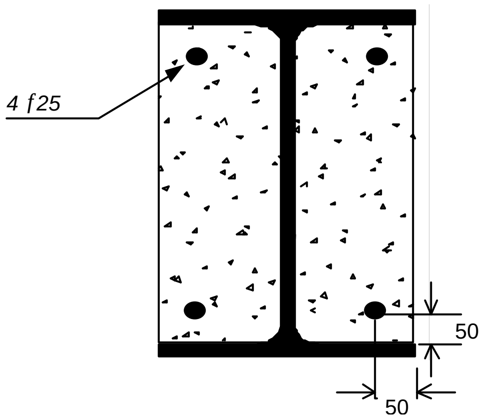


Figure 21 Cross-section of the column

Buckling resistance for ambient-temperature design:

$N_{b,Rd,z}$ = 7973,9 kN

Loads

	Characteristic values		Load fact.	Design values
Dead load	4,65 kN/m^2	4,65. 6,0 .8,0 . 7 = 1562,4 kN	γ_G = 1,35	2109,24 kN
Imp. load	6,10 kN/m^2	6,1 . 6,0 . 8,0 . 7 = 2049,6 kN	γ_{Q1} = 1,5	3074,40 kN
Total	**10,75 kN/m^2**	**3612,0 kN**		**5183,64 kN**

The load in the fire situation (estimated in the same way as previously):

$$\sum g_{GA}.G_k + y_{1,1}.Q_{k,1} + \sum y_{2,j}.Q_{k,j} + \sum A_d$$

so

$$N_{fi,Sd} = 1,0.1562,4 + 0,7.2049,6 = 2997,12 kN$$

The column is to be designed for resistance at time 90minutes, so

$$N_{fi,Sd} \le N_{fi,Rd} \rightarrow 2997,12 kN \le N_{fi,Rd}$$

The buckling resistance at fire duration 90 min is calculated below:

Simple calculation model

Restrictions on the application of Annex F, and the actual values, are as follows: *Annex F*

- *l_q £ 13,5b = 13,5.300 = 4050 mm* | *l_q = 2000 mm* *4.3.6.2*
- *230 mm £ h £ 1100 mm* | *h = 600 mm*

▪ *230 mm £ b £ 500 mm*	*b = 300 mm*
▪ *S 235 £* steel grade *£ S 460*	*S 235*
▪ *C 20/25 £* concrete *£ C 50/60*	*C 20/25*
▪ *1 % £* reinforcement *£ 6 %*	*1,3 %*
▪ R ≤ 120 min	*R = 90 min*
▪ For R90-120: min (h, b) = 300mm	b = 300 mm
▪ For R60: $\left.\begin{matrix} 230 \le b \le 300 \\ h / b > 3 \end{matrix}\right\} \Rightarrow l_\theta \le 10b$	$\begin{cases} b = 300\,mm \\ h / b = 600 / 300 = 2 \end{cases}$
▪ For R90 - 120: *h/b* > 3 $\Rightarrow l_\theta \le 10\,b$	*h/b* = 2 < 3

Annex F

To calculate the buckling resistance in the fire situation the rules of Annex F are used. These are valid only for bending around the weak axis "z". The reduced cross-section and the reduced material properties are now calculated (Fig. 22).

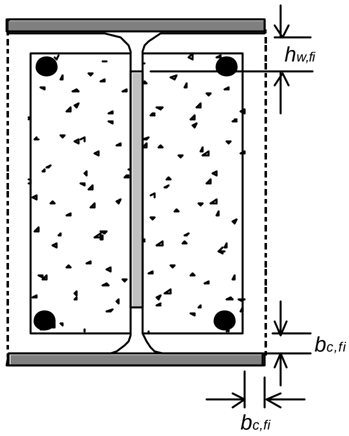


Figure 22 Reduced section

Flanges of the steel profile:

Annex F.2

The average flange temperature is

$$q_{f,t} = q_{0,t} + k_t \left(\frac{A_m}{V} \right)$$

where *t* is the fire duration.

$q_{0,t}$ = 805°C and k_t = 6,15

Table F.1

$$\frac{A_m}{V} = \frac{2(h+b)}{h.b} = \frac{2(0{,}3+0{,}6)}{0{,}3.0{,}6} = 10m^{-1}$$

$$q_{f,t} = 805 + 6{,}15(10) = 866{,}5°C$$

Yield strength and modulus of elasticity for temperature 866,5°C:

$$f_{a,max,f,q} = f_{ay,f,20°C} . k_{max,q} \qquad k_{max,866} = 0{,}0767$$

$$f_{a,max,f,q} = 235.0{,}0767 = 18{,}04\,MPa$$

$$\overline{E_{a,f,t}} = E_{a,f,20°C} . k_{E,q} \qquad k_{E,866} = 0{,}0751$$

$$\overline{E_{a,f,t}} = 210000.0{,}0751 = 15781{,}5\,MPa$$

Design value of the plastic resistance to axial compression in fire:

$$N_{fi,pl.Rd,f} = 2(b.e_f . f_{a,max,f,q}) / g_{M,fi,a}$$

$$N_{fi,pl.Rd,f} = 2(300.30.18{,}04) / 0{,}9 = 360{,}8\,kN$$

Effective flexural stiffness in fire:

$$(EI)_{fi,f,z} = \overline{E_{a,f,t}} (e_f b^3 / 6) = 15781{,}5.30.300^3 / 6 = 2130{,}5\,kNm^2$$

Web of the steel profile:

Neglected part of the web $h_{w,fi}$:

$$h_{w,fi} = 0{,}5\left(h - 2e_f\right)\left(1 - \sqrt{1 - 0{,}16\frac{H_t}{h}}\right);\ R90 \Rightarrow H_t = 1100$$

$$h_{w,fi} = 0{,}5\left(600 - 2.30\right)\left(1 - \sqrt{1 - 0{,}16\frac{1100}{600}}\right) = 43{,}03\,mm = 43\,mm$$

Annex F.3

Maximum stress level:

Table F.2

$$f_{a\,max,w,t} = f_{ay,20,w}\sqrt{1 - 0{,}16\frac{H_t}{h}}$$

$$f_{a\,max,w,t} = 235\sqrt{1 - 0{,}16\frac{1100}{600}} = 197{,}55\,MPa$$

Design value of plastic resistance to axial compression in fire:

$$N_{fi,pl.Rd,w} = e_w(h - 2e_f - 2h_{w,fi}).f_{a,max,w,q}\ /\ g_{M,fi,a}$$

$$N_{fi,pl.Rd,w} = 15{,}5(600 - 60 - 2.43).197{,}55 / 0{,}9 = 1544{,}6\,kN$$

Effective flexural stiffness in fire:

$$(EI)_{fi,w,z} = E_{a,w,20}\left(h - 2e_f - 2h_{w,fi}\right)e_w^3\ /\ 12 = 210000.454.15{,}5^3\ /\ 12 = 20{,}59\,kNm^2$$

Concrete:

Annex F.4

Neglected exterior layer $b_{c,fi}$:

For R90: $b_{c,fi} = 0{,}5\dfrac{A_m}{V} + 22{,}5$

Table F.3

$$b_{c,fi} = 0{,}5.10 + 22{,}5 = 27{,}5\,mm$$

Concrete temperature $q_{c,t}$ (R90):

$q_{c,t}$ = 357°C

Secant modulus of concrete for temperature 357°C:

Table F.4

$$E_{c.sec} = f_{c,q}\ /\ e_{cu,q} = f_{c,20}.k_{c,q}\ /\ e_{cu,q}$$

$k_{c,q} = 0{,}793$

$e_{cu}\ /\ 10^{-3} = 6{,}855$

3.2.2

$$E_{c.sec} = 20.0{,}793 / 6{,}855.10^{-3} = 2313{,}64\,MPa$$

Table 3.3

Design value of plastic resistance to axial compression in fire:

$$\begin{aligned} N_{fi,pl.Rd,c} &= 0{,}86\left\{\left(h - 2.e_f - 2b_{c,fi}\right)\left(b - e_w - 2b_{c,fi}\right) - A_s\right\}.0{,}85.f_{c,20}.k_{c,q}\ /\ g_{M,fi,c} \\ &= 0{,}86\left\{(600 - 2.30 - 2.27{,}5)(300 - 15{,}5 - 2.27{,}5) - 1964\right\}.0{,}85.20.0{,}793 / 1 \\ &= 1267{,}7\,kN \end{aligned}$$

Effective flexural stiffness in fire:

$$(EI)_{fi,c,z} = E_{c,sec,q}\left[\left\{(h-2.e_f-2b_{c,fi})\left((b-2b_{c,fi})^3-e_w^3\right)/12\right\}-I_{s,z}\right]$$

$$I_{s,z} = \frac{A_s}{2}.100^2.2 = 1964.100^2 = 19,64.10^6\ mm$$

$$(EI)_{fi,c,z} = 2313,64\left[\left\{(600-60-2.27,5)\left((300-2.27,5)^3-15,5^3\right)/12\right\}-1964.10^6\right]$$
$$= 4137,7\,kNm^2$$

Reinforcement:

$$\left.\begin{array}{l} u = 50mm \\ R90 \end{array}\right\} k_{y,t} = 0,572$$

Tab. F.5

$$\left.\begin{array}{l} u = 50mm \\ R90 \end{array}\right\} k_{E,t} = 0,406$$

$$u = \sqrt{u_1.u_2} = \sqrt{50.50} = 50$$

Tab. F.6

Design value of plastic resistance to axial compression in fire:

$$N_{fi,pl.Rd,s} = A_s.k_{y,t}.f_{sy,20}/g_{M,fi,s}$$

$$N_{fi,pl.Rd,s} = 1964.0,572.325/1 = 365,11\,kN$$

Effective flexural stiffness in fire:

$$(EI)_{fi,s,z} = k_{E,t}.E_{s,20}.I_{s,z} = 0,406.210000.19,64.10^6 = 1674,5\,kNm^2$$

Design axial buckling resistance at elevated temperature:

The reduced section after 90 minutes' fire duration is shown in Fig. 23.

Design value of plastic resistance to axial compression in fire:

$$\boldsymbol{N_{fi,pl.Rd} = N_{fi,pl.f} + N_{fi,pl.w} + N_{fi,pl.c} + N_{fi,pl.s}}$$

$$N_{fi,pl.Rd} = 360,8 + 1544,6 + 1267,7 + 365,11$$
$$= 3538,2\ kN$$

Tab. F.7

For fire resistance R 90:

$$j_{f,q} = 0,8;\ j_{w,q} = 1,0;\ j_{c,q} = 0,8;\ j_{s,q} = 0,8$$

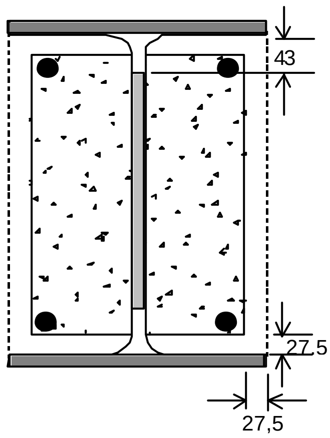


Figure 23 Reduced cross-section

Design value of effective flexural stiffness in fire:

$$(EI)_{fi,eff,z} = j_{f,q}\,(EI)_{fi,f,z} + j_{w,q}\,(EI)_{fi,w,z} + j_{c,q}\,(EI)_{fi,c,z} + j_{s,q}\,(EI)_{fi,s,z}$$

$$(EI)_{fi,eff,z} = 0,8.2130,5 + 1,0.29,58 + 0,8.4137,73 + 0,8.1674,5 = 6388,77\ kNm^2$$

Euler critical buckling load:

$$N_{fi,cr,z} = p^2.(EI)_{fi,eff,z}/(l_q)^2 = p^2.6388,77/(2)^2 = 15751,3\,kN$$

Non-dimensional slenderness:

$$\overline{l_q} = \sqrt{\frac{N_{fi,pl,R}}{N_{fi,cr,z}}} \qquad N_{fi,pl,R} = N_{fi,pl.Rd} \text{ wheng } _{M,fi,j} = 1{,}0$$
$$= 3347{,}7\,kN$$

$$\overline{l_q} = \sqrt{\frac{3347{,}67}{15751{,}3}} = 0{,}461$$

Design axial buckling resistance in fire:

$$N_{fi,Rd,z} = c_z . N_{fi,pl.Rd} \qquad c_z^c = 0{,}865$$

$$N_{fi,Rd,z} = 0{,}865.3538{,}2 = 3060{,}5\,kN$$

Check of the column:

$$N_{fi,Rd,z} \geq N_{fi,Sd}\,.$$ *EC4 Part 1.1*
$$3060{,}5\,kN > 2997{,}12\,kN$$

So the column satisfies the conditions for R90 fire resistance.

6.2.2 Use of tabular data for column

ENV 1994-1-2
4.2.3.3

Conditions for the use of tabular data:

Table 4.6

$$\left.\begin{array}{ll} min(h,b) = 300 & [300] \\ min\ u_s = 50 & [50] \\ min\ \dfrac{e_w}{e_f} = 0{,}7 & \left[\dfrac{15{,}5}{30} = 0{,}52\right] \end{array}\right\}$$ For R 90 it is not possible to use tabular data.

6.2.3 Steel section with total concrete encasement.

Concrete cover is 50 mm (fig. 24)

Concrete with only insulating function: *ENV 1994-1-2*

Table 4.5

c = 50 mm , R_{c2}=180 minutes.

Composite column with totally encased steel section - Tabular data:

If $$\left.\begin{array}{l} min(h_c, b_c) = 400mm \\ c = 50mm \\ u_s = 30mm \end{array}\right\}$$ R'_{b3}=120 minutes.

4.2.3.2
Table 4.4

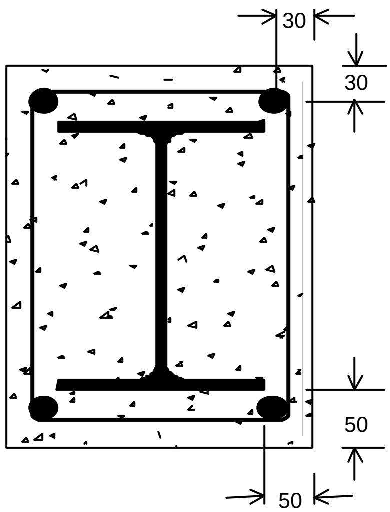


Figure 24 Totally encased column

7 Concluding summary

- Traditional fire protection of steelwork is usually achieved by covering it with an insulating material during construction. However it may be possible under EC4 to use a combination of strategies to ensure fire resistance.
- EC4 calculation of fire resistance takes account of the loading level on the element. However the safety factors applied are lower than in those used in strength design.
- EC4 provides simple calculations for the load resistance in fire of common types of elements. In case of composite beams lateral-torsional buckling is neglected, and for columns the buckling fire resistance can be estimated according to code rules only for the case of braced frames.
- Fire resistance of composite beams comprising steel beam and concrete or composite slab may be calculated in terms of time, as a load-bearing resistance at a certain time, or as a critical element temperature appropriate to the load level and required time of exposure. Other members (composite slabs, composite beams comprising steel beams with partial concrete encasement, composite columns with partially encased steel sections and concrete-filled hollow sections) are examined in terms of the required fire resistance time.
- EC4 provides tabular design data for some structural types which are not easily addressed by simplified calculation methods.
- To assure the composite action during the fire exposure and the transmission of the applied forces and moments in the beam to column connections some constructional requirements must be fulfilled.